A SIMPLE WAY TO MEASURE THE DYNAMIC VISCOSITY OF A FLUID

M. El Malki\textsuperscript{a,*}, A. Bria\textsuperscript{b}, S. Amraqui\textsuperscript{b}, D. Bria\textsuperscript{a}

\textsuperscript{a}Laboratory of Materials, Waves, Energy and Environment, Department of Physics, Faculty of Sciences, Mohammed First University, Oujda 60000, Morocco.

\textsuperscript{b}Laboratory of Mechanics & Energetics, Department of Physics, Faculty of Sciences, Mohammed First University, Oujda 60000, Morocco.

Abstract

In this paper, we measured the viscosity of different fluids, including edible oils and soaps, at a constant temperature, using two experimental methods. Namely, the falling ball and the oscillation of the mass-spring system inside a fluid. Three different masses were used to evaluate the viscosity obtained by the falling ball method. The results were very similar. More precision of the viscosity value is done using a calibration curve. The results are compared with those measured by the mass-spring system and they show good agreement. This simple way of viscosity measurement using mechanical physics concepts can be used for educational purposes such as the practical work of a bachelor.

Keywords: Viscosity; Experimental Methods; Calibration Curve; Practical Work.

*Corresponding author.
E-mail address: m.elmalki@ump.ac.ma

1. Introduction

Viscosity is an important physical property that allows the characterization of the friction inside a fluid produced from the molecular attraction which opposes its flow [1]. This property is generally used for the quality control of products such as oils, cosmetics products, and paints [2]. Different methods have been proposed to measure viscosity. Rao has measured using the falling ball viscometer the viscosity of different fluids [3]. Namely, petroleum products, pharmaceutical beverages, silicate glass, and food products. The experiment measures the speed of the fall of a ball that rolls and slides in a cylindrical tube filled with the fluid and then extracts its viscosity. Kim et al. have developed a double capillary tube scanning viscometer (SDCV) which is used to measure the viscosity of Newtonian and non-Newtonian fluids [4], such as blood. This type of viscometer is based on the evaluation of the emptying time, i.e., the time it takes for a certain quantity of liquid to flow through the capillary. Heuser G et al. have proposed an axial flow Couette viscometer to expose blood as a studied fluid to fixed shear stresses of short duration [5], the results lead to the possibility to
control the flow conditions to improve the quality of the fluid. Münstedt et al. used a universal extensional rheometer to study the elongation properties of molten polymers [6], the proposed system is based on the study of the deformation and flow of the material under external stress by applying shear to the material. The working modes of the system are tested on a polystyrene material, the viscosity and the recoverable strain in the stable elongation state were measured over three decades of tensile stress. Izumo et al. [7] have developed a viscometer called "Sine Wave Vibro Viscometer" based on the study of the amortized oscillatory motion of a mass attached to a spring, this type of sinusoidal wave Vibro-viscosimeter is characterized by a limited amount of material to be measured and a large range of measurements without replacing the sensor. The viscosity of various materials has been measured with this Vibro-viscometer such as the cloud point of a surfactant, the concentrations of ethanol solutions, the polymerization process of a silicone adhesive, and the coagulation process. Other experimental methods are proposed in [8-20].

In this paper, we present simple methods to measure dynamic viscosity for educational purposes. Namely, the falling ball in a liquid. The results are compared with those obtained by the vibration of a mass-spring system in a liquid to evaluate the best method for more realistic values.

2. Theoretical study

Viscosity is very important for a whole range of products; without adequate viscosity, we risk wasting products. The study of viscosity allows distinguishing between fluids, and it has several uses in the productive field, the commercial companies use viscosity to have products with precise properties reach their commercial objectives. Keeping materials at the right viscosity requires making the right calculations and managing storage conditions, such as temperature. Viscosity is a property used to measure the flow of a fluid. Newtonian mechanics gives several methods to calculate this viscosity which can give exact values, but because of technology.

In this section, we briefly present the theory of the falling ball and the vibration of the mass-spring system in a liquid to calculate its viscosity.

1.1 Falling ball in a liquid

The falling Ball in a liquid is a method of measuring the viscosity of a fluid according to the stock law [1]. The method is based on the measurement of the constant displacement velocity of a small ball in a liquid studied at a constant temperature where the viscosity of fluids is proportional to the flow time.

The ball with mass \( m_{\text{ball}} \) and radius \( r_{\text{ball}} \) is dropped without initial velocity into a vertical tube. The ball then moves under the action of gravity. Hence, the ball is submitted to its weight \( P \), the Archimedes force \( F_A \) and the braking force \( F_b \) exerted by the fluid on the ball.

If we consider a vertical axis oriented downward, these forces are written as:

- the weight of the ball \( P \):

\[
P = \rho_{\text{ball}} \times \left( \frac{4}{3} \right) \pi r_{\text{ball}}^3 \times g
\]  

(1)
• the Archimedes force $F_A$:

$$F_A = -\rho_{\text{fluid}} \times \left(\frac{4}{3}\right) \pi r_{\text{ball}}^3 \times g \tag{2}$$

• the braking force $F_b$:

$$F_b = -6\pi \times r_{\text{ball}} \times \mu \times \nu \tag{3}$$

where $m_{\text{ball}}, \rho_{\text{ball}},$ and $V_{\text{ball}}$ are the mass, density, and volume of the ball, respectively. While, $\rho_{\text{fluid}}$ is the density of the fluid, $\mu$ is the dynamic viscosity of the fluid, $g$ is the gravity acceleration, and $\nu$ is the limiting velocity for the ball to fall through the fluid.

When the ball, reaches its maximum speed of fall, its movement is then rectilinear and uniform. The principle of dynamics allows us to determine the expression of the dynamic viscosity of the fluid given by:

$$\mu = \frac{\left(\rho_{\text{ball}} - \rho_{\text{fluid}}\right) \times \left(\frac{4}{3}\right) \pi r_{\text{ball}}^3 \times g}{6\pi \times r_{\text{ball}} \times \nu} \tag{4}$$

2.2. Mass-spring vibration

The mass-spring vibration system is used to determine the value of the viscosity coefficient $\mu$ of a liquid by studying the damped oscillatory motion of a mass hanging to the spring. When a mass is suspended from a spring, the latter deforms and exerts a force on the object responsible for its deformation, this force is called spring tension.

The cylindrically shaped mass of radius $r$, height $h$, and density $\rho$ is subject to its weight $P$, the Archimedes force $F_A$, and the spring return force $F$. If we consider a vertical axis oriented downward these forces are written as:

• the weight of the ball $P$:

$$P = \rho \times \pi \times r^2 \times h \times g \tag{5}$$

• the Archimedes force $F_A$:

$$F_A = -\rho_{\text{fluid}} \times \pi \times r^2 \times h \times g \tag{6}$$

• the return force of a spring $F$:

$$F = -k \times X \tag{7}$$

where $X$ and $k$ are respectively the elongation and stiffness of the spring.

When the mass emerged in the fluid, it is submitted to a frictional force given by Stockes law:

$$f_f = -6\pi \times r \times \mu \times \nu \tag{8}$$

Then according to the fundamental principle of dynamics, we obtain the following viscosity equation:

$$\mu = \frac{\left(\rho - \rho_{\text{fluid}}\right) \times \pi \times r^2 \times h \times g - kX}{6\pi \times r \times \nu} \tag{9}$$
3. Experimental Study

Here, we experimentally verify the viscosity of different liquids by the falling ball in a liquid and the mass-spring vibration. We have already presented in section 2 the analytical calculation that makes it possible to determine the viscosity of a liquid. In this section, the measurement is carried out for five different fluids in which two types of edible oils and two types of liquid soap have been studied to see the difference between them in terms of viscosity. Temperature affects the measurement of viscosity. However, its effect is not taken into account in this study. The viscosity of liquids decreases a lot when the temperature increases. An example of the variation of viscosity as a function of temperature is shown in table 1. For the example of water, we can see that as the temperature increases the viscosity decreases.

Table 1. Variation of viscosity as a function of temperature.

<table>
<thead>
<tr>
<th>Fluid</th>
<th>( \mu ) (Pa.s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>water at 0 °C</td>
<td>1,787 ( 10^{-3} )</td>
</tr>
<tr>
<td>water at 20 °C</td>
<td>1,002 ( 10^{-3} )</td>
</tr>
<tr>
<td>water at 100 °C</td>
<td>0,2818 ( 10^{-3} )</td>
</tr>
<tr>
<td>H₂ at 20 °C</td>
<td>0,860 ( 10^{-5} )</td>
</tr>
<tr>
<td>O₂ at 20 °C</td>
<td>1,95 ( x 10^{-5} )</td>
</tr>
</tbody>
</table>

The use of the two viscosity measurement methods allows us to compare the results obtained in order to validate our results. To put into practice the experience of the viscometer based on the falling ball and the mass-spring system, we used the equipment presented in Fig 1.

![Figure 1. The equipment used during the experiments.](image)

3.1. Experiment 1: falling ball viscometer

The purpose of this experiment is to measure the viscosity of different liquids in order to understand the principle of the theory of the fall of a ball in a liquid. The experiment requires the use of the following equipment:

- Test tube filled with the studied fluid.
3.1.1 Experiment protocol

We established an experimental protocol that allowed us to obtain all the data necessary to measure the viscosity of different liquids. In particular, water, two edible oils, and two liquid soaps. To determine the dynamic viscosity of a fluid, the ball is released, without initial velocity in the fluid contained in the cylindrical tube (Fig 2). Using a timer, the time of ball fall $\Delta t$ between the two marks A and B is measured, which allows us to calculate the speed of the ball and determine the dynamic viscosity. To specify the value of $\Delta t$ we also used a video recording. The experiments were carried out around the months of May/June 2022 at a temperature close to 20-25°C and 1 bar pressure.

![Figure 2. The equipment used during the falling ball experience of a fluid.](image)

3.1.2 Results and discussion

The viscosity measurement obtained was carried out for the five liquids in which three different masses were released. Microsoft Excel software is used for data processing and sigmaPlot to plot the curves. Table 2 groups the entire measurement procedure used to calculate the viscosity for a falling small ball with a mass of 7g and a radius of 0.045cm.

<table>
<thead>
<tr>
<th>Fluid</th>
<th>Time (s)</th>
<th>Velocity (m/s)</th>
<th>Viscosity (Pa. s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water</td>
<td>0.021</td>
<td>8.095238095</td>
<td>0.994×10⁻³</td>
</tr>
<tr>
<td>Edible oil type 1</td>
<td>0.19</td>
<td>0.894736842</td>
<td>9.00×10⁻¹</td>
</tr>
<tr>
<td>Edible oil type 2</td>
<td>0.176</td>
<td>0.965909091</td>
<td>8.34×10⁻¹</td>
</tr>
<tr>
<td>Soap type 1</td>
<td>0.32</td>
<td>0.53125</td>
<td>1.51</td>
</tr>
</tbody>
</table>
The same procedure is repeated for two other balls of medium and large mass. Measuring Δt allows us to calculate the viscosity of each liquid. Tables 3 and 4 bring together all the results obtained. It is known that the viscosity of water equals 1×10⁻³Pa.s. At the time of practice, we noticed that the difference in time (Δt) of the small ball is 0.021s which is greater than that of the large ball 0.00014s thanks to the video recording. The experience shows also that the viscosity equals 0.994×10⁻³Pa.s, 0.995×10⁻³Pa.s, and 1.08×10⁻³Pa.s for the different balls respectively. At constant temperature, the viscosity is constant regardless of the mass used. The small difference between the values measured by the three masses is due to errors that can be made during measuring time. On average, the experimental viscosity of water is equal to 1.023×10⁻³Pa.s which is very close to the exact value.

<table>
<thead>
<tr>
<th>Fluid</th>
<th>Time (s)</th>
<th>Velocity (m/s)</th>
<th>Viscosity (Pa. s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water</td>
<td>0.017</td>
<td>10</td>
<td>0.995×10⁻³</td>
</tr>
<tr>
<td>Edible oil type 1</td>
<td>0.15</td>
<td>1.133333333</td>
<td>8.79×10⁻¹</td>
</tr>
<tr>
<td>Edible oil type 2</td>
<td>0.13</td>
<td>1.307692308</td>
<td>7.61×10⁻¹</td>
</tr>
<tr>
<td>Soap type 1</td>
<td>0.2</td>
<td>0.85</td>
<td>1.17</td>
</tr>
<tr>
<td>Soap type 2</td>
<td>1.96</td>
<td>0.086734694</td>
<td>1.15×10⁻¹</td>
</tr>
</tbody>
</table>

Table 2. Viscosity measurement by a medium ball of 10.3 g (Ball 2).

<table>
<thead>
<tr>
<th>Fluid</th>
<th>Time (s)</th>
<th>Velocity (m/s)</th>
<th>Viscosity (Pa. s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water</td>
<td>0.00014</td>
<td>1214.285714</td>
<td>1.08×10⁻³</td>
</tr>
<tr>
<td>Edible oil type 1</td>
<td>0.11</td>
<td>1.545454545</td>
<td>8.49×10⁻¹</td>
</tr>
<tr>
<td>Edible oil type 2</td>
<td>0.1</td>
<td>1.0625</td>
<td>7.72×10⁻¹</td>
</tr>
<tr>
<td>Soap type 1</td>
<td>0.16</td>
<td>1.0625</td>
<td>1.23</td>
</tr>
<tr>
<td>Soap type 2</td>
<td>1.58</td>
<td>0.107594937</td>
<td>1.22×10⁻¹</td>
</tr>
</tbody>
</table>

Table 2. Viscosity measurement by a large ball 16,5g (Ball 3).

Similarly, for edible oil type 1, the values obtained experimentally are very close. In particular, 0.90Pa.s, 0.79Pa.s and 0.84Pa.s for the three balls respectively. On average, we can say that the type 1 viscosity is equal to 0.84Pa.s. While the viscosity of type 1 edible oil is 0.78Pa.s on average. A comparison between the two types of table oil shows that type 1 is a more viscous fluid than type 2. we can also notice that the table oil is more viscous than water.

For soap type 1, its viscosity is around the following value 1.51Pa.s, 1.17Pa.s, and 1.23Pa.s for the three balls which is 1.20Pa.s on average. While soap type 2 is the most viscous fluid among our liquids. Its viscosity is equal to 12Pa.s on average. It is concluded that the viscosity value remains constant and does not depend on the ball used.

To further increase the precision, we performed the following procedure:
According to equation (4) we have
\[
\mu = A \times \frac{1}{p}
\]
where
\[
A = \frac{2}{9} \left( \rho_{\text{ball}} - \rho_{\text{fluid}} \right)
\]
and
\[
\mu = \frac{v}{r^2}
\]

We can draw a calibration curve that represents the variation of the \(v/r^2\) ratio for all types of liquids. The calculation of the slope of the curve \(\Delta v/\Delta r^2\) gives us a result that allows the precision of the experimental value of the viscosity of the liquids. The slope is calculated using two points included in the calibration curve.

Figure 3 represents the variation in the speed of the falls of the three balls as a function of the radius \(r^2\) of the balls for the two types of oils and soaps.

![Graphs showing the variation of speed of falling balls](image)

**Figure 3.** Variation of the speed of the falling three balls as a function of the square radius \((r^2)\) of three different balls. (a) case of Edible oil type 1. (b) case of edible oil type 2. (c) case of soaps type 1. (d) case of soap type 2.

In the case of Edible oil type 1, the calibration curves can show the errors made during the measurements since points are not linear. The curve passes through the maximum of the points in such a way as to approximate the value of the viscosity in Eq. 10. We performed the calculation for the four liquids. The following table gathers all the results obtained.
Table 5. Measurement of viscosity by the calibration curve.

<table>
<thead>
<tr>
<th>Fluid</th>
<th>$v_1$</th>
<th>$v_2$</th>
<th>$r_1^2$</th>
<th>$r_2^2$</th>
<th>Slope</th>
<th>Viscosity (Pa. s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Edible oil type 1</td>
<td>0.89</td>
<td>1.89</td>
<td>0.00002</td>
<td>0.00000423</td>
<td>44843.04933</td>
<td>0.863700949</td>
</tr>
<tr>
<td>Edible oil type 2</td>
<td>1</td>
<td>2.1</td>
<td>0.00002</td>
<td>0.00000423</td>
<td>49327.35426</td>
<td>0.785732297</td>
</tr>
<tr>
<td>Soap type 1</td>
<td>0.53</td>
<td>1.12</td>
<td>0.00002</td>
<td>0.00000423</td>
<td>26457.3991</td>
<td>1.456206203</td>
</tr>
<tr>
<td>Soap type 2</td>
<td>0.065</td>
<td>0.139</td>
<td>0.00002</td>
<td>0.00000423</td>
<td>3318.38565</td>
<td>11.63292753</td>
</tr>
</tbody>
</table>

The viscosity values obtained for the four liquids are more precise compared with the results obtained before. We note that measurements of the viscosity of viscous liquids such as the two types of soaps are made with fewer errors, which can give values more precise.

3.2 Experiment 2: mass-spring system in a liquid

The purpose of this experiment is to determine the viscosity of different liquids using a mass-spring system. To practice the mass-spring system experiment, the following equipment was used:

- A spring;
- Mass of cylindrical shape;
- A timer;
- A graduated ruler;
- Beaker filled with studied fluid.

3.2.1 Experiment protocol

To measure the viscosity of the same liquids used in experiment 1, we followed these steps:

- The mass is pulled, without initial speed in the fluid, contained in the cylindrical tube, without friction with its wall in such a way as to cause vibrations of the mass used.
- Using a timer, we measure the oscillation time which allows us to calculate $\Delta t$ in order to determine the viscosity.

The experiments were carried out around 20-25°C and pressure of 1 bar.
3.2.2 Results and discussion

In this experiment, we had a mass attached to a spring. Before releasing the masse inside the liquids, we noted the time of 10 oscillator periods of a released spring without a mass. Then, the mass is released in the water and the two types of edible oils to measure the time of 5 oscillator periods. The results obtained in Table (6) show that the viscosity of water equals 0.97×10^{-3}Pa.s knowing that the exact value is normally 1.005×10^{-3}Pa.s. The measured velocity of type 1 and type 2 of edible oils are respectively 0.96Pa.s and 0.83Pa.s. These values are very close to values obtained using the falling ball method which are respectively 0.86Pa.s and 0.78Pa.s for the same liquids. It should be noted that the mass-spring method is valid for less viscous liquids. Measuring the viscosity of liquids such as soap is difficult because the mass used is lighter so the vibrations of the mass are very low to be measured experimentally. Hence, more measurement errors can be done during the experiment.

Table 6. Viscosity measurement by a mass-spring method.

<table>
<thead>
<tr>
<th>Number periods</th>
<th>Fluid</th>
<th>Stiffness k (N/M)</th>
<th>Ball masse (m)</th>
<th>Time (s)</th>
<th>Velocity (m/s)</th>
<th>Viscosity (Pa.s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>Water</td>
<td>7.135803077</td>
<td>0.05</td>
<td>0.02</td>
<td>3.095238</td>
<td>0.969264768×10^{-3}</td>
</tr>
<tr>
<td>5</td>
<td>Edible oil type 1</td>
<td>7.18076865</td>
<td>0.05</td>
<td>0.01</td>
<td>3.611111</td>
<td>0.836033568</td>
</tr>
<tr>
<td>5</td>
<td>Edible oil type 2</td>
<td>7.187558006</td>
<td>0.05</td>
<td>0.01</td>
<td>3.823529</td>
<td>0.790333807</td>
</tr>
</tbody>
</table>

As a result, the following table regroups the measurement of the viscosity obtained by the falling ball (Method 1) and the mass-spring system (Method 2) in a liquid.

Table 7. Comparison of the results of the viscosity measurements obtained by the two methods.

<table>
<thead>
<tr>
<th>Fluid</th>
<th>Viscosity obtained by Method 1 (Pa.s)</th>
<th>Viscosity obtained by Method 2 (Pa.s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water</td>
<td>1.023×10^{-3}</td>
<td>0.969264768×10^{-3}</td>
</tr>
<tr>
<td>Edible oil type 1</td>
<td>0.863700949</td>
<td>0.836033568</td>
</tr>
<tr>
<td>Edible oil type 2</td>
<td>0.785732297</td>
<td>0.790333807</td>
</tr>
</tbody>
</table>

4. Conclusion

In this work, we simply measured the viscosity of different fluids including edible oils and soaps, at a constant temperature, by two simple experimental methods based on mechanical physics concepts. These are the falling ball and the oscillation of the mass-spring system inside a fluid. The evaluation of the viscosity obtained by the falling ball is made by using three different masses to compare the results obtained. A second precision of the viscosity value is used to approximate the values of each
mass by the use of a calibration curve. The results obtained were in good agreement with those measured by the mass-spring system.

The oscillation of the mass-spring system inside fluids has a limit, the vibrations of the mass are very weak to detect experimentally for more viscous liquids. It is therefore necessary to increase the mass used, which also necessitates an increase in the mass of the liquid. However, this operation cannot be practically considered inside the laboratory. Generally, the falling ball method is simpler than the mass-spring system and it gives more accurate values.

Currently, the measurement of viscosity is done in automatic, direct, and simple ways using electrical equipment (probes) and motion sensors, the latter methods have higher efficiencies. From perspective of this contribution, the measurement of viscosity by the fall of a ball and the mass-spring system can be used in practical work for educational works.

References


