The effect of mixture lengths of vehicles and braking on the traffic flow behavior in two-lane roadway

Abdelaziz Mhirech*

Université Mohammed V-Agdal, Faculté des Sciences, B.P. 1014, Rabat, Morocco.
Département de Physique, Laboratoire de Magnétisme et de la Physique des Hautes énergies, URAC 12.

Abstract:
Using a numerical simulation, we study the effect of the mixture lengths of vehicles on a traffic flow in a two-lane roadway, for periodic boundaries in parallel dynamics. A deterministic cellular automaton which is an extended version of the one-dimensional asymmetric model is used to take into account the exchange of the vehicles between the two lanes. The variation of mean velocity and flow in a two-lane roadway versus the initial concentration C_1, of vehicles in first lane, are studied for different values of the initial ratio *f*, between cars in lane 2 and vehicles in lane 1, and initial concentration *n* of trucks in lane 1. The presence of trucks has an important effect on phase transition between the maximal velocity phase and high density one. Indeed, the maximal current value decreases with increasing the concentration of trucks, and occurs at higher values of the initial density of vehicles in lane 1. The phase diagram (f,C_1) established for different values of *n*. The maximal velocity phase increases by increasing the concentration of trucks in the circuit. Also, to approach more the reality, we have taken into account the human factor by introducing random rates. The random noises have a great effect on the results compared to those obtained in the deterministic case. Indeed, the current flow decreases and the first-order transition (LDP)-(HDP) is replaced by a second-order one in the presence of random noises.
PACS: 89.40.+k Transportation. - 05.70.Fh. Phase transition: general studies - 05.40. Fluctuation phenomena, Random processes and Brownian motion.
Keywords: Traffic flow, vehicles, cars, trucks, two-lane roadway, phase diagram, mean velocity, current flow.

**Corresponding author E-mail address:* mhirech@fsr.ac.ma

I. Introduction

Recently, there have been considerable investigations concerning the traffic flow using methods of statistical physics [1-4]. Different models have been proposed for the description of traffic flow, in particular cellular automaton (CA) for their computational simplicity [5-8]. In spite of in some complex systems CA models give only some general qualitative features, they provide quantitative information in other cases. A much known prototype of deterministic CA model describing traffic flow on a linear road is the Wolfram's rule number 184 [9]. The one dimensional exclusion model is one of the simplest examples of a driven diffusive system and it is used easily to study the microscopic systems.

There are two methods for modeling traffic flow. Namely the macroscopic models which are based on fluid-dynamical description and the microscopic ones which focused it attention on individual vehicles treated as particles. In microscopic description systems, the idealized single-lane models are generalized to develop CA models of two-lanes [9,10], three-lanes [12], and 2-D traffic flow [13]. For the two-lane traffic, the study can be symmetric or asymmetric by respect to the lanes. If there are two types of vehicles like cars and trucks, for examples, the study can be symmetric or asymmetric by respect to the vehicles.

Our aim interest in this paper is to study numerically the effect of the mixture lengths of the vehicles, and random jump rates (braking) associated with particles on the mean velocity and on the current flow of vehicles in the case of a two-lane model, for periodic boundaries in parallel dynamics. We introduce two kinds of vehicles on the circuit. Here, we will denote by "cars" the vehicles which occupy one cell and by "trucks" those which occupy two cells. We initially incorporate the trucks in the first lane.

The paper is organized as follows: in the following section we define the model, the section 3 is reserved for results and discussions. The conclusion is given in section 4.

II. Definition of the model

Recently, several models have been proposed in two-lane traffic flow [14-17]. Nagatani have presented a simple model with one specie of vehicles for description of traffic flow on a two-lane roadway [14]. This model is an extension of the 1-D asymmetric exclusion model, which takes into account the exchange of vehicles between the first and second lane, in the limit of interaction between the two lanes. Moussa and Daoudia have proposed a new version of Nagatani model [14], which avoids the oscillation behavior in the high density phase. Our model is an extension of Nagatani's one and is as follow: In addition to the cars which occupy one site, we initially incorporate in the first lane of the circuit a second type of vehicles (trucks), which occupy two sites. Each vehicle in each lane moves ahead by one site unless it

is blocked by another vehicle. To avoid the oscillation behavior, if a vehicle in lane 1 (lane 2) is blocked ahead by another vehicle it shifts to the second lane (first lane) under the following conditions: A car on road 1 (road 2) moves vertically to the nearest neighboring site on road 2 (road 1) if this site is empty and its backward and forward neighboring sites are empty (these three sites being in road 2 (road 1)). A truck situated in sites *i* and *i*+1 on road 1 (road 2) moves vertically to the nearest neighboring sites on road 2 (road 1) if sites i-1, *i*, *i*+1 and *i*+2 in lane 2 (lane 1) are empty (see figure 1).

Figure1: The CA model for traffic flow on a two-lane roadway. The cars are designed by a single-solid circles whereas the trucks are the double-solid squares. At an odd (even) time, the vehicles in the first lane move ahead, shift to the second (first) lane, or stop according to the CA rules.

Indeed, there is no reason for a vehicle to change the lane if it can't move ahead. We reproduce the 1-D asymmetric model in a two-lane roadway without interaction between these lanes. The vehicles jump at unit rate with probability 1 to the vacant neighboring site. On the other hand, we introduce the braking linked to the vehicles. Precisely, in this later case, the vehicles jump to vacant adjacent sites in the right or vertically with a probability p_k in each discrete time step, where p_k is a quenched random variable associated with vehicle k (its intrinsic jump rate). The random rates (braking) represent the human factor in driving.

To illustrate the situation in our model, let us consider a circuit with a double-ring of 2×N sites, where:

C_{11} is the initial density of sites occupied by cars in lane 1.

C_{12} is the initial density of sites occupied by trucks in lane 1.

C_1 is the initial density of sites occupied by cars and trucks in lane 1 ($C_1 = C_{11} + C_{12}$).

C_2 is the initial density of sites occupied by cars in lane 2.

In order to make the study easy, we consider the parameters *n* and *f*, where:

$n = \dfrac{C_{12}}{C_1}$ and $C_{11} = (1 - n)C_1$,

and $f = \dfrac{C_2}{C_1}$.

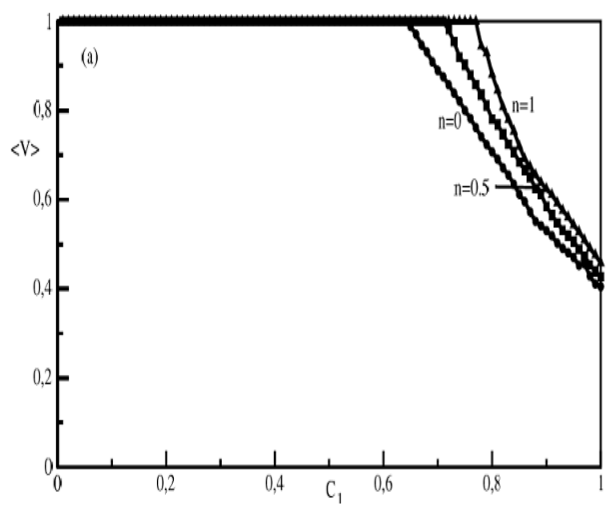
n represent the initial concentration of trucks in the road 1 and *f* is the initial ratio between cars in lane 2 and vehicles in lane 1.

In our model, the total number of vehicles on a two-lane roadway is conserved. However, the total number of vehicles in each lane is varied since the vehicles have the possibility to change the lane when the conditions discussed above are verified.

III. Results and Discussion

In our simulations, we have considered a two one-dimensional lattice of 2×1000 sites. We use the same time-treatment as Nagatani [14]. Hence a time step is divided in two times: an odd-time where we update the states of the first lane and an even-time where we update the states of the second lane. The vehicles are initially distributed randomly on lanes (1) and (2) with initial densities C_1 and C_2 of occupied sites, respectively. The system run for 2×10^{4} time steps to ensure that steady state is reached. In order to eliminate fluctuations 10 initial configurations are randomly chosen.

The figure 2-a gives the variation of mean velocity $\langle V\rangle$ of vehicles versus the initial concentration C_1 in the lane 1, for the ratio f = 0.5 and for different values of the initial concentration, n, of trucks in lane 1. The mean velocity $\langle V\rangle$ of vehicles in the two-unit of time interval is defined to be the number of vehicles which move ahead in one time interval divided by the total number of vehicles. Obviously, for all values of *n*, $\langle V\rangle$ is constant and equal to 1 for C_1 below than a critical value $C_{1c}(n,f)$. Indeed, for $C_1 < C_{1c}(n,f)$, the system is the low density phase (LDP) and therefore all vehicles can move. This critical value $C_{1c}(n,f)$ depends on *n* and *f*. For a fixed value of ratio *f*, $C_{1c}(n,f)$ increases when increasing *n*.



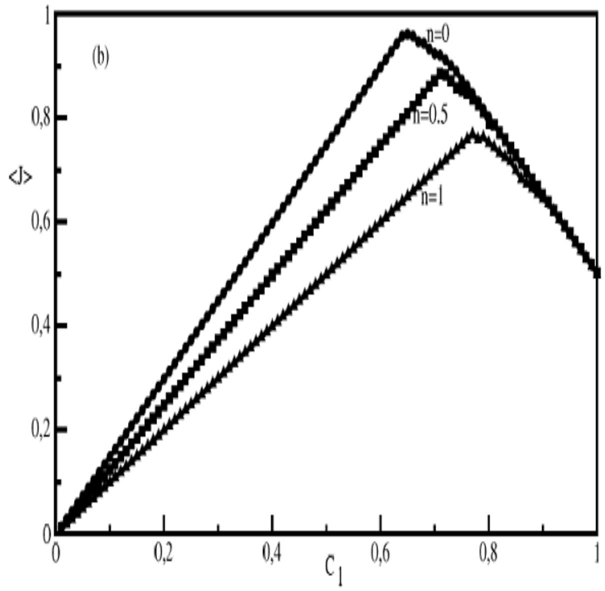


Figure 2: The variation of mean velocity <V> of vehicles (a) and the current flow (b) versus the initial concentration C_1 in the lane 1, for f = 0.5 and for different values of rapport n, without random rates.

Summary, for $C_1 < C_{1c}(n,f)$ the mean velocity of vehicles remains constant when increasing C_1 . While for $C_1 > C_{1c}(n,f)$ the mean velocity $<V>(C_1)$ decreases depending on the initial concentration of trucks. There is a phase transition between the maximal velocity phase (low density phase) in which <V> is constant and a high-density one where <V> decreases. In fact, when the number of trucks (*n*) increases in the circuit, the low density phase increases. Indeed, the trucks takes lot of time in passing the local detectors due to their length in contrast to the cars. We note that the phase transition in this case is a first-order transition because the transition (LDP)-(HDP) is sharp.

Numerically, we can explain the increase in $C_{1c}(n,f)$ versus n by the fact that without trucks in the road (n = 0), if the gap (distance between two consecutive cars) of a car is zero, the speed of this car is zero. Therefore, the system detaches from the case of (LDP). For against, in the presence of trucks, two consecutive occupied sites by a truck does not mean that the system is not in the case of (LDP).

Otherwise, in the absence of trucks (n = 0, only cars move in the roads), we retrieve the results of the new version for the Nagatani model proposed by Daoudia and Moussa [15].

In figure 2-b, the current flow <J> of vehicles is plotted versus the initial concentration of vehicles C_1 in the first lane for a moderate value of the ratio between cars in lane 2 and vehicles in lane 1 (f = 0.5) and for different values n of the initial concentration of trucks. The critical value $C_{1c}(n,f)$, observed in figure 2-a, corresponds to the maximal current flow of vehicles. For a given value of *f*, the maximal flow decreases and shifts to the high values of the initial concentration C_1 when increasing *n*. It is clear that increasing the number of trucks on the highway leads to a decrease in the maximum flow of traffic.

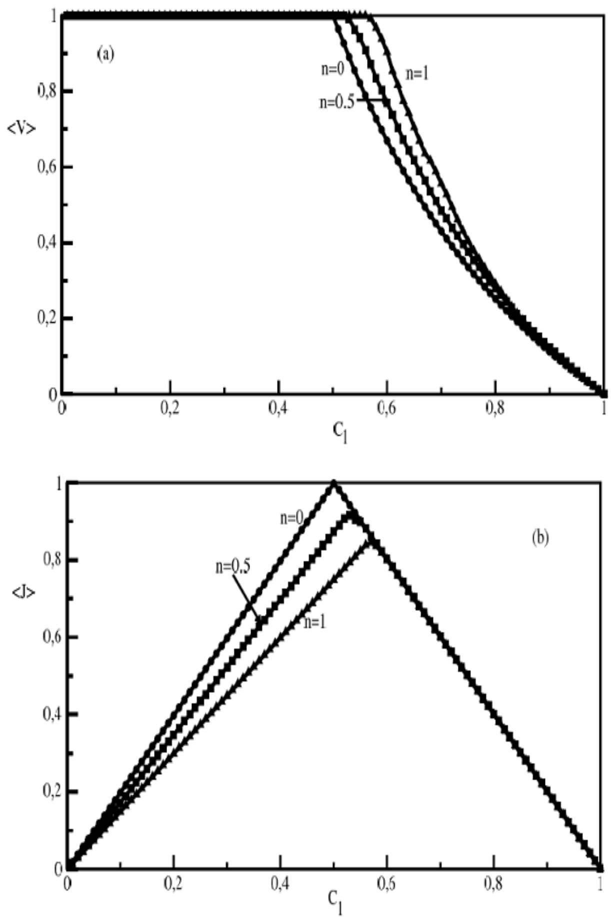


Figure 3: The variation of mean velocity (a) and flow (b) versus the initial concentration of vehicles in lane 1 for $f = 1$ and for different values of *n*, without random rates.

In figures 3-a and 3-b, we have plotted, respectively, the mean velocity and the current flow of vehicles in the road for the same initial distribution of concentration in both lanes (f = 1). We retrieve the analogous results found for one-dimensional road [18]. Indeed, when we consider only cars in the rings (n = 0), the current flow versus the initial concentration in lane 1 (C_1) presents a symmetry by respect to $C_{1c}(n,f)$ = 0.5. In figure 3-a, $C_{1c}(n,f)$= 0.5 corresponds to the critical value which separates the low density phase (<V> = 1) to the high density one (<V> < 1) [14,15]. As soon as we initially introduce a small concentration of trucks in lane 1, the break of the symmetry holes particles takes place (figure 3-b). The flow decreases by increasing n when $C_1 < C_{1c}(n,f)$. As in figures 2-a and 2-b, $C_{1c}(n,f)$ increases by increasing n. Whereas, for $C_1 > C_{1c}(n = 1,f)$, the current flux decreases linearly, independently of *n*. Indeed, for low densities ($C_1 < C_{1c}(n,f)$, the mean distance between vehicles is greater than 1 and there are too far apart to interact mutually which leads to an linear increase of current. This increase is controlled by the factor *n*.

Now we study the effect of the ratio (f) between cars in lane 2 and vehicles in lane 1 on <V> and <J>. The behavior of mean velocity and flow are presented, respectively, in figures 4-a and 4-b versus the initial concentration of vehicles C_1 in lane 1, for different

values of the ratio f and for a fixed value of the initial fraction of trucks in the circuit (n = 0.5). These figures show that when the second lane is initially free (f = 0), <V> remains constant and the current flow increases linearly for all values of C_1. Indeed, the gaps between vehicles are greater than one so all vehicles move without constraints, because the lane 2 is initially empty.

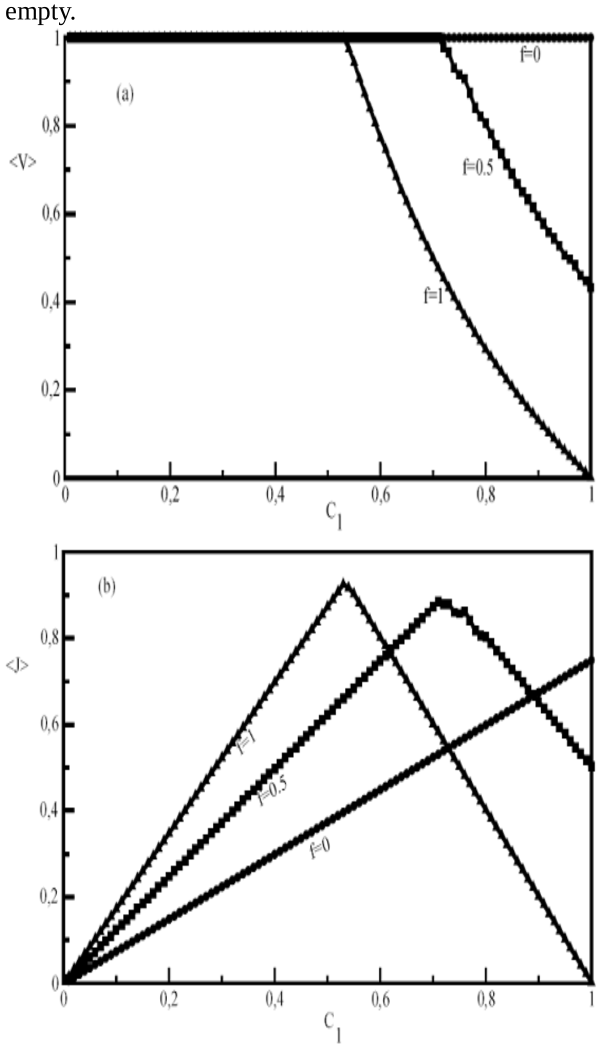


Figure 4: The variation of mean velocity (a) and flow (b) versus the initial concentration of vehicles in lane 1, for different values of the ratio f and for a fixed value of the initial fraction of trucks in the circuit (n = 0.5), without random rates.

For f > 0, figures 4-a and 4-b show two phases. Namely, low density phase for $C_1 < C_{1c}(n,f)$ where <V> remains equal to 1 and flow linearly increases and the high density phase for $C_1 > C_{1c}(n,f)$ where both <V> and flow decrease .

Figures 4-a and 4-b show, respectively, that <V> and flow increase in low density phase and decrease in high density one by increasing f. Obviously, in low density phase, no vehicles are ever blocked in the roadway so an increasing in global density (increasing of f) increases <V> and flow. On the other hand, in high density phase an increase of vehicles in the circuit leads to decrease of mean velocity and flow. Indeed, the number of mobile vehicles decreases due to congestion.

In contrast to the increase of $C_{1c}(n,f)$ versus the parameter n (figures 2 and 3), this critical value decreases when increasing the ratio f. Obviously, increasing the ratio f means an increase in the density of vehicles in the circuit so the system falls rapidly in the high density phase.

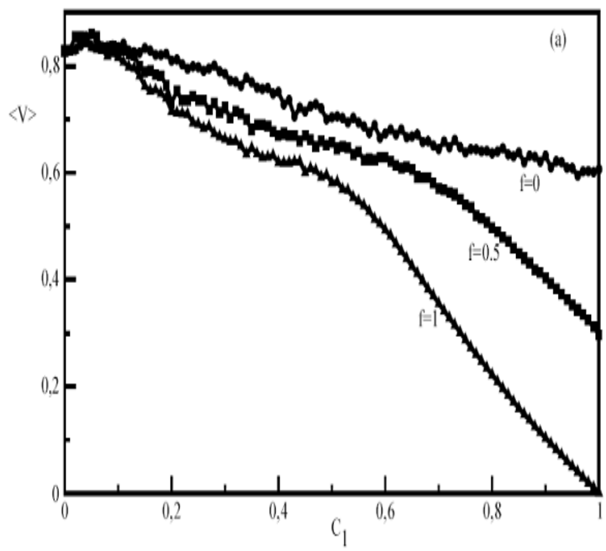
The case n = 1 and f = 0, where all vehicles in the lattice are trucks and the second lane is initially free, is also studied but not illustrated. This case gives <V> = 0.5 and current flow increases for all values of C_1. This means that there is only the low density phase (no phase transition) and all trucks move.

From the study above, we deduct that the critical value $C_{1c}(n,f)$ increases when increasing n and/or decreasing f.

In figures 5-a and 5-b, the variations of <V> and flow are, respectively, plotted against the initial concentration of vehicles in lane 1 for n= 0.5 and for different values of f with random rates. As we said above, the random rates reflect the state of drivers and is noted p_k. p_k is a probability that is randomly assigned to the vehicle k and remains linked to it. For each vehicle k and every time t, a number is randomly drawn. If this number is less than p_k, this vehicle can move otherwise it remains in its place.

By comparison to the figure 4-a, figure 5-a shows that the first-order transition (LDP)-(HDP) is replaced by a second-order one in presence of random rates. Indeed, even in low density phase there are vehicles which are blocked because of braking imposed by random rates. Consequently, we observe a continuous transitions (LDP)-(HDP).

Comparing figures 4-b and 5-b, we remark that random rates have a great effect on the current flow. Obviously, the flow is less important in the presence of random rates by respect to the deterministic case (figure 4-b). In the presence of random noises, a breakdown of the linearity is observed in the variation of the current flow versus C_1.



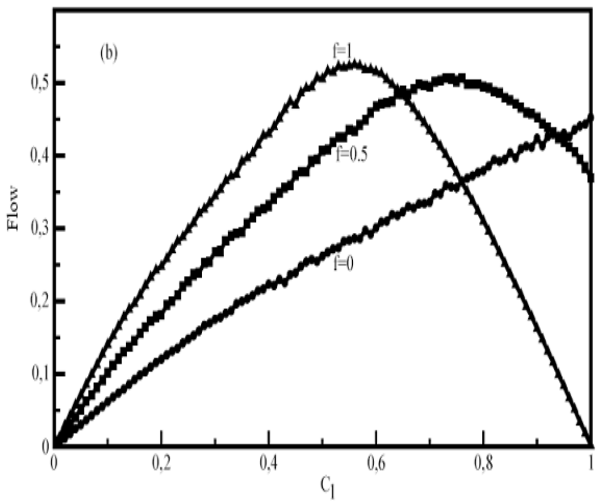


Figure 5: The variation of mean velocity (a) and flow (b) versus the initial concentration of vehicles in lane 1 for n = 0.5 and for different values of f, with random rates.

Figures 5-a and 5-b show that in low density phase, the current flow increase with proportion of cars in lane 2 by contrast to the high density phase.

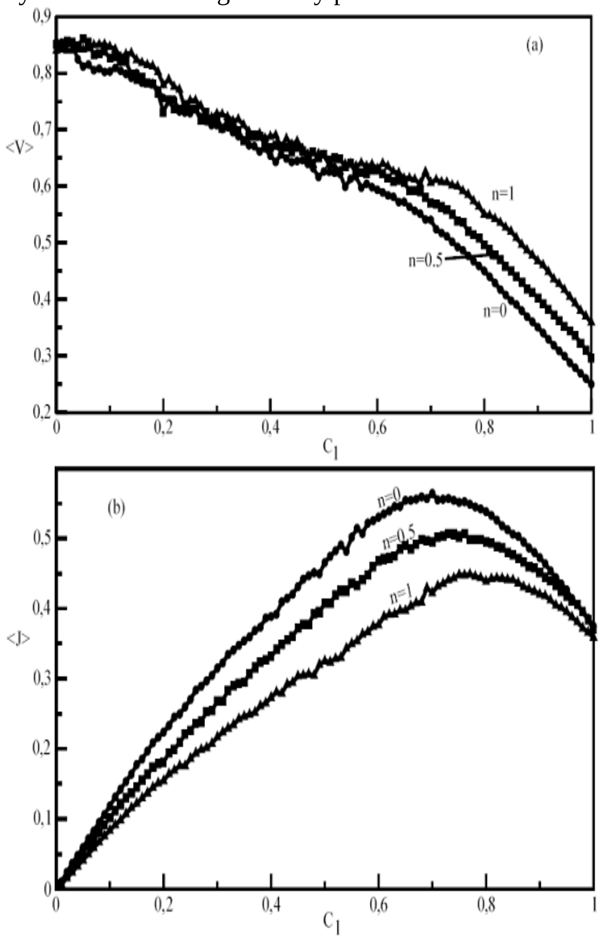


Figure 6: The variation of mean velocity (a) and flow (b) versus the initial concentration of vehicles in lane 1 for f = 0.5 and for different values of n, with random rates.

Figures 6-a and 6-b give, respectively, the variation of <V> and flow against C_1 for a fixed value of ratio between cars in lane 2 and vehicles in lane 1 (f = 0.5) and for different values of n, in presence of random rates. The mean velocity decreases versus C_1 for any values of n. Both <V> and flow decrease by comparing the figures 6-a and 6-b to the figures 2-a and 2-b. Also the first-order transition (LDP)-(HDP) is replaced by a second-order one in the presence of random noises.

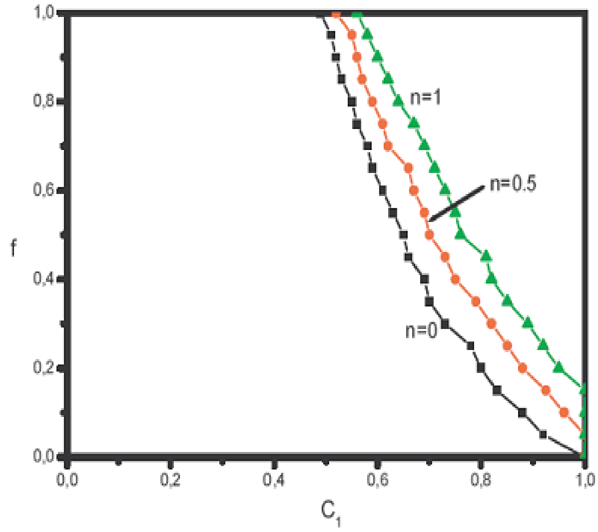


Figure 7: The phase diagram in (f,C_1)-plane for different values of n, in the deterministic case.

Figure 7 gives the phase diagram in (f,C_1)-plane for different values of the initial concentration of trucks in the first lane, without random rates. The maximal mean velocity (low density) phase increases by increasing n. Indeed, for a fixed value of f, the critical value $C_{1c}(n,f)$} increases when increasing n and leads to an increase of maximal velocity phase. On the other hand, for a fixed value of n, the maximal mean velocity phase decreases when increasing f, because the lane 2 tends to fill.

IV. Conclusion

We have studied the effect of mixture lengths of vehicles on traffic flow in two one-dimensional roadway using numerical simulation. We have defined two kinds of vehicles depending on their lengths, namely type 1 the small (cars) which occupy one cell and the long ones (trucks) takes two. We focused our study on the extension of the 1-D asymmetric exclusion model in parallel dynamic with and without random rates. Initially, the trucks are introduced only in first lane. In each time step, the vehicles can move ahead, stopped in each lane or shifted to the other lane according to the cellular automaton rules. The two main parameters examined in this paper are the initial fraction of the trucks in first road (n) and the initial ratio (f) between cars in lane 2 and vehicles in lane 1. The variation of mean velocity and flow of vehicles are studied against the initial concentration of vehicles in lane 1 for different values of n and f. In the deterministic case, the model presents two phases for f > 0, namely maximum velocity (low density) phase and high density one. In the first phase, <V> remains equal to 1 for any value of n and f because all vehicles can move. While the current flow increases linearly with C_1 depending on n and/or f. Indeed, <J> decreases

when increasing *n* because trucks take longer than cars to exceed the detector. By against, in high density phase, <V> decreases depending on *n*, while <J> decreases linearly and is independent of *n* for a fixed value of *f*.

When increasing f (*n* and C_1 are fixed), the flow increases in low density phase, because the density of vehicles increases, and decreases in high density one because the exchange of vehicles between the two lanes becomes difficult and the free movement fades.

The phase diagram in (f,C_1)-plane for different values of n shows that the maximal mean velocity phase increases by increasing the initial concentration of trucks in the lane 1.

In presence of random rates (braking) linked to the vehicles, which are introduced in the aim to take into account the human factor, the first-order transition (LDP)-(HDP) is replaced by a second-order one. However, both mean velocity and flow are reduced.

References

[1] R. Herman and K. Gardels, Sci. Am. (France) **209 (6)** (1963) 35.

[2] D. E. Wolf, M. Schreckenberg and A. Bachen, (eds),Traffic and Granular Flow (Word Scientific, Singapore, 1996).

[3] Transportation and Traffic Theory, edited by N. H. Gartner, N. H. M. Wilson (New York, Elsevier, 1987).

[4] M. Schreckenberg and D. E. Wolf, (eds), Traffic and 97 (Springer, Singapore, 1998).

[5] K. Nagel and M. Schreckenberg, J. Phys. I (France) **2** (1992) 2221.

[6] M. Bramson, J. Stat. Phys. **51** (1988) 863.

[7] A. Schadchneider and M. Schreckenberg, J. Phys. A, **26** (1993) 1679.

[8] M. Schreckenberg, K. Nagel and N. Ito, Phys. Rev. E, **51** (1995) 2939.

[9] S. Wolfram, Theory and application of Cellular Automata (World Scientific, Singapore, 1986).

[10] O. Biham, A. A. Middleton and D. Levine Phys. Rev. A, **46** (1992) R6124.

[11] W. Knospe, L. Santen, A. Scahdschneider and M. Schreckenberg, J. Phys. A: Math. Gen. **35** (2002) 3369.

[12] A. K. Daoudia and N. Moussa, Chinese J. of Phys. **41** (2003) 671.

[13] A. Benyoussef, H. Chakib and H. Ez-Zahraouy, Phys. Rev. E **68** (2003) 026129.

[14] T. Nagatani, J. Phys. A: Math. Gen. **26** (1993) L781.

[15] A. K. Daoudia and N. Moussa, Chinese J. of Phys., **40** (2002) 484.

[16] P. Wagner, K. Nagel and D. E. Wolf, Physica A, **234** (1997) 1997.

[17] N. Moussa and A. K. Daoudia, Eur. Phys. J. B. **31** (2003) 413.

[18] H. Ez-Zahraouy, K. Jetto and A. Benyoussef, Cond-mat/0401647 **v1 30 jan** (2004) .