Odd Number of Coupled Antiferromagnetic Anisotropic Heisenberg systems: Linear Spin Wave Analysis

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The effect of anisotropies on the energy gap and magnetization for odd number of coupled quantum spin-1/2 antiferromagnetic anisotropic Heisenberg systems is investigated using the linear spin wave theory. However, for the chains, the enrgy gap opens above a critical anisotropy. While, for the planes, in the isotropic case, such system exhibits a lon-range order and no energy gap, whereas in the anisotropic case the energy gap opens above a critical anisotropic value. Known results of the isotropic case have been obtained.

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I. INTRODUCTION

The discovery of materials such as $(VO)_2P_2O_7$ [1] and $Sr_2Cu_4O_6$ [2] which contain weakly coupled arrays of metal-oxide-metal ladders, has stimulated interest in coupled chain Heisenberg, Habbard, and t-J systems.

From the theoretical point of view, it has been suggested that systems with an even number of isotropic chains are gapped spin liquids, while those with an odd number of isotropic chains are gapless[3]. This idea was supported by limited exact diagonalization results for three chains[4], mean field results for two and four chains[5], and density matrix renormalization group (DMRG) results for odd and even number of isotropic chains[6]. To our knowledge the anisotropic case has not been studied up to now.

The quasi-bidimensional quantum Heisenberg model has received considerable attention in recent literature, perhaps due to the advent of high-temperature superconductivity in the compounds $La_{2-x}Sr_xCuO_4$ and $YBa_2Cu_3O_{7-x}$. The spin pseudogap observed in underdoped $YBa_2Cu_3O_{7-x}$ is one of the fascinating characteristics of the high-Tc cuprates. Neutron scattering experiments showed the decrease of low-energy magnetic excitation with decreasing temperature and found the precursor of a finite spin gap[12]. It has been pointed out that these astonishing experimental results can be explained provided that there is a spin pseudogap in the normal state of high-Tc materials. These phenomena indicating the spin pseudogap, however, have not been observed in $La_{2-x}Sr_xCuO_4$ systems[13] Therefore it is speculated that the number of CuO_2 planes between the insulating planes is essential for the formation of this gap although a successful theory has not been presented[14-19]. Furthermore, the anisotropic properties are very important since the exchange interaction in real materials is anisotropic to some extent. For instance it is pointed out that a small anisotropy exists in $La_{2-x}Sr_xCuO_4$[20].

From the theoretical point of view, it has been suggested that sustems with two coupled isotropic planes are gapped spin liquids above a critical interplane coupling value[21,22]. The linear spin wave theory has been applied to various low dimensional systems[23-26]. As far as we know, however, this theory has not been applied to a model with an odd number of coupled antiferromagnetic anisotropic Heisenberg systems, in spite of the importance of this model.

This paper is a review of studies which are published in references 27 and 28. Our aim in this paper is to investigate the odd number of coupled quantum spin-1/2 antiferromagnetic anisotropic Heisenberg systems, using the linear spin wave theory[7].

II. ODD NUMBER OF COUPLED ANTIFERROMAGNETIC ANISOTROPIC HEISENBERG CHAINS

The antiferromagnetic anisotropic Heisenberg model on the system (Fig. 1) is denoted as follows,

$$\begin{aligned} H = J_{//}^{xy} \sum_{\langle i,j\rangle_{//}} \left(S_i^x S_j^x + S_i^y S_j^y + \alpha_{//} S_i^z S_j^z\right) \\ + J_{\perp}^{xy} \sum_{\langle i,j\rangle_{\perp}} \left(S_i^x S_j^x + S_i^y S_j^y + \alpha_{\perp} S_i^z S_j^z\right) \end{aligned} \tag{1}$$

where the sums run over nearest neighbours $\langle i,j\rangle_{//}$ along the chains and $\langle i,j\rangle_{\perp}$ perpendicular to the chains. $J_{//}^{xy}$ and $J_{\perp}^{xy}$ are the positive antiferromagnetic exchange constants. $\alpha_{//} = J_{//}^{z} / J_{//}^{xy}$ and $\alpha_{\perp} = J_{\perp}^{z} / J_{\perp}^{xy}$ are the chain and perpendicular anisotropy parameters, respectively.

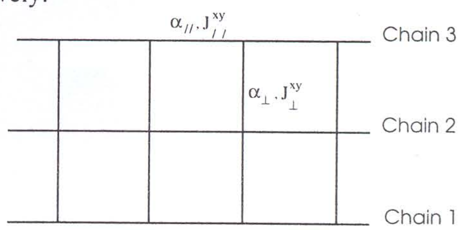

FIG. 1. The antiferromagnetic anisotropic system (Three coupled spin chains).

The antiferromagnetic spin wave theory by Anderson and Kubo[7] is applied to the Hamiltonian (1). Thus, we divide a lattice with NM sites (N is the sites number of the chain, and M is the anisotropic chains number) into two sublattices (A) and (B) such that each site of (A) is adjacent only to sites of (B). Then S_l and S_m denote the spin operators of the sublattices (A) and (B), respectively. The Holstein-Primakoff transformation applied to these operators. This transformation is defined by

$$S_l^z = S - a_l^+ a_l, \quad S_l^- = \sqrt{2S}\, a_l^+ \sqrt{1 - a_l^+ a_l / 2S},$$
$$S_m^z = -S + b_m^+ b_m, \quad S_m^- = \sqrt{2S}\sqrt{1 - b_m^+ b_m / 2S}\, b_m, \tag{2}$$

where a_l, a_l^+, b_m and b_m^+ are the creation and annihilation operators of spin deviations for sublattices (A) and (B), respectively, satisfying the Bose commutation relations, namely

$$\left[a_l, a_{l'}^+\right] = \delta_{l,l'}, \quad \left[b_m, b_{m'}^+\right] = \delta_{m,m'},$$
$$\left[a_l, b_m^+\right] = \left[a_l^+, b_m\right] = \left[a_l, b_m\right] = \left[a_l^+, b_m^+\right] = 0. \tag{3}$$

We substitute eq. (2) into eq. (1) and keep the part of only lowest order terms is denoted by H^{LSWT}. Then

$$H^{LSWT} = S \sum_{\substack{i=1,\dots,N \\ j=1,2,3}} \Bigg[\left(a_{i,j} b_{i+1,j} + a_{i,j}^+ b_{i+1,j}^+\right) + \alpha_{//}\left(a_{i,j}^+ a_{i,j} + b_{i+1,j}^+ b_{i+1,j}\right)\Bigg]$$
$$+ \alpha^{xy} S \sum_{i=1,\dots,N} \Bigg[\Big(a_{i,1} b_{i,2} + a_{i,3} b_{i,2} + a_{i,1}^+ b_{i,2}^+ + a_{i,3}^+ b_{i,2}^+\Big) + \alpha_\perp \Big(a_{i,1}^+ a_{i,1} + a_{i,3}^+ a_{i,3} + 2 b_{i,2}^+ b_{i,2}\Big)\Bigg] \tag{4}$$
$$- 3NS^2 \alpha_\perp - 2NS^2 \alpha^{xy} \alpha_\perp,$$

where α^{xy} $(= J_\perp^{xy} / J_{//}^{xy})$ is the anisotropy parameter.

We apply the Fourier transformation

$$a_l = \sqrt{\frac{2}{MN}} \sum_k e^{-ikl} a_k, \quad b_m = \sqrt{\frac{2}{MN}} \sum_k e^{ikm} b_k, \tag{5}$$

where k is a vector in a reciprocal lattice of a sublattice. Using eq. (5) the commutation relations become

$$\left[a_k, a_{k'}^+\right] = \delta_{k,k'}, \quad \left[b_k, b_{k'}^+\right] = \delta_{k,k'}, \tag{6}$$

$\left[a_k, b_{k'}^+\right] = \left[a_k^+, b_{k'}\right] = \left[a_k, b_{k'}\right] = \left[a_k^+, b_{k'}^+\right] = 0.$ Then eq. (4) is given by

$$H^{LSWT} = 2S \sum_k \Bigg[\left(\alpha_{//} + \frac{2}{3}\alpha^{xy}\alpha_\perp\right)\left(a_k^+ a_k + b_k^+ b_k\right) + \left(\cos(k_x) + \frac{2}{3}\alpha^{xy}\cos(k_y)\right)\left(a_k^+ b_k^+ + a_k b_k\right)\Bigg]$$
$$- 3NS^2\left(\alpha_{//} + \frac{2}{3}\alpha^{xy}\alpha_\perp\right). \tag{7}$$

Therefore, eq. (7) can be generalized to an arbitrary odd number of coupled anisotropic chains. So, we get

$$H^{LSWT} = 2S \sum_k \Bigg[\left(\alpha_{//} + \frac{M-1}{M}\alpha^{xy}\alpha_\perp\right)\left(a_k^+ a_k + b_k^+ b_k\right) + \left(\cos(k_x) + \frac{M-1}{M}\alpha^{xy}\cos(k_y)\right)\left(a_k^+ b_k^+ + a_k b_k\right)\Bigg]$$
$$- MNS^2\left(\alpha_{//} + \frac{M-1}{M}\alpha^{xy}\alpha_\perp\right). \tag{8}$$

This Hamiltonian can be diagonalized by the Bogoliubov transformation

$$U = \exp\left[-\sum_k \theta_k \left(a_k^+ b_k^+ - a_k b_k\right)\right],$$
$$\alpha_k = U a_k U^+ = a_k \cosh\theta_k + b_k^+ \sinh\theta_k, \tag{9}$$
$$\beta_k = U b_k^+ U^+ = a_k \sinh\theta_k + b_k^+ \cosh\theta_k.$$

As eq. (9) is unitary transformation, the commutation relations (6) are preserved, namely

$$\left[\alpha_k, \alpha_{k'}^+\right] = \delta_{k,k'}, \quad \left[\beta_k, \beta_{k'}^+\right] = \delta_{k,k'}, \tag{10}$$
$$\left[\alpha_k, \beta_{k'}^+\right] = \left[\alpha_k^+, \beta_{k'}\right] = \left[\alpha_k, \beta_{k'}\right] = \left[\alpha_k^+, \beta_{k'}^+\right] = 0.$$

Then Hamiltonian (8) becomes

$$H^{LSWT} = 2S \sum_k \Bigg[-2S\left(\alpha_{//} + \frac{M-1}{M}\alpha^{xy}\alpha_\perp\right) + W^{LSWT}(k)\left(\alpha_k^+ \alpha_k + \beta_k^+ \beta_k\right) + W^{LSWT}(k)\Bigg]$$
$$- MNS^2\left(\alpha_{//} + \frac{M-1}{M}\alpha^{xy}\alpha_\perp\right), \tag{11}$$

where the dispersion relation is given by

$$W^{LSWT}(k) = 2S\left[\left(\alpha_{//} + \frac{M-1}{M}\alpha^{xy}\alpha_{\perp}\right)^2 - \left(\cos(k_x) + \frac{M-1}{M}\alpha^{xy}\cos(k_y)\right)^2\right]^{1/2} \quad (12)$$

Therefore α_k, α_k^+, β_k and β_k^+ are the creation and annihilation operators of the elementary excitation and the ground state energy is their vacuum state. Then the spin deviation averaged in the ground state, denoted by ΔS, has the form

$$\Delta S = \left\langle a_1^+ a_1 \right\rangle = -\frac{1}{2} + \frac{1}{2}\int_{-\pi}^{\pi}\frac{dk_x}{2\pi}\left(\frac{1}{M}\sum_{\substack{k_y=-\pi+2j\pi/M \\ (j=1,\dots,M)}} \times \frac{2S(\alpha_{//} + [(M-1)/M]\alpha^{xy}\alpha_{\perp})}{W^{LSWT}(k_x,k_y)}\right). \quad (13)$$

The ground state energy per spin is

$$\frac{E_{gs}^{LSWT}}{NM} = \frac{1}{2}\int_{-\pi}^{\pi}\frac{dk_x}{2\pi}\left(\frac{1}{M}\sum_{\substack{k_y=-\pi+2j\pi/M \\ (j=1,\dots,M)}} W^{LSWT}(k_x,k_y)\right) - S(S+1)\left(\alpha_{//} + \frac{M-1}{M}\alpha^{xy}\alpha_{\perp}\right). \quad (14)$$

The energy gap has the form

$$E_g = W^{LSWT}(k_x = 0, k_y = 0) = 2S\left[\left(\alpha_{//} + \frac{M-1}{M}\alpha^{xy}\alpha_{\perp}\right)^2 - \left(1 + \frac{M-1}{M}\alpha^{xy}\right)^2\right]^{1/2}.$$

From Fig. 2, the spin wave theory results for the quantum spin-1/2 antiferromagnetic anisotropic Heisenberg chain are in good agreement with the known results given by Parkinson et al.[8]. Indeed, the energy gap has a linear (15) behaviour for $\alpha_{//} \geq \alpha_{//}^{0} = 3$ with the estimated critical value of the anisotropy $\alpha_{//}$ above which the spin wave theory estimate of the energy gap is accurate; while for $1 \leq \alpha_{//} \leq \alpha_{//}^{0}$, it has a simple power law form,

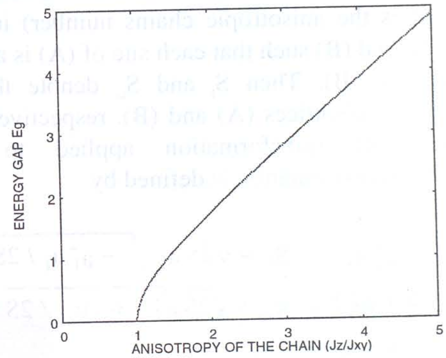

FIG. 2: The dependence of the energy gap Eg on the anisotropy $\alpha_{//}$.

$$E_g \approx c(\alpha_{//} - 1)^g, \quad (16)$$

where $c \approx \sqrt{2}$ is a constant and the exponent g=1/2.

This sipin deviation, which is displayed in Fig. 3, shows that the three coupled chains system is disordered for any value of α^{xy} at fixed values of the chain and perpendicular anisotropies, $\alpha_{//} = \alpha_{\perp} = 1$. While for M>3, the systems exhibits two finite critical anisotropy values $\alpha_{c_1}^{xy}$ and $\alpha_{c_2}^{xy}$; for $\alpha_{c_1}^{xy} < \alpha^{xy} < \alpha_{c_2}^{xy}$ the system is ordered, but elsewhere it is disordered. Furthermore, for infinite M, we get the case of the square lattice which presents a finite anisotropic value $\alpha_c^{xy} = 0.033$ below which the square lattice is disordered, while it is ordered for $\alpha^{xy} > \alpha_c^{xy}$. These results are in excellent agreement with those of Saki et al.[9]. In addition a relevant result is also obtained; the spin deviation diverges for a single chain, while it is finite for any finite odd number of chains.

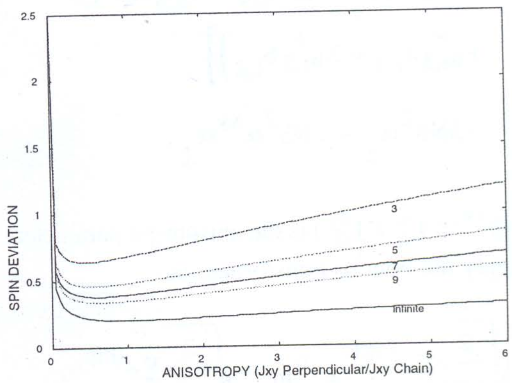

FIG. 3. The dependence of the spin deviation ΔS at fixed values of the chain and perpendicular anisotropies ($\alpha_{//} = \alpha_{\perp} = 1$). The number accompanying each curve denotes the number of coupled chains.

The ground state energy given in eq. (14) decreases with increasing value of the anisotropy α^{xy}. For $\alpha^{xy}=0$, we recover the usual spin-wave energy per spin for the isotropic chain ($E_{gs}^{LSWT}=-0.4317$) rather close to the exact result ($E_{gs}=-.4432$)[10]. At the point $\alpha^{xy}=1$, for infinite M, we get $E_{gs}^{LSWT}=-0.658$ which is in qualitative agreement with Monte Carlo calculations ($E_{gs}=-0.669$)[11].

The effect of the $\alpha_{//}$ and $\alpha_{\perp}$ anisotropies on the energy gap is given in Fig. 4. Here there exists a critical anisotropic value ($\alpha_c^{xy}=[(1-\alpha_{//})/(\alpha_{\perp}-1)]M/(M-1)$) for each anisotropy value $\alpha_{\perp}>1$ at a fixed value $\alpha_{//}\leq 1$, from which the energy gap opens. The energy gap has a linear behaviour for $\alpha^{xy}>>\alpha_c^{xy}$, while near α_c^{xy} it has a simple power law form, namely

$$E_g(\alpha^{xy})=C_1(\alpha^{xy}-\alpha_c^{xy})^{g_1}\quad , \qquad (17)$$

where the constant is given by

$$C_1\approx\left[\frac{M-1}{M}(\alpha_{\perp}-1)(\alpha_{//}+1\right.$$
$$\left.+\alpha_c^{xy}\frac{M-1}{M}(\alpha_{\perp}+1))\right]^{1/2}$$

and the exponent $g_1=1/2$. To locate the region where the power law of the energy gap eq. (17) as a function of the anisotropy α^{xy} is valid,

we have solved the inequality $\left|[E_g-E\ (\alpha^{xy})]/E_g\right|\leq\varepsilon$, where ε is an estimated value (in our calculation $\varepsilon\approx 10^{-5}$). Then for $\alpha_c^{xy}\leq\alpha^{xy}\leq\alpha_0^{xy}$ the energy gap has a simple power law, eq. (17), while for $\alpha^{xy}\geq\alpha_0^{xy}$ the energy gap has a linear behaviour. So, α_0^{xy} is a function of $\alpha_{//}$ and $\alpha_{\perp}$. This dependence is represented in Fig. 5. However, α_0^{xy} decreases when increasing $\alpha_{\perp}$ and/or increasing $\alpha_{//}$. Then the critical anisotropic value α_c^{xy} decreases with increasing perpendicular anisotropy $\alpha_{\perp}$. For $\alpha_{//}=\alpha_{\perp}=1$, the energy gap vanishes. This results is in excellent agreement with the density matrix renormalization group (DMRG) results[6].

Equivalent results are obtained for any odd number of anisotropic chains. However, the anisotropic chain number favours the energy gap opening much more at fixed values of the $\alpha_{//}$ and $\alpha_{\perp}$ anisotropy parameters.

In conclusion, using the linear spin wave theory we have studied the energy gap for an odd number of anisotropic chains. The energy gap opens for $\alpha^{xy}>\alpha_c^{xy}$. The known results of the isotropic case have been obtained.

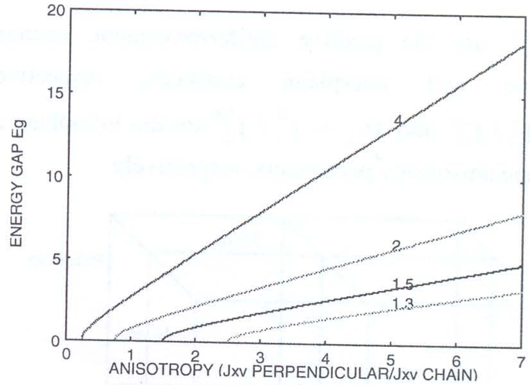

FIG. 4. The dependence of energy gap E_g on the anisotropy α^{xy} at a fixed value of the chain anisotropy ($\alpha_{//}=0.5$). The number accompanying each curve denotes the value of the perpendicular anisotropy $\alpha_{\perp}$.

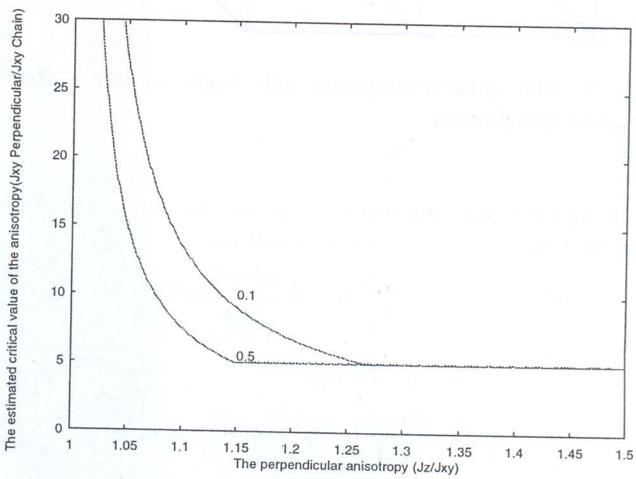

FIG. 5. The dependence of the estimated critical value of the anisotropy α_0^{xy} on the perpendicular anisotropy $\alpha_{\perp}$ at a fixed value of the chain anisotropy $\alpha_{//}$. The number accompanying each curve denotes the value of the chain anisotropy $\alpha_{//}$.

III. ODD NUMBER OF COUPLED ANTIFERROMAGNETIC ANISOTROPIC HEISENBERG PLANES

The quasi-bidimensional antiferromagnetic anisotropic Heisenberg model on the system (Fig. 6) is denoted as follows,

$$H=J_{//}^{xy}\sum_{\langle i,j\rangle_{//}}\left(S_i^xS_j^x+S_i^yS_j^y+\alpha_{//}S_i^zS_j^z\right)$$
$$+J_{\perp}^{xy}\sum_{\langle i,j\rangle_{\perp}}\left(S_i^xS_j^x+S_i^yS_j^y+\alpha_{\perp}S_i^zS_j^z\right)\qquad (18)$$

where the sums run over nearest neighbours $\langle i,j\rangle_{//}$ along the planes and $\langle i,j\rangle_{\perp}$ perpendicular to the planes. $J_{//}^{xy}$

and $J_{\perp}^{xy}$ are the positive antiferromagnetic exchange intraplane and interplane constants, respectively. $\alpha_{//} = J_{//}^{z} / J_{//}^{xy}$ and $\alpha_{\perp} = J_{\perp}^{z} / J_{\perp}^{xy}$ are the intraplane and interplane anisotropy parameters, respectively.

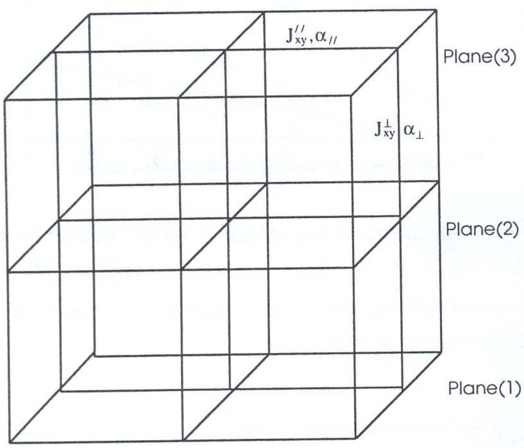

FIG. 6: The antiferromagnetic anisotropic system (three coupled spin planes).

Using the linear spin wave theory defined above, eq. (18) can be wtitten in Fourier space as follows

$$\begin{aligned} H^{LSWT} = 4 J_{//}^{xy} S \sum_{k} \Bigg[& \left(\alpha_{//} + \frac{M-1}{2M} \alpha^{xy} \alpha_{\perp} \right) \\ & \times \left(a_k^{+} a_k + b_k^{+} b_k \right) \\ & + \left(\gamma_k^{//} + \frac{M-1}{2M} \alpha^{xy} \gamma_{k)}^{\perp} \right) \left(a_k^{+} b_k^{+} + a_k b_k \right) \Bigg] \\ & - MN J_{//}^{xy} 2S^2 \left(\alpha_{//} + \frac{M-1}{2M} \alpha^{xy} \alpha_{\perp} \right). \end{aligned} \tag{19}$$

This Hamiltonian can be diagonalized by Bogoliubov transformation defined in section (2). So, eq. (19) becomes

$$\begin{aligned} H^{LSWT} = \sum_{k} \Bigg[& -4 J_{//}^{xy} S \left(\alpha_{//} + \frac{M-1}{M} \alpha^{xy} \alpha_{\perp} \right) \\ & + W^{LSWT}(k) \left(\alpha_k^{+} \alpha_k + \beta_k^{+} \beta_k \right) + W^{LSWT}(k) \Bigg] \\ & - MN J_{//}^{xy} 2S^2 \left(\alpha_{//} + \frac{M-1}{M} \alpha^{xy} \alpha_{\perp} \right), \end{aligned} \tag{20}$$

where the dispersion relation is given by

$$\begin{aligned} W^{LSWT}(k) = 4 J_{//}^{xy} S \Bigg[& \left(\alpha_{//} + \frac{M-1}{2M} \alpha^{xy} \alpha_{\perp} \right)^2 \\ & - \left(\gamma_k^{//} + \frac{M-1}{2M} \alpha^{xy} \gamma_k^{\perp} \right)^2 \Bigg]^{1/2}. \end{aligned} \tag{21}$$

The spin deviation averaged in the groud state has the form

$$\begin{aligned} \Delta S = \left\langle a_l^{+} a_l \right\rangle = -\frac{1}{2} + \frac{1}{2} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \frac{dk_x dk_y}{(2\pi)^2} \Bigg(& \frac{1}{M} \\ \sum_{\substack{k_z = -\pi + 2j\pi/M \\ (j=1,\dots,M)}} & \frac{4 J_{//}^{xy} S (\alpha_{//} + [(M-1)/2M] \alpha^{xy} \alpha_{\perp})}{W^{LSWT}(k_x, k_y)} \Bigg). \end{aligned} \tag{22}$$

The ground state energy per spin is

$$\begin{aligned} \frac{E_{gs}^{LSWT}}{NM} = \frac{1}{2} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \frac{dk_x dk_y}{(2\pi)^2} \Bigg(& \frac{1}{M} \\ & \sum_{\substack{k_z = -\pi + 2j\pi/M \\ (j=1,\dots,M)}} W^{LSWT}(k) \Bigg) \\ & - S(S+1) \left(\alpha_{//} + \frac{M-1}{2M} \alpha^{xy} \alpha_{\perp} \right). \end{aligned} \tag{23}$$

The energy gap has the form

$$\begin{aligned} E_g &= W^{LSWT}(k_x = \pi, k_y = \pi, k_z = \pi) \\ &= 4 J_{//}^{xy} S \Bigg[\left(\alpha_{//} + \frac{M-1}{2M} \alpha^{xy} \alpha_{\perp} \right)^2 \\ & \quad - \left(1 + \frac{M-1}{2M} \alpha^{xy} \right)^2 \Bigg]^{1/2}. \end{aligned} \tag{24}$$

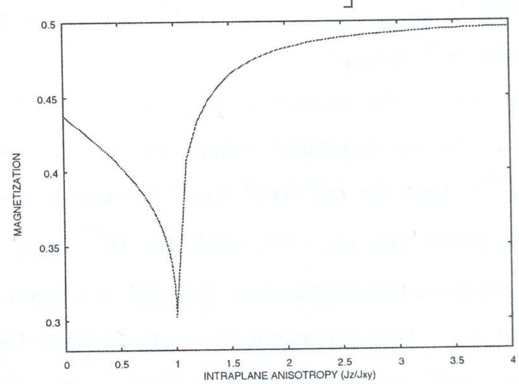

FIG. 7: The dependence of the magnetization on the intraplane anisotropy $\alpha_{//}$.

From Fig. 7, the spin wave theory results for quantum spin-1/2 antiferromagnetic anisotropic Heisenberg plane give two excitation specta $W^{LSWT} = 4 J_{//}^{xy} S (\alpha_{//}{}^2 - \gamma_k^{//2})^{1/2}$ and $W^{LSWT} = 2 J_{//}^{xy} S [(1 - \gamma_k^{//})(1 + \alpha_{//} \gamma_k^{//})]^{1/2}$ which correspond to $\alpha_{//} \geq 1$ and $\alpha_{//} \leq 1$, respectively. These results of the magnetization are in good qualitative agreement with the known results obtained from Monte Carlo simulations[26] and the schwinger Boson mean field theory[27] for $\alpha_{//} \leq 1$. Indeed the plane is ordered for any value of the intraplane anisotropy $\alpha_{//}$. For $\alpha_{//} \leq 1$ the magnetization decreases with increasing $\alpha_{//}$, while for $\alpha_{//} \geq 1$ it increases with increasing $\alpha_{//}$. However, the plane is gapless in the

isotropic case $\alpha_{//} = 1$, this result is in excellent agreement with those obtained by Parola[11]; while in the anisotropic case $\alpha_{//} \neq 1$, it exhibits an energy gap (Fig. 8). Indeed, the energy gap has a linear behaviour for $\alpha_{//} >> 1$ and for $\alpha_{//} < 1$ the energy gap decreases with increasing $\alpha_{//}$; while near $(\alpha_{//})_c = 1$, it has a simple power law form,

$$E_g(\alpha_{//}) \approx c(\alpha_{//} - 1)^g \qquad (25)$$

where $c = 2\sqrt{2}$ is the constant and the exponent g=1/2.

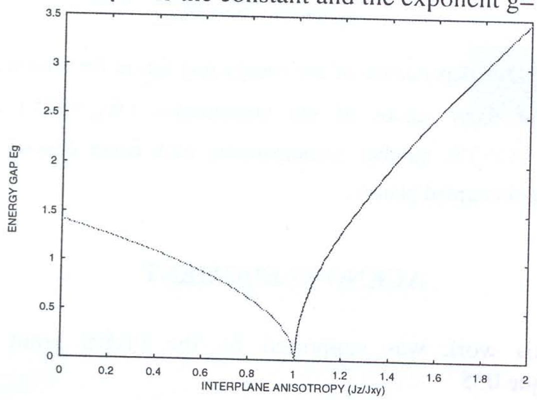

Fig. 8.: The dependence of the energy gap Eg on the intraplane anisoptropy

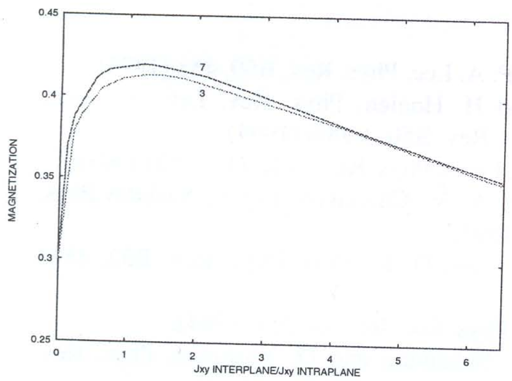

FIG. 9: The dependence of the magnetization at fixed values on the interplane and intraplane anisotropies ($\alpha_{//} = \alpha_{\perp} = 1$). The number accompanying each curve denotes the number of coupled planes.

The magnetization, which is displayed in Fig. 9, shows that the odd number of coupled planes system is ordered for any value of α^{xy} at fixed values of the interplane and intraplane anisotropies, $\alpha_{//} = \alpha_{\perp} = 1$. Furthemore, for infinite M, we get the case of a simple cubic lattice which presents a long-range order for any value of α^{xy} with $\alpha_{//} = \alpha_{\perp} = 1$. Such result is in good agreement with experimental results obtained in three-dimensional systems[28]. Especially, we have suggested a conjecture from this study on systems type in analogy from a prediction of even and odd coupled isotropic chains given by White et al.[6]. Our suggestion is written as follows: For odd number of coupled isotropic planes, there is a long-range order and no gap, while for even number of coupled planes, there a transition from ordered phase to disordered phase with a gap in the disordered phase; these latter results (two coupled planes) are given by a variety of analytical studies[21,22,25].

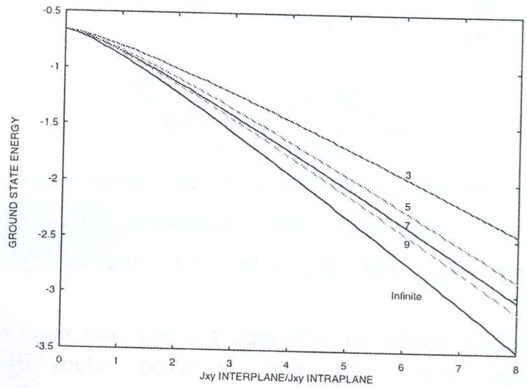

FIG. 10: The dependence of the ground state energy on the anisotropy α^{xy} at fixed values of the interplane and intraplane anisotropies ($\alpha_{//} = \alpha_{\perp} = 1$). The number accompanying each curve denotes the number of coupled planes.

The ground state energy given in Fig. 10 decreases with an increasing number of planes, or an increasing value of the anisotropy α^{xy}. For $\alpha^{xy} = 0$, we recover the usual spin-wave energy per spin for the isotropic plane ($E_{gs}^{LSWt} = -0.658$) which is in qualitative agreement with numerical Monte Carlo calculations ($E_g = -0.669$)[11].

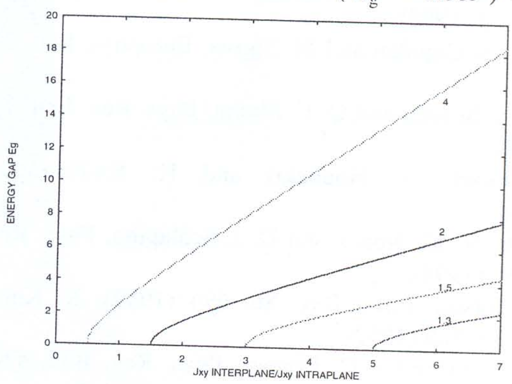

FIG. 11: The dependence of the energy gap Eg on the anisotropy α^{xy} at fixed values of the intraplane anisotropy ($\alpha_{//} = 0.5$) and M=3. The number accompanying each curve denotes the value of the interplane anisotropy.

The effect of the $\alpha_{//}$ and $\alpha_{\perp}$ anisotropies on the energy gap is given in Fig. 11. There exists a critical anisotropic value $[\alpha_c^{xy} = (1 - \alpha_{//} / \alpha_{\perp} - 1)(2M / M - 1)]$ for each anisotropy value $\alpha_{\perp} > 1$ at a fixed value $\alpha_{//} \leq 1$, from which the energy gap opens. The energy gap opens. The energy gap has a linear behaviour for

$\alpha^{xy} >> \alpha_c^{xy}$, while near α_c^{xy}, it has a simple power law form, namely

$$E_g(\alpha^{xy}) \approx c_1(\alpha^{xy} - \alpha_c^{xy})^{g_1} \qquad (26)$$

where the constant is given by

$$C_1 = 2\left[\frac{M-1}{2M}(\alpha_\perp - 1)(\alpha_{//} + \alpha_c^{xy}\frac{M-1}{2M}(\alpha_\perp + 1))\right]^{1/2}$$

and the exponent g_1=1/2. However, the critical anisotropic value α_c^{xy} decreases with increasing $\alpha_\perp$ and/or increasing $\alpha_{//}$. For $\alpha_{//} = \alpha_\perp = 1$, the energy gap vanishes.

equivalent results are obtained for any odd number of anisotropic planes. However, at fixed values of the anisotropies $\alpha_{//}$ and $\alpha_\perp$, an inctreasing number of anisotropic planes favours the opening of the energy gap (Fig. 12).

In conclusion, using a linear spin-wave theory we have studied the energy gap for odd number of coupled anisotropic planes. The energy gap opens for $\alpha^{xy} > \alpha_c^{xy}$. Results of both single plane and simple cubic systems have been obtained

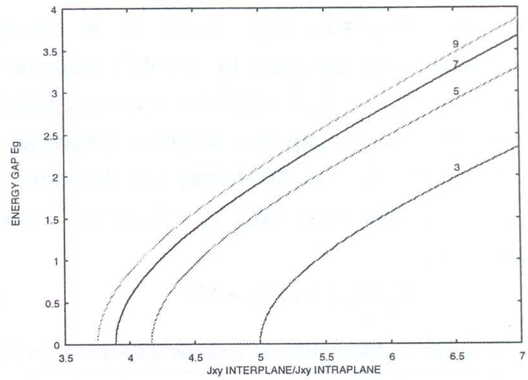

FIG. 12: The dependence of the energy gap Eg on the anisotropy α^{xy} at fixed values of the anisotropies ($\alpha_{//} = 0.5$) and ($\alpha_\perp = 1.3$). The number accompanying each curve denotes the number of coupled planes.

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