Identification of Materials' Mechanical parameters

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Abstract: The purpose of this study is to set the numerical bases before approaching the study of the identification of the mechanical parameters of materials' elastic behavior.
Indeed, most part of the mechanical tests do not allow to identify these mechanical parameters, the use of finite elements method calculations to design structures is limited by a poor knowledge of the mechanical properties. It is in this context that arises the inverse analysis problematic [1] [2]. For the parameters of the behaviour laws of material ; what information can be obtained from the in situ measures ? Besides what are the numerical techniques needed to obtain a determination of these parameters precisely and systematically.
In this work we present a new way of proceeding by proposing an easy useful formulation by a treatment of the inverse problem. The problem so found is a differential system instead of a partial derivative problem.
The resolution of the direct problem leads to obtain convincing results. These latters are in agreement with the simulation by a commercial code. This will allow us afterward to approach, without apprehension, the inverse problem. This is achieves through proposing a technique of systematic identification by using the database beforehand definite [3].

Keywords: mechanical parameters, identification, direct problem, inverse problem.

Introduction

The mechanical properties of materials are of great interest since they affect not only the forming problems, but also their behavior in extremely diverse industrial applications. The choice of material in any industrial part depends on their mechanical properties, strength, hardness, ductility... It is thus necessary to measure these physical sizes by mechanical tests.
The precise determination of these properties is of growing interest of the scientists. Indeed, in the field of the mechanical knowledge and design, the mechanical properties of materials are necessary for a more realistic modelling. Firstly, we will treat the direct problem in two dimensions by the finite element method. This will help us to simplify the setting of the equilibrium equation in a differential equation easily resolvable and useful for the inverse problem. This can be extended to materials of more complex behaviour such as composites.
The mechanical characterization of materials is a precondition for the establishment of bases required in mechanical tests. The objective is to elaborate a simple, fast, reliable and non-destructive protocol.

Position of the problem

Let's consider a homogeneous solid ; a rectangular plate, with thickness e, length L and width h. The distribution of displacements in the plate is estimated in 2-D. The plate occupies the interval [0, L] of the Ox axis and [0, h] of the Oy axis [5, 8].

Weak Formulation

In linear elasticity the behavior law has the following shape:

$$y = Ax = \lambda tr(x)e + 2\mu x$$

A simple calculation leads to determine the matrix of this law in an adequate base formed by one deviatoric and two spherical matrix.

$$A = \begin{bmatrix} 2\mu & 0 & 0 \\ 0 & 2\mu & 0 \\ 0 & 0 & 2(\lambda+\mu) \end{bmatrix}$$

Where λ and μ are the Lamé coefficients.

Matrix Formulation

By using the Voigt matrix notation [6, 7], the displacement, deformation and stress are noted respectively:

$$u=\begin{pmatrix}u_x\\u_y\end{pmatrix},\varepsilon=\begin{pmatrix}\varepsilon_x\\\varepsilon_y\\2\varepsilon_{xy}\end{pmatrix},\ \sigma=\begin{pmatrix}\sigma_x\\\sigma_y\\\sigma_{xy}\end{pmatrix}$$

The resolution of a problem of plane elasticity consists in looking for the displacement field [9].

The energy formulation of the problem to be resolved can be returned to a variational formulation where the deformation energy is:

$$\delta w=\int_v \delta\varepsilon_{ij}\sigma_{ij}dv=\int_v\{\delta\varepsilon\}^t\{\sigma\}dv=\int_v\{\delta u\}^t[\partial]^t[A][\partial]\{u\}dv$$

$[\partial]$ is the derivatif operator written also under matrix shape.

And the work of the external forces:

$$\delta\tau=\int_v f_i\delta u_i dv+\int_{s\sigma}\delta u_i F_i ds \text{ with } F_i=\sigma_{ij}n_j$$

We are thus looking for a solution approached by the finite elements methode which consists in discretizing the domain in finite elements:

$$\Omega=\bigcup_{e=1}^{ne}\Omega_e$$

Vectors and matrix at the level of the element:

$\{u\}=[N]\{q^e\}$, where $\{q^e\}$ are the nodal displacements, $[N]$ the elementary matrix of interpolation functions and $\{u\}$ the displacement in the domain.

The expression of the deformation energy allowed us to write the elementary rigidity matrix :

$\left[k^e\right]=\left[B^e\right]^t[A]\left[B^e\right]\dfrac{J}{2}$, with J the jacobian and $\left[B^e\right]=[\partial][N]$ is a matrix gradient of interpolation functions.

The assembly at the level of the whole structure can be made by introducing the location matrices $\left[P^e\right]$ relatif a shape (e).

$$\{q^e\}=\left[P^e\right]\{Q\}$$

$\{Q\}$ is the displacement of the structure level.

Thus :

$$\left[\delta q^e\right]=\left[P^e\right][\delta Q]$$

The elementary work at the level of the element of the strucutre

$$\delta w^e=\{\delta q\}^t\left[P^e\right]^t\left[k^e\right]\left[P^e\right]\{Q\}$$

One can set:

$$\left[K^e\right]=\left[P^e\right]^t\left[k^e\right]\left[P^e\right]$$

as the elementary rigidity matrix at the level of the global structure.

The global energy of deformation is written as:

$$\delta w=\sum_e\delta w^e=\{\delta q\}^t\sum_e K^e\{Q\}$$

After assembly the global rigidity matrix is:

$$[K]=\sum_e K^e=\sum_e\left[P^e\right]^t\left[k^e\right]\left[P^e\right]$$

The force vector due to a surfasic force of intensity $\{f^e\}$ applied to one of the borders of an element is equal to:

$$\{f^e\}=e\int_{-1}^{1}\left[N(\xi)\right]^T p\frac{1}{J_s}\begin{Bmatrix}J_y\\-J_x\end{Bmatrix}J_s d\xi$$

$$J_x=\frac{x_2-x_1}{2},\ J_y=\frac{y_2-y_1}{2},$$

$$\{f\}=\frac{p}{J_s}\begin{Bmatrix}J_y\\-J_x\end{Bmatrix}$$

By taking three nodes triangles as elements and p the distribution of the force by unit surface on the border.

We suppose in our case $p=cte$ and $\begin{Bmatrix}J_x\\J_y\end{Bmatrix}$ constant

$$\{f\}=ep\begin{Bmatrix}J_y\\-J_x\\J_y\\-J_x\end{Bmatrix}$$

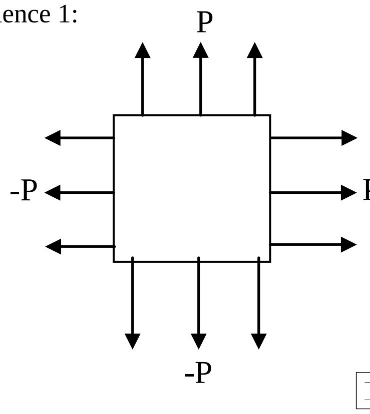
After assembly we find the total force:

$$\{F\}=\sum_e\left[P^e\right]^T\{f\}$$

Finally, the determination of the displacement field means resolving the linear differential equation $[K]\{q\}=\{F\}$. This equation is much more accessible from its simplicity. The validation was applied to several simple cases. The comparison with results given by a commercial code (Castem) confirms our approach (Figures1-6).

Results

Experience 1:



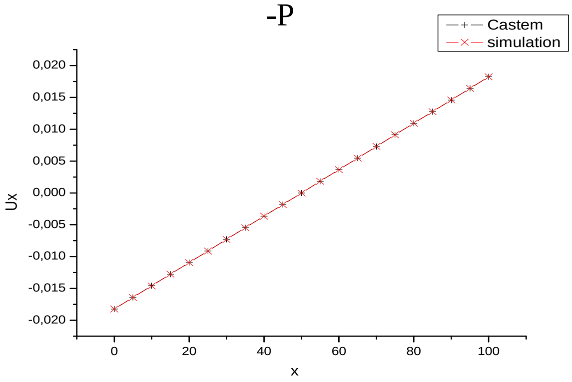


Fig 1. Axial Displacement function of x

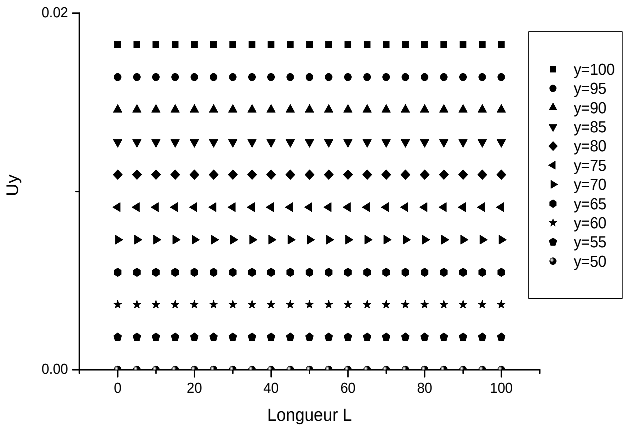
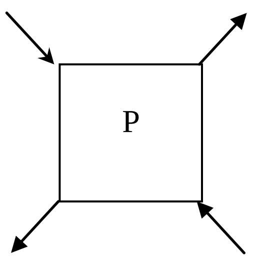


Fig 2. Displacement along y axis

Experience 2:



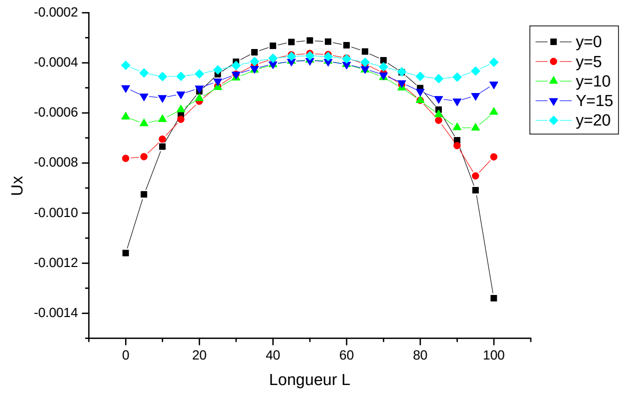


Fig 3. Axial displacement function of x

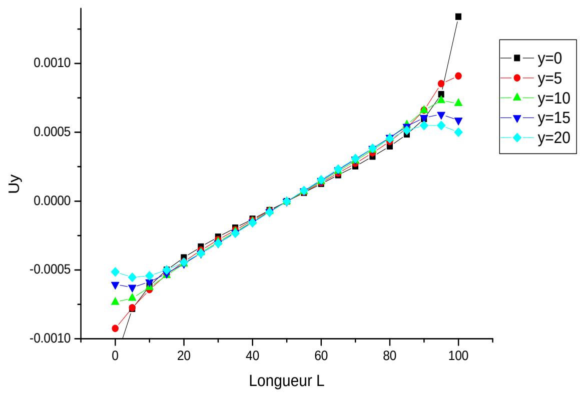
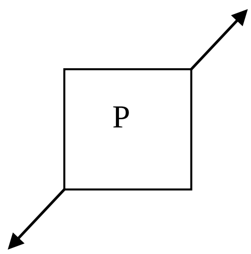


Fig 4. Displacement along y axis

Experience 3:



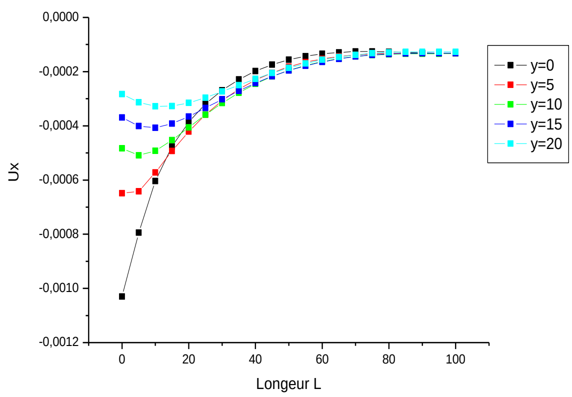


Fig 5. Axial displacement function of x

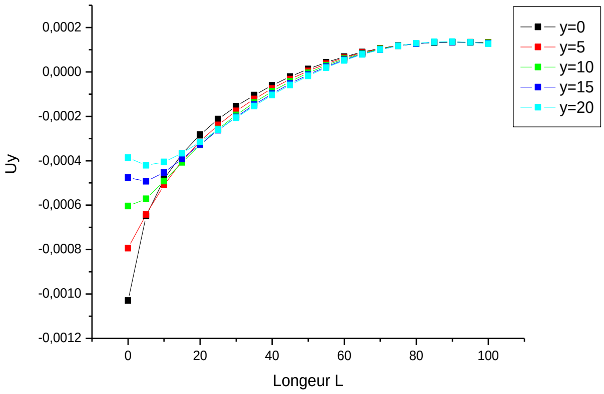


Fig 6. Displacement along y axis

Results found by our calculations are convincing and identical to those found by the commercial code Castem.

Conclusion

This work has allowed us to validate our approach based on the finite elements' method and to build a useful database in the inverse problem. The identification, modelling of the systems from experimental data, constitutes an active purpose of research in several domains and more particularly in materials. The objective of the identification is then to supply an estimation of the mathematical model of the system considered to simulate command or determine the physical parameters of a material in a systematic way.

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