Maghrebian Journal of Pure and Applied Science
ISSN : 2458-715X
http://revues.imist.ma/?journal=mjpas&page=index
https://doi.org/10.48383/IMIST.PRSM/mjpas-v7i1.26090

The Modeling of the Alternative Mechanical Engines, the mathematical brake work and the mechanical definition of the availability

Akram Louiz[1*]

[1] The Higher Institute for Maritime Studies, Casablanca, Morocco.
[*]Corresponding Author: louiz.akram@gmail.com
Received 28 February 2021, Revised 28 April 2021, Accepted 05 June 2021

Abstract Engines are nowadays mandatory for the domestic as for the industrial use. However, they need enough control and maintenance in order to make their usefulness last longer. Some works have been done for this purpose but they all propose mechanisms and methods that consume a big amount of useful energy from the engine and thus stops it from being operational during the control, and thus, these studies can't allow testing continuously the operation of the used engine. Other works propose thermodynamic experiments that are too expensive and difficult to be done in order to provide the characteristics of engines. However, the equations of real gases can only be approximate and the studied gases can't be considered ideal gases at the high pressures inside engines. This paper proposes new accurate formulas and methods to detect cylinder problems by allowing continuous control of each cylinder brake work without consuming any energy from the operating engine. This work proposes also a new mechanism to measure the availability of each cylinder of the studied engine and thus introduces a new method to test the second law efficiency for engine. This work explains also and proves all the formulas and methods after modeling the connecting rod and crankshaft operation. Consequently, this is a very useful work for the simulations of engine experts as for the simple understanding of beginners.

Keywords. The internal combustion engine, connecting rod, crank, crankshaft, the brake work, the availability, efficiency, energetic study.

1. Introduction

Necessity is the mother of invention. The industrial revolution caused competitiveness so strong that it imposed the use of cheap means of production and transportation in all industries.

Therefore, engines started to appear in the 18th century society by the Watt steam engine that allowed efficient semi-automated factories in places where the power of water was not available.

During the beginning of the 19th century, the internal combustion piston engines started to be tested, as advanced theoretically by Carnot, then the efficiency of the Carnot cycle started to surpass the Watt steam engine efficiency and thus started to spread around the world for many transportation applications. The more efficient Diesel engines appeared afterward to be used especially for the heavy

transportation means then they were improved to the widely used turbo Diesel engines.

Many other kinds of engines appeared with different technologies namely the engines with horizontally opposed pistons and the rotary engines, but they haven't been used widely.

This work starts with a modeling of the crank and connecting rod mechanism in order to allow a simulation of the operation that is also present in displacement pumps like piston pumps, and to find useful formulas that allow the energetic study of the internal combustion piston engines.

The readers are invited to discover the part of this work about the related articles where many energetic studies are mentioned in order to understand the usefulness of this work results.

This paper proposes a better method to measure different kinds of engine energies in order to make a more accurate and reliable control of each cylinder of the operating engine for failure prevention.

2. Background and literature review:

2.1. The internal combustion engine:

The internal combustion engine (ICE) is a heat engine that transforms the chemical energy of a fuel into a useful mechanical energy thanks to the combustion process.

The internal combustion piston engines use the crank and connecting rod mechanism to transform the heat energy into a useful mechanical energy.

The combustion starts at the combustion chamber while the piston is at the top dead center (TDC), then the high pressure of combustion pushes the piston until the bottom dead center (BDC).

The crank transforms the straight movement of the piston into a useful rotation. However, this transformation is not complete since there is a lost energy that the reader can remark after reading the part of this work about the half pinion crankshaft.

The fuel is injected through an injection system in the combustion chamber at the top inside the cylinder which facilitates a mixture between the fuel and the oxidizer (usually air). Then, the combustion happens thanks to a spark in the petrol engines, or thanks to the high pressure like in the Diesel engines. The crankshaft is related to the camshaft and to other accessories that allow the fuel supply, the lubrication, the cooling and the oxidizer renewal for each cylinder.

This paper concerns mainly two kinds of internal combustion engines which are the two-stroke engines and the four-stroke engines.

During one crankshaft revolution, a two-stroke engine completes a power cycle with two strokes through up and down movements of the piston, whereas the four-stroke engine completes a power cycle with four strokes of the piston. These two additional strokes are mandatory to renew the oxidizer inside the cylinder otherwise the engine will need the turbocharging in order to make the scavenging (the oxidizer renewal) operation naturally inside the cylinder of the two-stroke engine.

The turbocharging uses the exhaust gases to compress the air (oxidizer) at a pressure higher to the atmospheric pressure which allows the natural scavenging [1].

2.2. Related articles

Many articles make the modeling of the internal combustion engines operation for their energetic study, but most of those papers give only approximate formulas and methods that don't allow an accurate control of the engine.

For instance, in "International Review of Mechanical Engineering (IREME)" [2] the authors proposed formulas made through an algorithm in order to find an approximate work given or consumed by each engine stroke, but the reader should remark that the compression and scavenging strokes consume different energies since the air reaches a high pressure during the compression. For instance, the air pressure is 2 Mpa at the end of compression of a petrol engine, and it reaches 3,5 Mpa for a diesel engine. Consequently, it is impossible that both strokes have the same yield equaling $\frac{1}{2}$. Furthermore, the other proposed yields don't take into account the energy consumed by the camshaft and the friction.

In "Internal Combustion Engine (ICE) Fundamentals" [3], the work is calculated only for the half

of the combustion stroke even if the piston continues its movement until the bottom dead center (BDC) by the gases pressure and by inertia. However, the reader should notice that this work defines the stroke as the double of the crank radius. Furthermore, the final proposed formulas of the piston velocity and acceleration are only approximate whereas the new technologies allow the process of long and complicated formulas for a higher precision of engine control.

Grimaldi, C.N. and Millo, F, in their article [3], give however many useful definitions. For instance, P_b the brake power (per engine cycle) is defined as the power that can be measured by using a dynamometer or a brake on the crankshaft which consumes an energy from the engine during the measure and may stop it from being useful. The brake work W_b can be deduced then by using this formula: $P_b = \frac{n \times W_b \times \dot{\varphi}}{R}$ where n is the number of cylinders, R is the crankshaft revolutions per engine cycle and $\dot{\varphi}$ is the crankshaft angular velocity.

The indicated work W_i is also defined as the work by the gas on the piston head, it can be also measured experimentally by using an experimental graph of the cylinder pressure p in order to integrate in this formula: $W_i = \int p\,.\,dV$ where V is the volume between the piston head and the top inside the cylinder. The brake power can be deduced then by using this formula: $P_i = \frac{n \times W_i \times \dot{\varphi}}{R}$. Whereas the given mechanical efficiency formula is: $\eta_m = \frac{W_b}{W_i}$.

This related article makes a definition to W_f the friction work as the work necessary to drive all the engine accessories by overcoming the mechanical friction of all the engine mechanism (the mechanical losses) and its formula is: $W_f = W_i - W_f$.

The friction work can be also found directly by summing the works consumed by the camshaft, and by the other engine accessories like explained by Petrescu, F. I., & Petrescu, R. V in these articles: “Cam gears dynamics in the classic distribution” [4], “Forces and efficiency of cams” [5], “Determining the dynamic efficiency of cams” [6] and in this article about the energies involved in the injection systems “Modeling of the Automatic Mechanical Injection System” [7], without forgetting the compression stroke and friction consumed energies. However, the technologies used in the camshafts and injectors are evolving and now different from an engine to another which makes this method difficult and not general like explained in “Cams with high efficiency” [8].

“The application of availability and energy balances to a diesel engine” [9] is another useful article that defines the availability as the issue of maximum extractable work output from the engine during a thermodynamic state. The first law efficiency of the engine is also defined as the ratio of brake power to input fuel power, whereas the second law efficiency is the ratio of the brake power to the corresponding maximum extractable power.

However, the articles: “The application of availability and energy balances to a diesel engine” by Alkidas, A. C, and also “Operating characteristics of a spark-ignition engine using the second law of thermodynamics: effects of speed and load” and “The thermodynamic characteristics of high efficiency, internal-combustion engines” by Caton, J. A [9,10,11] make a thermodynamic energetic study to the engine while there are thermo-fluid-dynamic losses defined as the losses by incomplete combustion, heat transfer to the walls, leakages and gas exchange [3]. It is therefore a difficult and expensive study especially that the combustion gases can’t be considered ideal gases since they can reach 10 Mpa at the combustion.

This work can be very useful in order to understand accurately the relation between the performance and the thermodynamic chemical emissions and in order to elaborate a better control of the engine’s pollution for different fuels and situations like required by the environment new regulations [12,13,14].

The engineer can therefore use this work as a starting ground in order to understand the full engine operation before starting the experimental work about the thermal study [15,16,17] and about the chemical emissions [18,19,20].

3. The Proposed approach:

3.1. The geometric study:

In the considered crank and connecting rod system, l is the rod length and r is the crank radius.

x_A is the height upon the piston inside the cylinder that changes by the movement of the point A. V_A is not the velocity of A but is the velocity of variation of that height that we get by deriving x_A, and γ_A is the acceleration that we get by deriving V_A. They are both always parallel to an axis $\vec{\iota}$.

The axis $\vec{\iota}$ is always in the same movement orientation of the point A, hence the velocity of A should be always positive but not necessarily V_A.

Θ *is* the connecting rod angle of rotation that varies in the interval: $[-\Theta_{max}, \Theta_{max}]$

And φ is the crankshaft angle of rotation that varies in the interval: $[\frac{-\pi}{2}, \frac{\pi}{2}]$

φ can be detected by a rotation captor on the crankshaft, whereas Θ can be detected by a rotation captor at the big end or at the small end of each connecting rod.

The change of the orientation of V_A obliges to divide the operation of the hole system into two steps in order to study the sign of γ_A correctly. Also, the change of the rotation Θ orientation obliges to divide each one of the two steps into two other steps. Finally, the studied system gets divided into four steps.

$|\varphi|$and $|\theta|$are used as absolute values in the calculations when triangles angles are used in the formula proof.

$(\vec{\iota},\vec{\jmath},\vec{k})$ represents the used direct orthonormal basis oriented in each one of the four steps in a manner allowing always that:

- Θ increases and thus always $\dot{\theta} > 0$.
- The axis $\vec{\iota}$ is in the same movement orientation of the point A that is always the center of the used direct orthonormal basis, hence the velocity of A should be always positive.

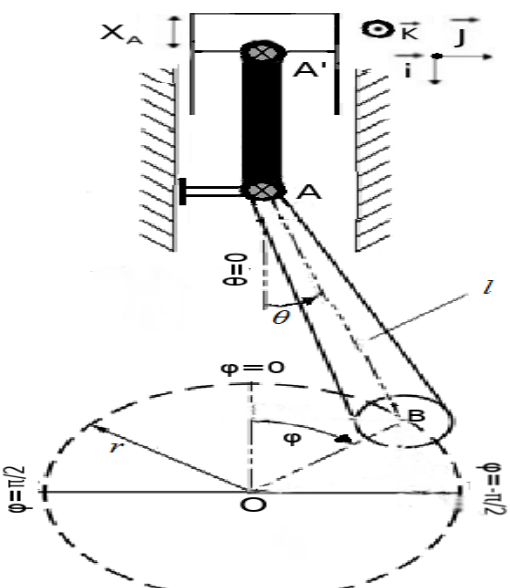

Figure 1. The first geometric step of an engine crank operating with its connecting rod.

In the step 1 where $\frac{-\pi}{2} < \varphi < 0$ and $0 < \theta < \theta_{max}$:

φ is in the opposite clockwise orientation and thus it decreases, consequently: $\dot{\varphi} < 0$.

However, $|\varphi|$ increases and thus:

$$\frac{dsin(|\varphi|)}{dt} = -\dot{\varphi} \times cos(|\varphi|) \quad (1)$$

and:

$$\frac{dcos(|\varphi|)}{dt} = \dot{\varphi} \times sin(|\varphi|) \quad (2)$$

Θ is in the direct counterclockwise orientation and thus it increases, consequently: $\dot{\theta} > 0$. However, $|\theta|$ also increases.
Consequently:

$$\frac{dsin(|\theta|)}{dt} = \dot{\theta} \times cos(|\theta|) \quad (3)$$

and:

$$\frac{dcos(|\theta|)}{dt} = -\dot{\theta} \times sin(|\theta|) \quad (4)$$

Also:

$$r \times sin(|\varphi|) = l \times sin(|\theta|) \quad (5)$$

Hence:

$$sin(\theta) = \frac{-r}{l} \times sin(\varphi) \quad (6)$$

and:

$$\dot{\theta} \times cos(\theta) = \frac{-r}{l} \times \dot{\varphi} \times cos(\varphi) \quad (7)$$

and also:

$$\ddot{\theta} \times cos(\theta) - \dot{\theta}^2 \times sin(\theta) = \frac{-r}{l} \times \left(\ddot{\varphi} \times cos(\varphi) - \dot{\varphi}^2 \times sin(\varphi)\right) \quad (8)$$

Also:

$$cos(\varphi) = \sqrt{1 - \frac{l^2 \times \left(1 - cos(2\theta)\right)}{2 \times r^2}} \quad (9)$$

because: $cos(\varphi) > 0$

Furthermore:

$$x_A = l + r - |x_1| \quad (10)$$

Where:

$$x_1 = l \times cos(|\theta|) + r \times cos(|\varphi|) \quad (11)$$

and $x_1 > 0$.

Therefore:

$$x_A = l + r - \left(l \times cos(\theta) + r \times \sqrt{1 - \frac{l^2 \times \left(1 - cos(2\theta)\right)}{2 \times r^2}}\right) \quad (12)$$

where $x_A(t)$ is derivable.

x_A increases in the figure 1 and thus the velocity of the point A is exactly V_A. And the acceleration of the point A is therefore γ_A.

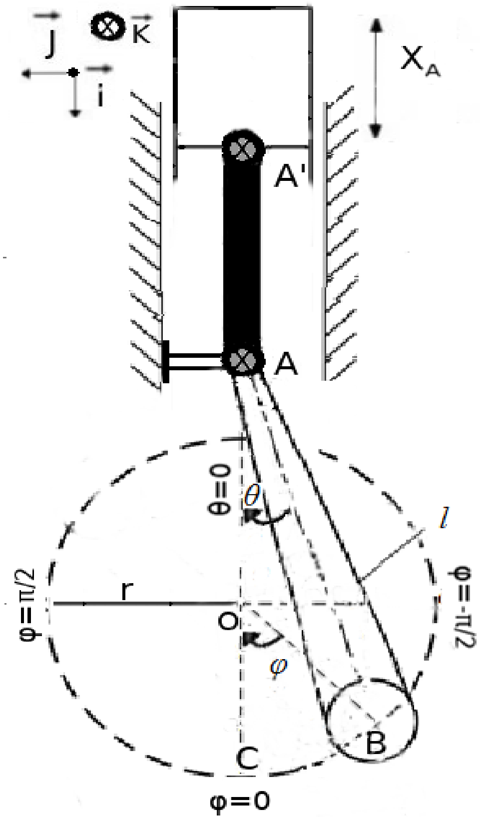

Figure 2. The second geometric step of an engine crank operating with its connecting rod.

In the step 2 where $\frac{-\pi}{2} < \varphi < 0$and $-\theta_{max} < \theta < 0$:
φ is in the direct counterclockwise orientation and thus it increases, consequently: $\dot{\varphi} > 0$.
However, $|\varphi|$ decreases and thus:

$$\frac{dsin(|\varphi|)}{dt} = -\dot{\varphi} \times cos(|\varphi|) \tag{13}$$

and:

$$\frac{dcos(|\varphi|)}{dt} = \dot{\varphi} \times sin(|\varphi|) \tag{14}$$

Θ is in the direct counterclockwise orientation and thus it increases, consequently:$\dot{\theta} > 0$.
However, $|\theta|$ decreases. Consequently:

$$\frac{dsin(|\theta|)}{dt} = -\dot{\theta} \times cos(|\theta|) \tag{15}$$

and:

$$\frac{dcos(|\theta|)}{dt} = \dot{\theta} \times sin(|\theta|) \tag{16}$$

and:

$$r \times sin(|\varphi|) = l \times sin(|\theta|) \tag{17}$$

Hence

$$sin(\theta) = \frac{r}{l} \times sin(\varphi) \tag{18}$$

and:

$$\dot{\theta} \times cos(\theta) = \frac{r}{l} \times \dot{\varphi} \times cos(\varphi) \tag{19}$$

and also:

$$\ddot{\theta} \times cos(\theta) - \dot{\theta}^2 \times sin(\theta) = \frac{r}{l} \times \left(\ddot{\varphi} \times cos(\varphi) - \dot{\varphi}^2 \times sin(\varphi)\right) \tag{20}$$

Also:

$$cos(\varphi)^2 = 1 - \frac{l^2 \times sin(\theta)^2}{r^2} \tag{21}$$

therefore:

$$cos(\varphi) = \sqrt{1 - \frac{l^2}{2r^2} \times \left(1 - cos(2\theta)\right)} \tag{22}$$

because: $cos(\varphi) > 0$.

Furthermore:

$$x_A = l + 2r - x_1 \tag{23}$$

where:

$$x_1^2 = l^2 + |BC|^2 - 2l \times |BC| \times cos\left(\frac{\pi}{2} - |\theta| + \left|\frac{\varphi}{2}\right|\right) = l^2 + |BC|^2 - 2l \times |BC| \times sin\left(|\theta| - \left|\frac{\varphi}{2}\right|\right) \tag{24}$$

by Al-kashi formula.
And

$$|BC| = 2r \times sin\left(\frac{|\varphi|}{2}\right) = -2r \times sin\left(\frac{\varphi}{2}\right) \tag{25}$$

And

$$sin\left(\frac{\pi}{2} - \left|\frac{\varphi}{2}\right|\right) \times |BC| = sin(|\theta|) \times l \tag{26}$$

Hence

$$cos\left(\frac{\varphi}{2}\right) \times |BC| = -sin(\theta) \times l \tag{27}$$

since $O\hat{C}B = \frac{\pi}{2} - \left|\frac{\varphi}{2}\right|$

And after simplification:

$$x_1^2 = 2l^2 - \frac{l^3}{r} - 2l^2 \times \left(1 - \frac{l}{r}\right) \times cos(\theta) + l^2 \times \left(1 - \frac{l}{r}\right) \times cos(2\theta) \tag{28}$$

And thus:

$$x_A = l + 2r - \left(2l^2 - \frac{l^3}{r} - 2l^2 \times \left(1 - \frac{l}{r}\right) \times cos(\theta) + l^2 \times \left(1 - \frac{l}{r}\right) \times cos(2\theta)\right)^{\frac{1}{2}} \tag{29}$$

where $x_A(t)$ is derivable.
We remark that we should always have:

$$\frac{2r-l}{r-l} > 2cos(\theta) - cos(2\theta) \quad (30)$$

where: $\frac{-\pi}{2} < \theta < 0$ since: $x_1^2$is positive.

x_A increases in the figure 2 and thus the velocity of the point A is exactly V_A. And the acceleration of the point A is therefore γ_A.

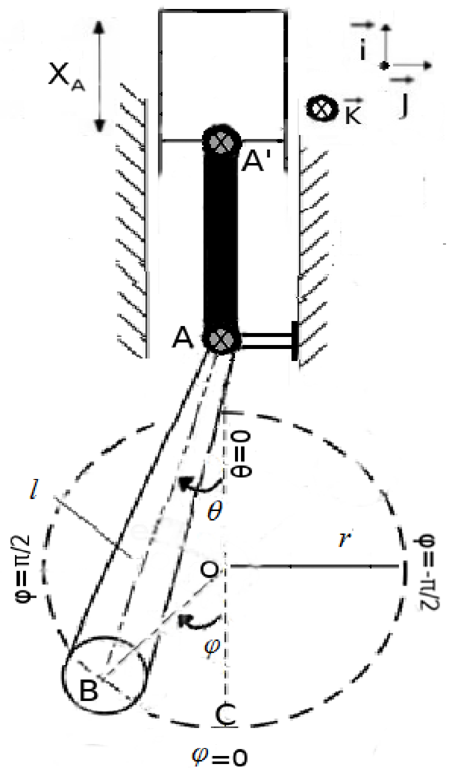

Figure 3. The third geometric step of an engine crank operating with its connecting rod

In the step 3 where $0 < \varphi < \frac{\pi}{2}$**and** $0 < \theta < \theta_{max}$**:**

φ is in the direct counterclockwise orientation and thus it increases, consequently: $\dot{\varphi} > 0$.
However, $|\varphi|$ increases and thus:

$$\frac{dsin(|\varphi|)}{dt} = \dot{\varphi} \times cos(|\varphi|) \quad (31)$$

and:

$$\frac{dcos(|\varphi|)}{dt} = -\dot{\varphi} \times sin(|\varphi|) \quad (32)$$

Θ is in the direct counterclockwise orientation and thus it increases, consequently:$\dot{\theta} > 0$.
However, $|\theta|$ also increases. Consequently:

$$\frac{dsin(|\theta|)}{dt} = \dot{\theta} \times cos(|\theta|) \quad (33)$$

and:

$$\frac{dcos(|\theta|)}{dt} = -\dot{\theta} \times sin(|\theta|) \quad (34)$$

Also:

$$r \times sin(|\varphi|) = l \times sin(|\theta|) \tag{35}$$

Hence:

$$sin(\theta) = \frac{r}{l} \times sin(\varphi) \tag{36}$$

and:

$$\dot{\theta} \times cos(\theta) = \frac{r}{l} \times \dot{\varphi} \times cos(\varphi) \tag{37}$$

and also:

$$\ddot{\theta} \times cos(\theta) - \dot{\theta}^2 \times sin(\theta) = \frac{r}{l} \times \left(\ddot{\varphi} \times cos(\varphi) - \dot{\varphi}^2 \times sin(\varphi)\right) \tag{38}$$

Also:

$$cos(\varphi)^2 = 1 - \frac{l^2 \times sin(\theta)^2}{r^2} \tag{39}$$

therefore:

$$cos(\varphi) = \sqrt{1 - \frac{l^2}{2r^2} \times \left(1 - cos(2\theta)\right)} \tag{40}$$

because: $cos(\varphi) > 0$

Furthermore:

$$x_A = l + 2r - |x_1| \tag{41}$$

where:

$$x_1^2 = l^2 + |BC|^2 - 2l \times |BC| \times cos\left(\frac{\pi}{2} - |\theta| + \frac{|\varphi|}{2}\right) = l^2 + |BC|^2 - 2l \times |BC| \times sin\left(|\theta| - \frac{|\varphi|}{2}\right) \tag{42}$$

by Al-Kashi formula.

And

$$|BC| = 2r \times sin\left(\frac{\varphi}{2}\right) \tag{43}$$

and

$$sin\left(\frac{\pi}{2} - \left|\frac{\varphi}{2}\right|\right) \times |BC| = sin(\theta) \times l \tag{44}$$

Hence

$$cos\left(\frac{\varphi}{2}\right) \times |BC| = sin(\theta) \times l \tag{45}$$

since $O\hat{C}B = \frac{\pi}{2} - \left|\frac{\varphi}{2}\right|$

And after simplification:

$$x_1^2 = 2l^2 - \frac{l^3}{r} - 2l^2 \times \left(1 - \frac{l}{r}\right) \times cos(\theta) + l^2 \times \left(1 - \frac{l}{r}\right) \times cos(2\theta) \tag{46}$$

And thus:

$$x_A = l + 2r - \left(2l^2 - \frac{l^3}{r} - 2l^2 \times \left(1 - \frac{l}{r}\right) \times cos(\theta) + l^2 \times \left(1 - \frac{l}{r}\right) \times cos(2\theta)\right)^{\frac{1}{2}} \qquad (47)$$

where $x_A(t)$ is derivable.

We remark that we should always have:

$$\frac{2r-l}{r-l} > 2cos(\theta) - cos(2\theta) \qquad (48)$$

where: $0 < \theta < \frac{\pi}{2}$ since: x_1^2 is positive.

x_A decreases in the figure 3 and thus the velocity of the point A is exactly ($-V_A$). And the acceleration of the point A is therefore ($-\gamma_A$).

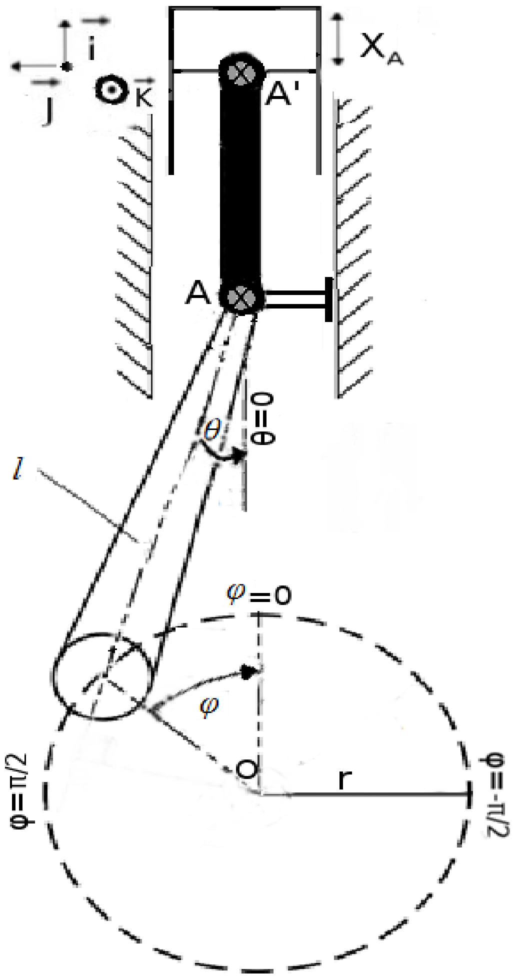

Figure 4. The fourth geometric step of an engine crank operating with its connecting rod.

In the step 4 where $0 < \varphi < \frac{\pi}{2}$ **and** $-\theta_{max} < \theta < 0$**:**

φ is in the opposite clockwise orientation and thus it decreases, consequently: $\dot{\varphi} < 0$. However, $|\varphi|$ decreases and thus:

$$\frac{dsin(|\varphi|)}{dt} = \dot{\varphi} \times cos(|\varphi|) \qquad (49)$$

and:

$$\frac{dcos(|\varphi|)}{dt} = -\dot{\varphi} \times sin(|\varphi|) \tag{50}$$

Θ is in the direct counterclockwise orientation and thus it increases, consequently: $\dot{\theta} > 0$.

However, $|\theta|$ decreases. Consequently:

$$\frac{dsin(|\theta|)}{dt} = -\dot{\theta} \times cos(|\theta|) \tag{51}$$

and:

$$\frac{dcos(|\theta|)}{dt} = \dot{\theta} \times sin(|\theta|) \tag{52}$$

Also:

$$r \times sin(|\varphi|) = l \times sin(|\theta|) \tag{53}$$

Hence:

$$sin(\theta) = \frac{-r}{l} \times sin(\varphi) \tag{54}$$

and:

$$\dot{\theta} \times cos(\theta) = \frac{-r}{l} \times \dot{\varphi} \times cos(\varphi) \tag{55}$$

and also:

$$\ddot{\theta} \times cos(\theta) - \dot{\theta}^2 \times sin(\theta) = \frac{-r}{l} \times \left(\ddot{\varphi} \times cos(\varphi) - \dot{\varphi}^2 \times sin(\varphi)\right) \tag{56}$$

Also:

$$cos(\varphi) = \sqrt{1 - \frac{l^2 \times \left(1 - cos(2\theta)\right)}{2 \times r^2}} \tag{57}$$

because: $cos(\varphi) > 0$

Furthermore:

$$x_A = l + r - |x_1| \tag{58}$$

Where:

$$x_1 = l \times cos(|\theta|) + r \times cos(|\varphi|) \tag{59}$$

and $x_1 > 0$.

Therefore:

$$x_A = l + r - \left(l \times cos(\theta) + r \times \sqrt{1 - \frac{l^2 \times \left(1 - cos(2\theta)\right)}{2 \times r^2}}\right) \tag{60}$$

where $x_A(t)$ is derivable.

x_A decreases in the figure 4 and thus the velocity of the point A is exactly ($-V_A$). And the acceleration of the point A is therefore ($-\gamma_A$).

3.2. Remark:

In order that the piston head doesn't touch the crank, always in the four steps:

$$l > 2r \Leftrightarrow \frac{r}{l} \prec \frac{1}{2} \Leftrightarrow sin(\Theta_{max}) < \frac{1}{2} \Leftrightarrow \theta_{max} < \frac{\pi}{6} \tag{61}$$

where:

$$sin(\theta_{max}) = \frac{r}{l} \tag{62}$$

and

$$cos(\theta_{max}) = \sqrt{1 - \frac{r^2}{l^2}} \tag{63}$$

Consequently in the four steps:

$$\frac{-\pi}{6} < \theta < \frac{\pi}{6} \tag{64}$$

And

$$sin(|\theta|) < \frac{1}{2} \tag{65}$$

And

$$cos(\theta) > \frac{\sqrt{3}}{2} \tag{66}$$

3.3. The calculations of: $\boldsymbol{V_A = V_A(\theta), \gamma_A = \gamma_A(\theta)}$:

We use Θ as the variable in the proof and not φ in order to calculate formulas that enable to study each cylinder isolated from the other cylinders. However, when we have enough data about the behavior of φ(t) especially at $\varphi = \frac{-\pi}{2}$ and $\varphi = \frac{\pi}{2}$ where Θ(t) is not continuous and not derivable, we can use φ as the variable of all the calculated final formulas by substituting using the demonstrated relations between the two rotation angles formulas.

In the step 1 where $\theta_{max} > \theta > 0$:

Since

$$\frac{dsin(|\theta|)}{dt} = \dot{\theta} \times cos(|\theta|) \tag{67}$$

and:

$$\frac{dcos(|\theta|)}{dt} = -\dot{\theta} \times sin(|\theta|) \tag{68}$$

then: $V_A = \dot{\theta} \times \left(l \times sin(|\theta|) + \frac{l^2}{2r} \times \left(1 - \frac{l^2}{2r^2} \times \left(1 - cos(|2\theta|)\right)\right)^{\frac{-1}{2}} \times sin(|2\theta|) \right)$

(69)

and since $\theta > 0$: $V_A = \dot{\theta} \times \left(l \times sin(\theta) + \frac{l^2}{2r} \times \left(1 - \frac{l^2}{2r^2} \times \left(1 - cos(2\theta)\right)\right)^{\frac{-1}{2}} \times sin(2\theta) \right)$

(70)

Where: $V_A > 0$as expected since $\theta_{max} > \theta > 0$.

And also:

$\gamma_A = l \times \ddot{\theta} \times sin(\theta) + l \times \dot{\theta}^2 \times cos(\theta) + \frac{l^4}{8r^3} \times \dot{\theta}^2 \times \left(1 - cos(4\theta)\right) \times \left(1 - \frac{l^2}{2r^2} \times \left(1 - cos(2\theta)\right)\right)^{\frac{-3}{2}} + \frac{l^2}{2r} \times \left(2\dot{\theta}^2 \times cos(2\theta) + \ddot{\theta} \times sin(2\theta)\right) \times \left(1 - \frac{l^2}{2r^2} \times \left(1 - cos(2\theta)\right)\right)^{\frac{-1}{2}}$

(71)

In the step 2 where $-\theta_{max} < \theta < 0$:

since: $\frac{dsin(|\theta|)}{dt} = -\dot{\theta} \times cos(|\theta|)$ (72)

and: $\frac{dcos(|\theta|)}{dt} = \dot{\theta} \times sin(|\theta|)$ (73)

$$V_A = l^2 \times \left(1 - \frac{l}{r}\right) \times \dot{\theta} \times \left(sin\left(|\theta| - sin(2|\theta|)\right)\right) \times \left(2l^2 - \frac{l^3}{r} - 2l^2 \times \left(1 - \frac{l}{r}\right) \times cos(\theta) + l^2 \times \left(1 - \frac{l}{r}\right) \times cos(2\theta)\right)^{\frac{-1}{2}} \quad (74)$$

and since $\theta < 0$:

$$V_A = l^2 \times \left(1 - \frac{l}{r}\right) \times \dot{\theta} \times \left(sin(2\theta) - sin(\theta)\right) \times \left(2l^2 - \frac{l^3}{r} - 2l^2 \times \left(1 - \frac{l}{r}\right) \times cos(\theta) + l^2 \times \left(1 - \frac{l}{r}\right) \times cos(2\theta)\right)^{\frac{-1}{2}} \quad 75)$$

Always: $l > 2r$ in order that the piston head doesn't touch the crank, and thus:$1 - \frac{l}{r} < 0$,

and $sin(2\theta) - sin(\theta) < 0$ (76)

consequently: $V_A > 0$ as expected.

And also:

$$\gamma_A = l^2 \times \left(1-\frac{l}{r}\right) \times \left(\ddot{\theta} \times \left(sin(2\theta) - sin(\theta)\right) + \dot{\theta}^2 \times \left(2 \times cos(2\theta) - cos(\theta)\right)\right) \times \left(2l^2 - \frac{l^3}{r} - 2l^2 \times \left(1-\frac{l}{r}\right) \times cos(\theta) + l^2 \times \left(1-\frac{l}{r}\right) \times cos(2\theta)\right)^{\frac{-1}{2}} + \left(l^2 \times \left(1-\frac{l}{r}\right) \times \dot{\theta} \times \left(sin(2\theta) - sin(\theta)\right)\right)^2 \times \left(2l^2 - \frac{l^3}{r} - 2l^2 \times \left(1-\frac{l}{r}\right) \times cos(\theta) + l^2 \times \left(1-\frac{l}{r}\right) \times cos(2\theta)\right)^{\frac{-3}{2}}$$

(77)

In the step 3 where $\theta_{max} > \theta > 0$:

since: $\frac{dsin(|\theta|)}{dt} = \dot{\theta} \times cos(|\theta|)$ (78)

and: $\frac{dcos(|\theta|)}{dt} = -\dot{\theta} \times sin(|\theta|)$ (79)

$$V_A = l^2 \times \left(1-\frac{l}{r}\right) \times \dot{\theta} \times \left(sin(2|\theta|) - sin(|\theta|)\right) \times \left(2l^2 - \frac{l^3}{r} - 2l^2 \times \left(1-\frac{l}{r}\right) \times cos(\theta) + l^2 \times \left(1-\frac{l}{r}\right) \times cos(2\theta)\right)^{\frac{-1}{2}} \quad (80)$$

and since $\theta > 0$:

$$V_A = l^2 \times \left(1-\frac{l}{r}\right) \times \dot{\theta} \times \left(sin(2\theta) - sin(\theta)\right) \times \left(2l^2 - \frac{l^3}{r} - 2l^2 \times \left(1-\frac{l}{r}\right) \times cos(\theta) + l^2 \times \left(1-\frac{l}{r}\right) \times cos(2\theta)\right)^{\frac{-1}{2}} \quad (81)$$

Always:$1-\frac{l}{r} < 0$ and$sin(2\theta) - sin(\theta) > 0$ (82)

consequently: $V_A < 0$ as expected.

And also:

$$\gamma_A = l^2 \times \left(1-\frac{l}{r}\right) \times \left(\ddot{\theta} \times \left(sin(2\theta) - sin(\theta)\right) + \dot{\theta}^2 \times \left(2 \times cos(2\theta) - cos(\theta)\right)\right) \times \left(2l^2 - \frac{l^3}{r} - 2l^2 \times \left(1-\frac{l}{r}\right) \times cos(\theta) + l^2 \times \left(1-\frac{l}{r}\right) \times cos(2\theta)\right)^{\frac{-1}{2}} + \left(l^2 \times \left(1-\frac{l}{r}\right) \times \dot{\theta} \times \left(sin(2\theta) - sin(\theta)\right)\right)^2 \times \left(2l^2 - \frac{l^3}{r} - 2l^2 \times \left(1-\frac{l}{r}\right) \times cos(\theta) + l^2 \times \left(1-\frac{l}{r}\right) \times cos(2\theta)\right)^{\frac{-3}{2}}$$

(83)

In the step 4 where $-\theta_{max} < \theta < 0$:

since: $\frac{dsin(|\theta|)}{dt} = -\dot{\theta} \times cos(|\theta|)$ (84)

and: $\frac{dcos(|\theta|)}{dt} = \dot{\theta} \times sin(|\theta|)$ (85)

$$V_A = -\dot{\theta} \times \left(l \times sin(|\theta|) + \frac{l^2}{2r} \times \left(1 - \frac{l^2}{2r^2} \times \left(1 - cos(|2\theta|)\right)\right)^{\frac{-1}{2}} \times sin(|2\theta|) \right) \quad (86)$$

and since $\theta < 0$:$V_A = \dot{\theta} \times \left(l \times sin(\theta) + \frac{l^2}{2r} \times \left(1 - \frac{l^2}{2r^2} \times \left(1 - cos(2\theta)\right)\right)^{\frac{-1}{2}} \times sin(2\theta) \right)$ (87)

Where $V_A < 0$ as expected since: $-\theta_{max} < \theta < 0$.

And also:

$$\gamma_A = l \times \ddot{\theta} \times sin(\theta) + l \times \dot{\theta}^2 \times cos(\theta) + \frac{l^4}{8r^3} \times \dot{\theta}^2 \times \left(1 - cos(4\theta)\right) \times \left(1 - \frac{l^2}{2r^2} \times \left(1 - cos(2\theta)\right)\right)^{\frac{-3}{2}} + \frac{l^2}{2r} \times \left(2\dot{\theta}^2 \times cos(2\theta) + \ddot{\theta} \times sin(2\theta)\right) \times \left(1 - \frac{l^2}{2r^2} \times \left(1 - cos(2\theta)\right)\right)^{\frac{-1}{2}} \quad (88)$$

3.4. The continuity of velocities and accelerations at the top dead center (TDC) where φ=0 and at the bottom dead center (BDC) where φ=-π:

ϴ and φ are continuous at the TDC and at the BDC where the axis $\vec{\iota}$ changes its orientation to the opposite.

At the TDC :

step1:

φ tends to 0^-⇒Θ tends to 0^+ Consequently: $\lim_{\theta \to 0^+} V_A = 0$ (89)

and: $\lim_{\theta \to 0^+} \gamma_A = \dot{\theta}_1^2 \times \left(l + \frac{l^2}{r}\right)$ (positive). (90)

step4:

φ tends to 0^+⇒Θ tends to 0^- Consequently: $\lim_{\theta \to 0^-} V_A = 0$ (91)

and: $\lim_{\theta \to 0^-} \gamma_A = \dot{\theta}_2^2 \times \left(l + \frac{l^2}{r}\right)$ (positive). (92)

There is indeed a deceleration of A before the TDC since V_A=0, but the acceleration of A in the step 4 is $(-\gamma_A)$.

3.5. Conclusion

V_A is continuous, and we should only compare $\dot{\theta}_1^2$ and $\dot{\theta}_2^2$ to investigate the continuity of γ_A.

However, in the step 1 and in the step 4: $\dot{\theta} \times cos(\theta) = \frac{-r}{l} \times \dot{\varphi} \times cos(\varphi)$ (93)

and thus at the TDC: $\dot{\theta} = \frac{-r}{l} \times \dot{\varphi}$ (94)

consequently: $\dot{\theta}$ is continuous only if the angular velocity of the crankshaft $\dot{\varphi}$ is considered continuous.

And thus γ_A is also continuous only if the angular velocity of the crankshaft $\dot{\varphi}$ is considered continuous.

The angular velocity $\dot{\varphi}$ can be controlled by a detector on the crankshaft.

At the BDC:

step2:

φ tends to 0^- ⇒ Θ tends to 0^- Consequently: $\lim\limits_{\theta \to 0^-} V_A = 0$ (95)

and: $\lim\limits_{\theta \to 0^-} \gamma_A = \dot{\theta}_3^2 \times \left(l - \frac{l^2}{r}\right)$ (negative) (96)

step3:

φ tends to 0^+ ⇒ Θ tends to 0^+ Consequently: $\lim\limits_{\theta \to 0^+} V_A = 0$ (97)

and: $\lim\limits_{\theta \to 0^+} \gamma_A = \dot{\theta}_4^2 \times \left(l - \frac{l^2}{r}\right)$ (negative). (98)

There is indeed an acceleration of A before the BDC since V_A=0, but the acceleration of A in the step 3 is $(-\gamma_A)$.

3.6. Conclusion

V_A is continuous, and we should only compare $\dot{\theta}_3^2$ and $\dot{\theta}_4^2$ to investigate the continuity of γ_A.

However, in the step 2 and in the step 3: $\dot{\theta} \times cos(\theta) = \frac{r}{l} \times \dot{\varphi} \times cos(\varphi)$ (99)

and thus at the BDC: $\dot{\theta} = \frac{r}{l} \times \dot{\varphi}$ (100)

consequently: $\dot{\theta}$ is continuous only if the angular velocity of the crankshaft $\dot{\varphi}$ is considered continuous.

And thus, V_A is also continuous only if the angular velocity of the crankshaft $\dot{\varphi}$ is considered continuous.

3.7. Verification

In the article: "Analysis of diagnostic utility of instantaneous angular speed fluctuation of diesel engine crankshaft" [21], the figure 3 shows the fluctuations shape of $\dot{\varphi}$. It looks continuous in the graph since it was taken continuously by a captor. However, the explosions in the cylinders is so strong that the angular velocity $\dot{\varphi}$can also be considered non-continuous.

3.8. Remark

This geometric study is also very useful for some positive displacement pumps like piston pumps.

3.9. Forces:

In the four following figures

The forces established on A are: $\{ \vec{g}, \vec{R}, \vec{F} \}$.

$\vec{g}$ is the force established by the metal on A and it is parallel to the axis $\vec{j}$. This force is exercised by foundations of the engine through the crosshead mechanism. It can also be exercised by a part of the liner on the piston head in the absence of a crosshead mechanism, and in this case A and A' are gathered one on the other as the piston head center of mass, but the results of this study remains valid in both cases.

$\vec{R}$ is the force of the crank on A, and $\vec{F}$ is the resulting sum of forces that are parallel to the axis $\vec{\imath}$.$\vec{F}$ is caused by the pressure P in the cylinder and by the weight of the piston head but it is lowered by R' that is the resulting sum of the piston head frictions in the cylinder therefore:

$$F = m \times G + P \times S - R' \qquad (101)$$

where G is the acceleration due to gravity and S is the area of the piston head surface and m is the piston head mass that is increased by a rod mass when the crosshead case is the one studied.

In the four steps:

$(\vec{\imath}, \vec{\jmath}, \vec{k})$ represents the used direct orthonormal basis oriented in each one of the four steps in a manner allowing always that:

- Θ increases and thus always $\dot{\theta} > 0$.
- The axis $\vec{\imath}$ is in the same movement orientation of the point A that is always the center of the used direct orthonormal basis, hence the velocity of A should be always positive.
- The used forces values in the four steps are the absolute values always: $F > 0, R > 0, g > 0$.

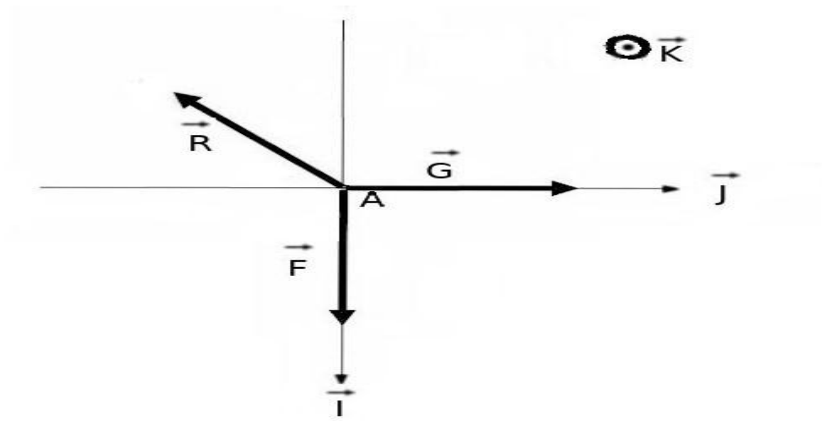

Figure 5. The forces established on A at the first geometric step.

In the step 1: where $\frac{-\pi}{2} < \varphi < 0$ ***and*** $0 < \theta < \theta_{max}$***:***

The velocity of A is exactly V_A and thus the acceleration of A is also γ_A.

Also: $\vec{F} = F \times \vec{\imath}$ (102)

and $\vec{G} = G \times \vec{\jmath}$ (103)

and: $\vec{R} = -R \times cos(|\theta|) \times \vec{\imath} - R \times sin(|\theta|) \times \vec{\jmath} = -R \times cos(\theta) \times \vec{\imath} - R \times sin(\theta) \times \vec{\jmath}$ (104)

The velocity of A along the axis $\vec{\jmath}$ is zero, consequently: $g \times \vec{\jmath} - R \times sin(|\theta|) \times \vec{\jmath} = \vec{0}$ (105)

and thus: $g = R \times sin(\theta)$ (106)

Also: $F \times \vec{\imath} - R \times cos(\theta) \times \vec{\imath} = m \times \gamma_A \times \vec{\imath}$ (107)

and thus: $\frac{F - R \times cos(\theta)}{m} = \gamma_A$ (108)

and also: $\gamma_A = \frac{F - \sqrt{R^2 - g^2}}{m}$ (109)

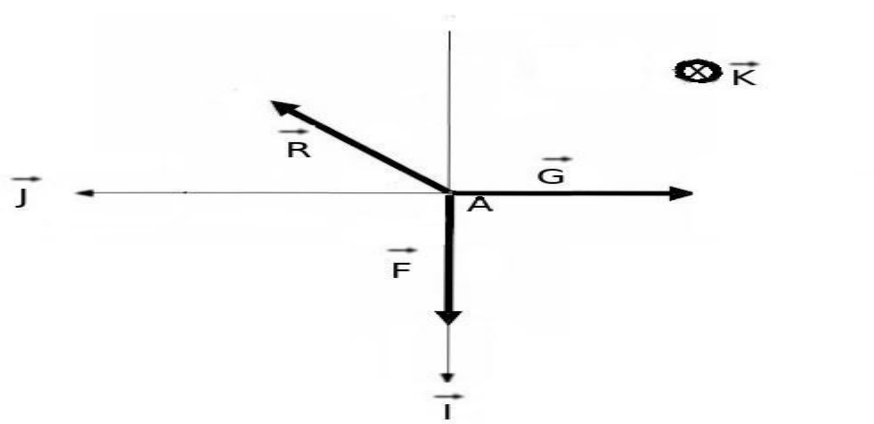

Figure 6. The forces established on A at the second geometric step.

In the step 2: where a $\frac{-\pi}{2} < \varphi < 0$ ***nd*** $-\theta_{max} < \theta < 0$***:***

Θ changed sign when φ reached $\frac{-\pi}{2}$ at the end of the step 1.

The velocity of A is exactly V_A and thus the acceleration of A is also γ_A.

Also: $\vec{F} = F \times \vec{\imath}$ (110)

and $\vec{G} = -G \times \vec{\jmath}$ (111)

and: $\vec{R} = -R \times cos(|\theta|) \times \vec{\imath} + R \times sin(|\theta|) \times \vec{\jmath} = -R \times cos(\theta) \times \vec{\imath} - R \times sin(\theta) \times \vec{\jmath}$ (112)

The velocity of A along the axis $\vec{\jmath}$ is zero, consequently: $g \times \vec{\jmath} - R \times sin(|\theta|) \times \vec{\jmath} = \vec{0}$ (113)

and thus: $g = R \times sin(|\theta|) = -R \times sin(\theta)$ (114)

Also: $F \times \vec{\imath} - R \times cos(\theta) \times \vec{\imath} = m \times \gamma_A \times \vec{\imath}$ (115)

and thus: $\frac{F - R \times cos(\theta)}{m} = \gamma_A$ (116)

And also: $\gamma_A = \frac{F - \sqrt{R^2 - g^2}}{m}$ (117)

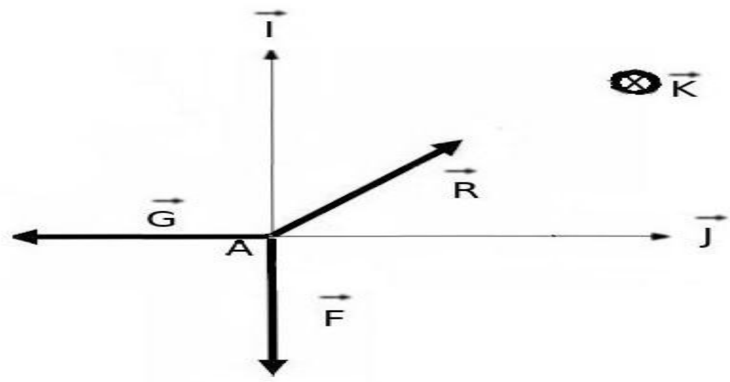

Figure 7. The forces established on A at the third geometric step.

In the step 3: where a $0 < \varphi < \frac{\pi}{2}$ ***and*** $0 < \theta < \theta_{max}$***:***

The velocity of A is exactly ($-V_A$) and thus the acceleration of A is also ($-\gamma_A$).

Also $\vec{F} = -F \times \vec{\imath}$ (118)

and $\vec{G} = -G \times \vec{\jmath}$ (119)

and: $\vec{R} = R \times cos(|\theta|) \times \vec{\imath} + R \times sin(|\theta|) \times \vec{\jmath} = R \times cos(\theta) \times \vec{\imath} + R \times sin(\theta) \times \vec{\jmath}$ (120)

The velocity of A along the axis $\vec{j}$ is zero, consequently: $g \times \vec{j} - R \times sin(|\theta|) \times \vec{j} = \vec{0}$ (121)

and thus: $g = R \times sin(\theta)$ (122)

Also: $-F \times \vec{\imath} + R \times cos(\theta) \times \vec{\imath} = -m \times \gamma_A \times \vec{\imath}$ (123)

and thus: $\frac{F - R\times cos(\theta)}{m} = \gamma_A$ (124)

And also: $\gamma_A = \frac{F - \sqrt{R^2 - g^2}}{m}$ (125)

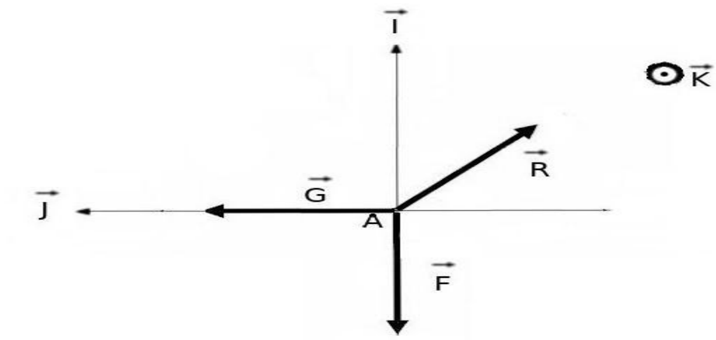

Figure 8. The forces established on A at the fourth geometric step.

In the step 4: where $0 < \varphi < \frac{\pi}{2}$ ***and*** $-\theta_{max} < \theta < 0$***:***

Θ changed sign when φ reached $\frac{\pi}{2}$ at the end of the step 3.

The velocity of A is exactly ($-V_A$) and thus the acceleration of A is also ($-\gamma_A$).

Also $\vec{F} = -F \times \vec{\imath}$ (126)

and $\vec{G} = G \times \vec{j}$ (127)

and: $\vec{R} = R \times cos(|\theta|) \times \vec{\imath} - R \times sin(|\theta|) \times \vec{j} = R \times cos(\theta) \times \vec{\imath} + R \times sin(\theta) \times \vec{j}$ (128)

The velocity of A along the axis $\vec{j}$ is zero, consequently: $g \times \vec{j} - R \times sin(|\theta|) \times \vec{j} = \vec{0}$ (129)

and thus: $g = R \times sin(|\theta|) = -R \times sin(\theta)$ (130)

Also: $-F \times \vec{\imath} + R \times cos(\theta) \times \vec{\imath} = -m \times \gamma_A \times \vec{\imath}$ (131)

and thus: $\frac{F - R\times cos(\theta)}{m} = \gamma_A$ (132)

And also: $\gamma_A = \frac{F - \sqrt{R^2 - g^2}}{m}$ (133)

3.10. One cylinder brake work given per engine cycle W_b :

During this study we will need a new frame of reference made by the vectors $\vec{u}$ and $\vec{v}$ in order to divide some forces into their projections like it is explained below.

In the step 1:

The figure 1 is also useful in order to understand the following demonstrations.
We have always: $m \times \overrightarrow{\gamma_A} = m \times \gamma_A \times \vec{\imath}$.
The useful forces are the forces that are established upon the connecting rod and thus also upon the crankshaft (in the point B) and cause the rotation of the crankshaft. They are the forces oriented downwards or oriented to the right exactly like the orientation of the Axis $\vec{\imath}$ and axis $\vec{\jmath}$ of the step 1.
The first useful force $\overrightarrow{f_1}$ is established straight along the connecting rod perpendicularly to the line (Oe) and influences the crankshaft (in the point B):

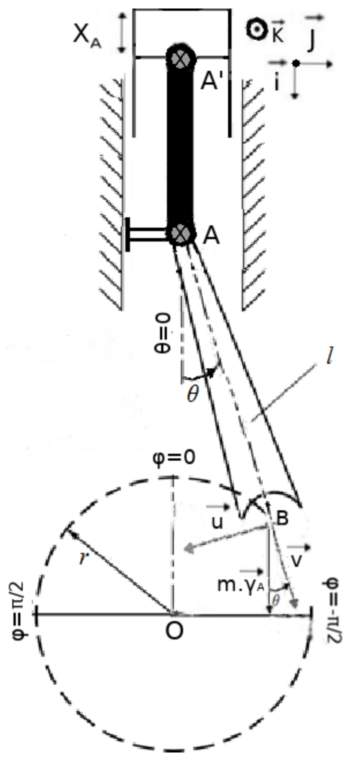

Figure 9. The first geometric step of an engine crank and connecting rod with the two frames of reference.

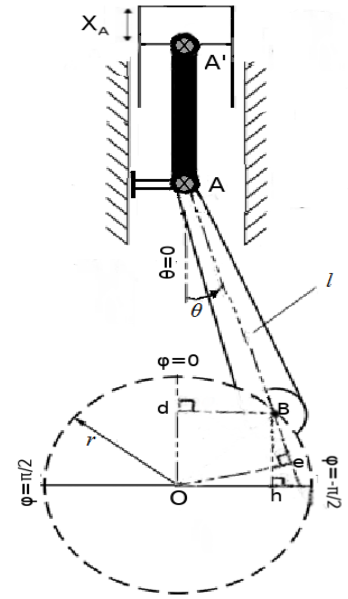

Figure 10. The first geometric step of an engine crank and connecting rod with the useful orthogonal projections.

$$\overrightarrow{f_1} = f_1 \times \vec{v} = m \times \gamma_A \times cos(\theta) \times \vec{v} \qquad (134)$$

Consequently, the first useful (motor) torque is: $M_1(O) = m \times \gamma_A \times cos(|\theta|) \times |Oe|$ (135)

where: $|Oe| = sin(|\theta|) \times \left(l \times cos(|\theta|) + r \times cos(|\varphi|)\right)$ (136)

consequently: $M_1(O) = m \times r \times \gamma_A \times cos(|\theta|) \times sin(|\theta|) \times \left(\frac{l}{r} \times cos(|\theta|) + \sqrt{1 - \frac{l^2}{2r^2} \times \left(1 - cos(2\theta)\right)}\right)$ (137)

and thus: $M_1(O) = \frac{1}{2} \times m \times r \times \gamma_A \times sin(2\theta) \times \left(\frac{l}{r} \times cos(\theta) + \sqrt{1 - \frac{l^2}{2r^2} \times \left(1 - cos(2\theta)\right)}\right)$ (138)

Therefore: $M_1(O) = \frac{1}{2} \times m \times l \times \gamma_A \times sin(2\theta) \times cos(\theta) + \frac{1}{2} \times m \times r \times \gamma_A \times sin(2\theta) \times \sqrt{1 - \frac{l^2}{2r^2} \times \left(1 - cos(2\theta)\right)}$
(139)

Hence: $M_1(O) = \frac{1}{2} \times m \times l \times \gamma_A \times \left(\frac{sin(3\theta)}{2} + \frac{sin(\theta)}{2}\right) + \frac{1}{2} \times m \times r \times \gamma_A \times sin(2\theta) \times \sqrt{1 - \frac{l^2}{2r^2} \times \left(1 - cos(2\theta)\right)}$ (140)

since: $\theta > 0$ and by using γ_A of the step 1.

In the step 1, there is a second force. It is established perpendicularly upon the connecting rod and influences the crankshaft (in the point B). It is: $\overrightarrow{f_2} = f_2 \times \vec{u} = m \times \gamma_A \times sin(\theta) \times \vec{u}$ (141)

The force $\overrightarrow{f_2}$ useful component downwards is $\overrightarrow{g_1}$ with: $\overrightarrow{g_1} = g_1 \times \vec{\imath} = f_2 \times sin(\theta) \times \vec{\imath}$ (142)

Where: $m \times \gamma_A \times sin(\theta)^2 = \frac{m\times\gamma_A\times(1-cos(2\theta))}{2}$

$\overrightarrow{g_1}$ is perpendicular to the line (Oh).

Consequently, the second useful (motor) torque is: $M_2(O) = \frac{m\times\gamma_A\times(1-cos(2\theta))}{2} \times |Oh|$ (143)

Where: $|Oh| = r \times sin(|\varphi|) = l \times sin(|\theta|) = l \times sin(\theta)$ (144)

Hence: $M_2(O) = \frac{m\times\gamma_A\times(1-cos(2\theta))}{2} \times l \times sin(\theta) = \frac{m\times l\times\gamma_A}{2} \times \left(\frac{3sin(\theta)}{2} - \frac{sin(3\theta)}{2}\right)$ (145)

since: $\theta > 0$ and by using γ_A of the step 1.

The force $\overrightarrow{f_2}$ component oriented to the left is $\overrightarrow{g_2}$ with: $\overrightarrow{g_2} = g_2 \times \vec{j} = -f_2 \times cos(\theta) \times \vec{j}$ (146)

Where: $m \times \gamma_A \times sin(\theta) \times cos(\theta) = \frac{m\times\gamma_A\times sin(2\theta)}{2}$

$\overrightarrow{g_2}$ is perpendicular to the line (Od). It is against the rotation and makes a torque that has a negative related work.

The absolute value of the torque is:

$$M_3(O) = \frac{m\times\gamma_A\times sin(2\theta)}{2} \times |Od| \quad (147)$$

Where: $|Od| = r \times cos(|\varphi|) = r \times \sqrt{1 - \frac{l^2\times(1-cos(2\theta))}{2\times r^2}}$ (148)

Hence: $M_3(O) = \frac{m\times r\times\gamma_A\times sin(2\theta)}{2} \times \sqrt{1 - \frac{l^2\times(1-cos(2\theta))}{2\times r^2}}$ (149)

since: $\theta > 0$ and by using γ_A of the step 1.

Finally, The useful (motor) work given in the first step by one piston and cylinder to the crank is:

$$W_{step1} = \int_0^{\theta_{max}} \big(M_1(O) + M_2(O) - M_3(O)\big)\, d\theta \quad (150)$$

Hence:

$$W_{step1} = \int_0^{\theta_{max}} \big(m \times l \times \gamma_1 \times sin(\theta)\big)\, d\theta \quad (151)$$

Where: $\gamma_1 = \gamma_A$ at the step 1 defined at $]0, \theta_{max}[$ and θ_{max}= arcsin($\frac{r}{l}$) .

In the step 2:

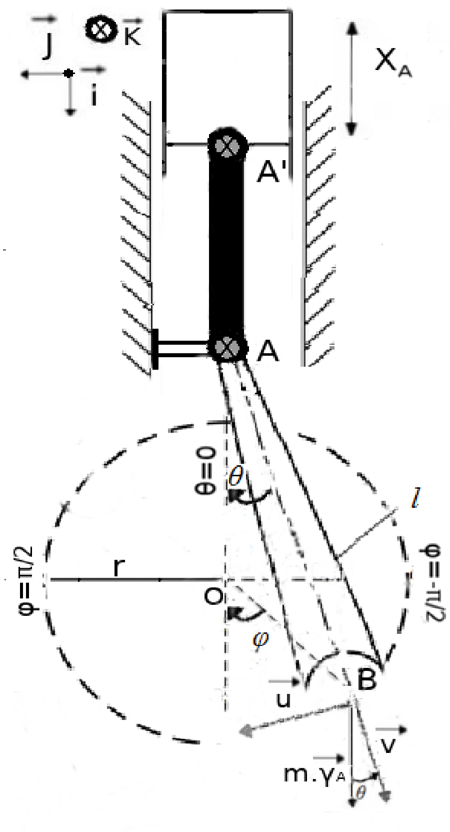

Figure 11. The second geometric step of an engine crank and connecting rod with the two frames of reference.

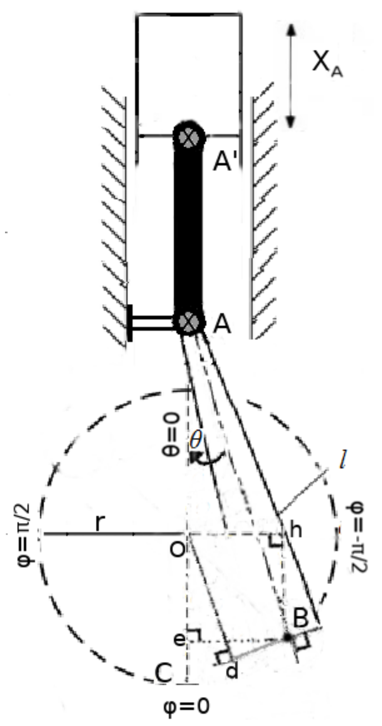

Figure 12. The second geometric step of an engine crank and connecting rod with the useful

orthogonal projections.

The figure 2 is also useful in order to understand the following demonstrations.

We have always: $m \times \overrightarrow{\gamma_A} = m \times \gamma_A \times \vec{\imath}$ and the two lines (AB) and (Od) are parallel. These two lines are also perpendicular to the line (Bd).

The useful forces are the forces that are established upon the connecting rod and thus also upon the crankshaft (in the point B) and cause the rotation of the crankshaft. They are the forces oriented downwards or oriented to the left exactly like the orientation of the Axis $\vec{\imath}$ and axis $\vec{\jmath}$ of the step 2.

The first useful force is: $\overrightarrow{f_1} = f_1 \times \vec{u} = -m \times \gamma_A \times sin(\theta) \times \vec{u}$ (152)

since: $\theta < 0$.

It is established perpendicularly upon the connecting rod and influences the crankshaft.

Consequently, the first useful (motor) torque is: $M_1(O) = -m \times \gamma_A \times sin(\theta) \times |Od|$ (153)

where: $|Od| = r \times cos(|\varphi| - |\theta|) = r \times cos(\varphi - \theta)$ (154)

consequently: $|Od| = \frac{l \times (1 - cos(2\theta))}{2} + r \times cos(\theta) \times \sqrt{1 - \frac{l^2}{2r^2} \times (1 - cos(2\theta))}$ (155)

Hence: $M_1(O) = -3\frac{m \times l \times \gamma_A \times sin(\theta)}{4} + \frac{m \times l \times \gamma_A \times sin(3\theta)}{4} - \frac{m \times r \times \gamma_A \times sin(2\theta)}{2} \times \sqrt{1 - \frac{l^2}{2r^2} \times (1 - cos(2\theta))}$ (156)

since: $\theta < 0$ and by using γ_A of the step 2.

In the step 2, there is a second force: $\overrightarrow{f_2} = f_2 \times \vec{v} = m \times \gamma_A \times cos(\theta) \times \vec{v}$ (157)

And it is established straight along the connecting rod perpendicularly to the line (Bd) and influences the crankshaft (in the point B).

The force $\overrightarrow{f_2}$ useful component downwards is $\overrightarrow{g_1}$: $\overrightarrow{g_1} = g_1 \times \vec{\imath} = f_2 \times cos(\theta) \times \vec{\imath}$ (158)

Where: $m \times \gamma_A \times cos(\theta)^2 = \frac{m \times \gamma_A \times (1 + cos(2\theta))}{2}$

$\overrightarrow{g_1}$ is perpendicular to the line (Oh).

Consequently, the second useful (motor) torque is: $M_2(O) = \frac{m \times \gamma_A \times (1 + cos(2\theta))}{2} \times |Oh|$ (159)

Where: $|Oh| = r \times cos\left(\frac{\pi}{2} - |\varphi|\right) = l \times sin(|\theta|) = -l \times sin(\theta)$ (160)

Hence: $M_2(O) = \frac{-m \times \gamma_A \times (1 + cos(2\theta))}{2} \times l \times sin(\theta) = \frac{-m \times l \times \gamma_A}{2} \times \left(\frac{sin(\theta)}{2} + \frac{sin(3\theta)}{2}\right)$ (161)

since: $\theta < 0$ and by using γ_A of the step 2.

The force $\overrightarrow{f_2}$ component oriented to the right is $\overrightarrow{g_2}$: $\overrightarrow{g_2} = g_2 \times \vec{\jmath} = f_2 \times sin(\theta) \times \vec{\jmath}$ (162)

since: $\theta < 0$.

With: $m \times \gamma_A \times sin(\theta) \times cos(\theta) = \frac{m \times \gamma_A \times sin(2\theta)}{2}$

$\overrightarrow{g_2}$ is perpendicular to the line (Oe).

It is against the rotation and makes a torque that has a negative related work.

The absolute value of the torque is:

$$M_3(O) = \frac{-m\times\gamma_A\times sin(2\theta)}{2} \times |Oe| \quad (163)$$

Where: $|Oe| = r \times cos(|\varphi|) = r \times \sqrt{1 - \frac{l^2\times(1-cos(2\theta))}{2\times r^2}}$ (164)

Hence: $M_3(O) = \frac{-m\times r\times\gamma_A\times sin(2\theta)}{2} \times \sqrt{1 - \frac{l^2\times(1-cos(2\theta))}{2\times r^2}}$ (165)

since: $\theta < 0$ and by using γ_A of the step 2.

Finally, The useful (motor) work given in the second step by one piston and cylinder to the crank is:

$$W_{step2} = \int_{-\theta_{max}}^{0} \big(M_1(O) + M_2(O) - M_3(O)\big)\, d\theta \quad (166)$$

Hence:

$$W_{step2} = \int_{-\theta_{max}}^{0} \big(-m \times l \times \gamma_2 \times sin(\theta)\big)\, d\theta \quad (167)$$

Where: $\gamma_2 = \gamma_A$ at the step 2 defined at $]-\theta_{max}$,0[and θ_{max}= arcsin($\frac{r}{l}$) (168)

3.11. Conclusion:

There is no useful (motor) torque given by the piston in the steps 3 and 4. Consequently, the total useful (motor) work given by one piston and cylinder to the crankshaft is:

$$W_b = W_{step1} + W_{step2} = m \times l \times (\int_0^{\theta_{max}} \big(\gamma_1 \times sin(\theta)\big)\, d\theta - \int_{-\theta_{max}}^{0} \big(\gamma_2 \times sin(\theta)\big)\, d\theta) \quad (169)$$

W_b is exactly the brake (useful) work given per engine cycle by one cylinder, because it is taken by observing a working engine while all its parts are working together in order to give the final useful work.

The brake power is conventionally taken experimentally by a dynamometer or a a brake from the studied engine crankshaft, then the brake work is deduced, but in this case, the brake and the dynamometer consume a useful energy or stop completely the engine from working and being useful during the measure. Furthermore, it is a difficult and expensive method.

The formula proposed above can be used to find the most exact W_b by using angular detectors and a computer. The used computer will be the electronic integrator that can for example use the graph surface made for $\gamma_1(\theta) \times sin(\theta)$ during the step 1 (Θ between 0 and Θ_{max}) and the graph surface made for $\gamma_2(\theta) \times sin(\theta)$ during the step 2 (Θ between 0 and $-\Theta_{max}$). The two surfaces are made by the computer thanks to the detectors for $\dot{\theta}(\theta)$ and $\ddot{\theta}(\theta)$ at the connecting rod, but if we use φ as the variable of all the calculated final formulas by using the relations demonstrated above, then the detectors should be placed on the crankshaft. However, in this case we won't differentiate easily the performances of many engine cylinders that work at the same time.

Also, the terms:{ γ_A, $cos(\Theta)$, $sin(\Theta)$, $cos(2\Theta)$, $sin(2\Theta)$, $cos(4\Theta)$ } can be substituted by their demonstrated equivalents in terms of forces in the final formulas of each step when we can have enough data or a continuous input of the forces established on the studied point A by detectors of the forces strength placed on the engine working piston, but in this case the engine will stop to be useful during the measure of the detectors.

By using the detectors for $\dot{\theta}(\theta)$ and $\ddot{\theta}(\theta)$ at a connecting rod, the brake work W_b can be taken while the engine works and is being useful just like the method used for the measure of the indicated power. Consequently, its is a very useful method that can take continuously the brake work of any chosen cylinder without consuming a significant energy from the studied engine.

3.12. The method proposed to test the brake work of an engine cylinder:

The application to twin-cylinder engines is obvious, and it is also easy in the case of engines made up of two groups of cylinders where each cylinder operates simultaneously in the same way as the other members of its group (synchronized operation). In this case, all the cylinders are linked to the same rotating crankshaft. And thus, by using the demonstrated formulas, the measurement of the work given by a cylinder will be identical to all those of the N members of its group, it is W_1: One cylinder average brake work. Consequently, in order to find W_b: the exact brake work given by this cylinder, we just have to stop its injection during the engine operation, and then, by using the demonstrated formulas, we use a cylinder of its group in order to measure W_2: One cylinder new average brake work.

We deduce consequently that: $W_b = N \times W_1 - (N-1) \times W_2$ (170)

In the case of engines made up of more than two groups of N synchronized cylinders, when one group A finishes its useful phase that gives the useful work, at least one other useful group of cylinders doesn't. In this case, at least one other group participates partially and differently in producing the work measured from the group A. Consequently, in order to test a cylinder X of the group A, we study it during all its useful phase by using the demonstrated formulas. However, even if the measure of W_1 will be identical to all the cylinders of the group A, it is actually the sum of the average brake work of one cylinder of the group A and the average partial brake works of one representative of each other participating group that doesn't finish its useful phase. Hence, in order to test the cylinder X, we should, during the engine operation, stop its injection, but also stop the injection of the set of cylinders S composed of one representative of each other participating (useful) group that doesn't finish its useful phase. After that, we should use a cylinder of the same group of A in order to measure the new sum W_2.

Hence, we deduce that: $W_b = N \times W_1 - (N-1) \times W_2$ (171)

where W_b is the exact brake work given during the measure by all the cylinders with stopped injections. It is a sum of the cylinder X complete exact brake work and the partial exact brake works of each member of the set S (the other stopped cylinders).

This method doesn't give accurate characteristics about the tested cylinder X but it allows us to compare accurately the exact brake work of the cylinder X with the exact brake work of a cylinder Y that is member of the same group A. We only have to follow the same steps to measure W'_b that is the sum of the cylinder Y complete exact brake work and the partial exact brake works of each member of the same set S. Obviously, during the study of the cylinder Y for the measure of W'_b, we will have to stop the injection of the same set S of cylinders.

Finally, the difference between the cylinder X and the cylinder Y exact and complete brake works is:

$$\Delta W = W_b - W'_b \quad (172)$$

In both cases, the measurement of the work given by a cylinder will be identical to all those of the N synchronized members of its group. Consequently, this method allows to test the brake work of any cylinder by measuring the work of any representative of its group. Hence, we only have to choose

arbitrarily one cylinder of each group in order to place the detectors of the angle ϴ, the angular speed θ and the angular acceleration θ. However, we remind that it is the injection of the tested cylinder that should be stopped during the application of this method.

3.13. The half pinion crankshaft:

In order to make an easy performance study of an engine cylinders, the ordinary crankshaft of the studied engine can be substituted by the following half pinion crankshaft.

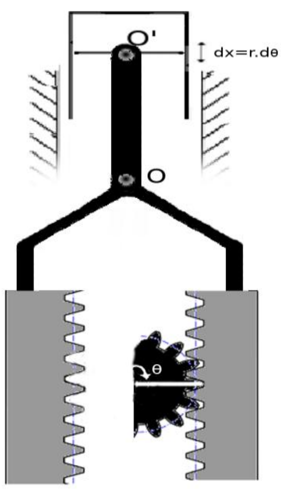

Figure 13. The system of The half pinion crankshaft

The mechanism is made of two toothed bars and a half pinion that is fixed with the crankshaft and never touches the vertical toothed bars both at the same time. The elementary displacement of the point O' is dx and the rotation of the crankshaft is the rotation of the half pinion ϴ where: dx=r.dϴ.

The half pinion system is so simple that its operation should be divided to only two steps as explained in the following simplified figures.

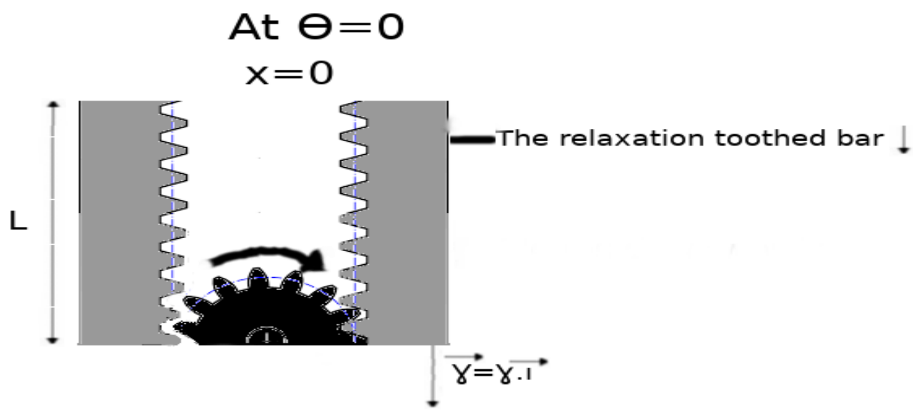

Figure 14. The system parts at the start of the cylinder gases relaxation.

In this figure, L is the length of each toothed bar. At this step $\Theta=0$ *and* x=0. Ɣ is the acceleration of the piston and thus Ɣ is also the acceleration of the relaxation toothed bar that is in-touch alone

with the half pinion.

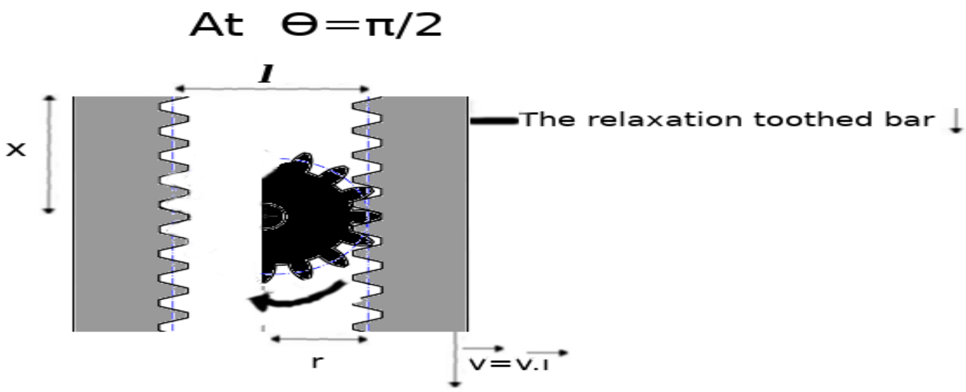

Figure 15. The system parts during the cylinder gases relaxation.

At this step, the relaxation toothed bar moves downwards with a velocity V and transmits the work to the crankshaft by making the contacted half pinion turn during the relaxation of the cylinder gases. The radius of the half circular pinion is r where l=2r.

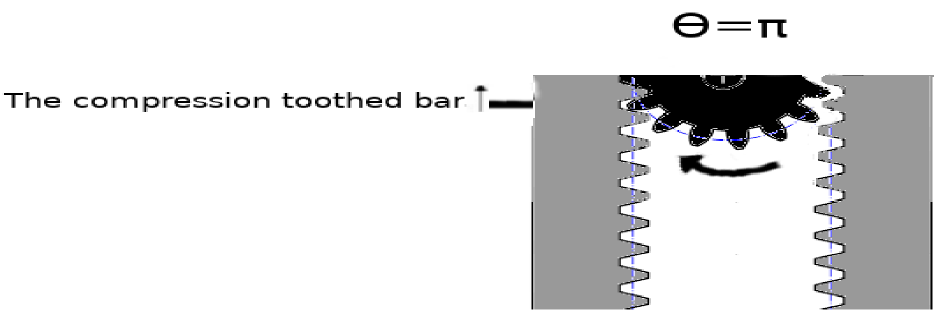

Figure 16. The system parts at the start of the compression of the cylinder air.

When x reaches the length L, the half pinion becomes in-touch with the compression toothed bar alone.

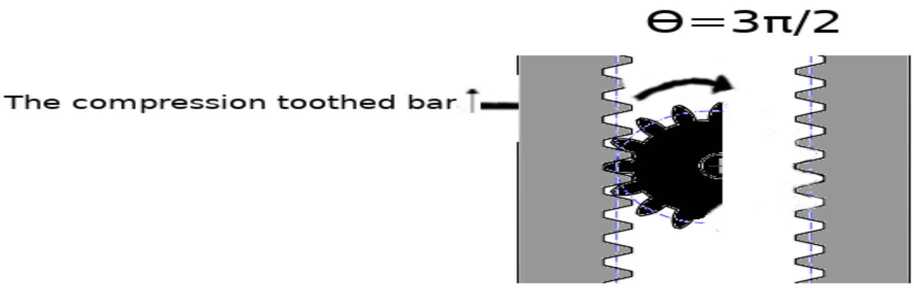

Figure 17. The system parts during the compression of the cylinder air.

At this step, The compression toothed bar moves upwards since it receives the work by the contacted half pinion that works like in a rack and pinion system in order to compress the air in the cylinder.

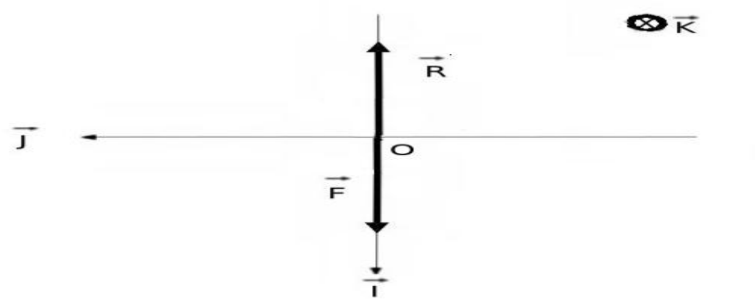

Figure 18. The forces established on the point O during the system operation.

By considering that L is also the cylinder stroke: $L = \frac{\pi \times l}{2}$ (173)

Consequently: $l = \frac{2L}{\pi} = 2r$ (174)

where: $r = \frac{L}{\pi}$ (175)

Θ always increases and thus always $\dot{\theta} > 0$.

F is the pressure force upon the piston that is entirely consumed thanks to the half pinion system and R is the resistance by the crankshaft to the force F.

3.14. Remark:

The mechanical work transmitted thanks to this system is mechanically maximal since the force F is always perpendicular to the crankshaft whereas the normal engine transmitted work changes by the

angle of the slanting connecting rod.

The half pinion and the two bars of the crankshaft could not stand the big load on the engine. Consequently, since the resistance of the materials used in the half pinion and the two bars is limited, this system is recommended to be used in the current generators where the load upon the engine doesn't increase dangerously.

The half pinion crankshaft can also be used to make highly efficient positive displacement pumps like piston pumps.

3.15. The engine cylinder availability:

The useful mechanical work of the first step where there is the gases relaxation in the studied cylinder is calculated such as the half pinion is half circular and always tangent to the bar that touches it and by considering that the half pinion has a high number of teeth.

By considering that W is the work given by one piston: $W = \int_0^{\pi} \big(F(\theta) - R(\theta)\big) \times r.d\theta$ (176)

Where: $F(\theta) = P(\theta) \times S$ (177)

such as P(Θ) is the pressure in the cylinder changing with Θ and S is the area of the piston head surface.

During the first step of the system operation, always$F > R$.

And since: $F - R = m.\gamma$ (178)

and $\gamma = r.\ddot{\theta}$ (179)

The useful work can also be written this way: $W = \int_0^{\pi} m \times r^2 \times \ddot{\theta}(\theta).d\theta = m \times r^2 \times \left(\dot{\theta}(\pi) - \dot{\theta}(0)\right)$ (180)

Where, in order to get a positive given work. we need $\dot{\theta}(\pi) > \dot{\theta}(0)$ (181)

Where m is the mass of the piston head and the two toothed rod.

$\dot{\theta}(\pi)$and $\dot{\theta}(0)$can be taken by two velocity detectors on the crankshaft.

or the useful work can be written this way: $W = \int_0^{L} \left(F\left(\frac{x}{r}\right) - R\left(\frac{x}{r}\right)\right).dx = m \times \int_0^{L} \gamma\left(\frac{x}{r}\right).dx = m \times r \times \left(V\left(\frac{L}{r}\right) - V(0^+)\right) = m \times r \times V\left(\frac{L}{r}\right)$ (182)

The integration is made only in the open interval]0,L[where $V = r \times \dot{\theta}$ (183)

V(0^+) is approximately zero at the starting of the gases relaxation in the studied cylinder.

$V\left(\frac{L}{r}\right)$ can be taken easily by one velocity detector on the relaxation toothed bar.

Remark: *If the system of the half pinion crankshaft works with very negligible friction, then the work W is exactly the availability of the studied engine cylinder (with piston).*

3.16. The second law efficiency:

By considering that W_b is the brake work given per engine cycle by one cylinder with the ordinary crankshaft, and W is the work given per engine cycle by the same cylinder to the half pinion crankshaft mechanism, the second law efficiency E of the studied engine cylinder is:

$$E = \frac{W_b}{W} = \frac{m \times l \times (\int_0^{\theta max} (\gamma_1 \times sin(\theta))d\theta - \int_{-\theta_{max}}^{0} (\gamma_2 \times sin(\theta))d\theta)}{m\prime \times r\prime \times V\left(\frac{L}{r\prime}\right)} = \frac{l}{r\prime} \cdot \frac{\int_0^{\theta max} (\gamma_1 \times sin(\theta))d\theta - \int_{-\theta_{max}}^{0} (\gamma_2 \times sin(\theta))d\theta}{V\left(\frac{L}{r\prime}\right)} \tag{184}$$

Where m is the mass of the piston head and its connecting rod, and m' is the mass of the piston head and the two toothed rod. The masses m and m' are considered equal. They can be made exactly equal for easy performance rate calculations. The radius of the half circular pinion is r' and the length of the connecting rod is l.

3.17. Conclusion

The second law efficiency E can be measured using this proposed method after the manufacturing of each system composed of a cylinder and a piston. Of course, the connected crankshaft will be replaced by the half pinion crankshaft during this study.

This study requires obviously that the engine stops of being exploited. However, this method gives the most exact result since most of the thermodynamic studies consider wrongly that the combustion gases are ideal gases or use only approximative thermodynamic formulas.

Thermodynamic experiments prove that E varies between 22 to 48 percent over the load range evaluated [9].

4. Summary and general conclusion:

This work started by describing the operation of the internal combustion engines and by demonstrating their geometric formulas related to Θ (the connecting rod angle of rotation) and φ (the crankshaft angle of rotation). Then the forces upon the piston head have been exposed in order to demonstrate the relations useful to substitute the angular formulas by the forces' formulas.

A new formula of the brake work W_b has been deduced after that in order to allow a continuous measure of this work without any significant energy consumption. This demonstrated brake work is useful to evaluate each cylinder apart in order to detect early and exactly the problems that may occur to a cylinder which allows to prevent the engine failure. This new formula can also allow to find easily and continuously the friction work W_f and the mechanical efficiency η_m for a given internal combustion engine.

Furthermore, the article exposes a useful method that allows to exploit the new brake work formula. This method steps can be easily made by professionals in order to know the state of their engines without stopping their operation. This method is very useful during the use of a big engine in energy stations or in large ships.

At the end, the half pinion crankshaft has been proposed as a new system that tests the availability at each cylinder without the thermodynamic difficult and expensive experiments especially that the engine gases are not ideal gases at very high pressures to make an easy thermodynamic study.

Acknowledgments

The author would like to thank The Higher Institute for Maritime Studies, Casablanca, Morocco For using their laboratories. The author reported no funding for this manuscript.

Conflict of interest

The authors declare that they have no conflict of interest

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