Effect of disorder in magnetic and biological systems

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Using a replica formalism, a generalization of a recent mean field model corresponding to the observed wrinkling transition in randomly polymerized membranes, and a generalization of Schneider and Pytte model to the l-component classical spin vector model are presented. In the first model, we study the effects of global fluctuations of the surface normal to the flat membrane, which can be introduced by a random local field. In absence of these global fluctuations, we show that, the model exhibits both continuous and discontinuous transitions between flat and wrinkled phases, contrary to what has been predicted by Bensimon et al and Attal et al. Phase diagrams both in replica symmetry and in breaking of replica symmetry in sense of Almeida and Thouless are given. We have also investigated the effects of global fluctuations on the replica symmetry phase diagram. We show that, the wrinkled phase is favored and the flat phase is unstable. For large global fluctuations, the transition between wrinkled and flat phases becomes first order. In the second model, effects of a Gaussian random field on the phase transition of the l-component classical spin vector model are investigated. The phase diagrams are obtained in the cases l=1 and l=3. In opposite to what has been predicted by Schneided and Pytte. The results we obtain, for l=1 and l=3 show that the model exhibits a second-order, tricritical point and a first-order transition depending on the value of the =random field.

I. INTRODUCTION

Flexible membranes are two dimensional generalization of linear polymer chains. The properties of a 2D membrane, embedded in three dimensional space, depend strongly on the internal order, crystalline, hexatic, or fluid. As in other realizations of 2D matter, defects, and their interactions, affect crucially the stability of a given phase. In contrast to linear polymers, crystalline membranes, also known as tethered or polymerized membranes are expected to exhibit quite different physical properties from their linear counterparts. In particular, they are predicted to have a remarkable low-temperature ordered phase. This ordered, or flat phase, is characterized by long-range order in the orientation of surface normal. At high temperature, or equivalently low bending rigidity, phantom (non-self-avoiding) crystalline membranes will entropically disorder and crumple. Separating these two phases should be a crumpling transition. So, different phases of a crystalline membrane can be distinguished by the behavior of the surface normal. In the flat phase the normal will have long-range order, while in the crumpled phase the normal eventually decorrelate. These systems have aroused considerable interest both from theoretical and experimental point of view [1].

Recent studies of membranes with defects and quenched random disorder have been prompted in part by experimental works [2-4]. These experiments show that partially polymerized membranes undergo, possibly first-order, a reversible phase transition from high-temperature phase characterized by a smooth, floppy surface, to a low-temperature phase. In this last phase the membranes appear rigid and highly wrinkled. It is important to notice that this phase is very different from the so-called crumpled phase of membrane [1], in which the local normal to the membrane fluctuate in time. However, in the wrinkled phase the normal are randomly frozen. Based on the mean field solution of a model for a heterogeneous (disordered) membrane, this transition has been linked to the spin glass transition of magnetic systems [3,5-7]. Alternatively, Nelson et al [8] and Morse et al [9] have analyzed the stability of the flat phase of disordered membranes by the field theoretical method ε=4-D expansion, Radzihovsky and Le Doussal [10] have studied this problem in the large d limit. As result of these works it appears that the randomness in the metric destabilizes a flat membrane towards a possible spin glass phase, whereas adding random curvature yield a new T=0 flat phase, with a crossover to a glassy phase at higher temperature. From the magnetic point of view a membrane model [11] is constrained spin system. In fact, the spin (surface normal) must be normal to the underlying surface, and the constraint is of course essential to the stability of the ordered phase. But within the mean field approximation [12], forfeiting all spatial information, where one is interested in obtaining the possible thermodynamic phases. The constraint may be relaxed as long as it is satisfied in all phases. With this drastic approximation, the disordered membrane model is akin to Heisenberg model with random Dzyaloshinky Moriya (DM) interactions [5,6]. Thus, it might be helpful to keep in mind the magnetic analogy where the ferromagnetic, paramagnetic and spin glass phases are the analog of flat, crumpled and wrinkled phases, respectively.

The effect of the disorder are important firstly because they are encountered in many practical problems, and secondly because they provide interesting and fundamental problems in statistical mechanics. Tow of

these systems, namely, spin glasses and the ferromagnet in a random magnetic field, have attracted special attention from many workers. The relationship between these tow systems is rather puzzling. The ordered phase of both systems is characterized by irreversibility [13] and long-time relaxation processes [14,15] du to multiple minima in the free-energy surface. The joint study of spin-glass and random-field problems has been proposed as an appropriate system for the description of mixed hydrogen-bonded ferroelectric and antiferroelectrics, the so called proton and deuteron glasses [16-20]. Also, experiments on the diluted antiferromagnet $Fe_xZn_{1-x}Fe$ in an external magnetic field have presented a crossover between random-field and spin-glass behaviors [21-28] for some range of concentration x.

However, the random field Ising model (RFIM) [29] has been found to have several interesting and unusual properties. The model was proposed originally [29] to study the effects of quenched random field on critical phenomena at a second-order ferromagnetic transition. It lower critical dimension d_l has been widely investigated by rigorous theoretical analysis[30], Renormalization-group arguments [31], high-temperature series analysis [32], numerical simulations [33], and experiments on bond-diluted antiferromagnets [34]. All these works point to $d_l=2$. Moreover, a lot of effort has been devoted to the RFIM [22-39]. Thus, in these works and for a bimodal distribution of random field, tricritical behavior is relevant feature [35-38]. While, for the gaussien distribution, Schneider and Pytte [40] have concluded that no such tricritical point exist. Recently, the analysis of the high-temperature susceptibility series for the last distribution demonstrates the existence of first-order transition below four dimension [39].

Firstly, we will generalize a solvable model proposed by Schneider and Pytte to an l-component spin vector model case. Using replica method, we give phase diagrams in the cases l=1 and l=3. In these phase diagrams we show that the model exhibits a second-order, tricritical point and a first-order transition. In the limit of our model we find the same expressions for the free energy and equations of state as in Ref. [40]; However, our phase diagram differs from their phase diagram basically in the existence of a tricritical point. The same feature is observed as the number of component l is large. Moreover, by setting l=3, for simplicity and adding a random DM interactions to this model, we recover a natural generalization of the recent mean field model corresponding to the observed wrinkling transition in randomly polymerized membrane. In this case, the local field can be interpreted as global fluctuations of the surface normal induced by inhomogeneities of elastic properties of membrane. Within the replica symmetry (RS) solution, we have calculated the free energy and constructed set of a self-consistent equations, as is well-known in the mean field procedure. In zero random field, we find that our phase diagram differs in part from that obtained by Bensimon et al [5] and Attal et al [6]. In fact, in our analyzes we show that the model exhibits both a first and second order transitions between flat (ferromagnetic) and wrinkled (spin glass) phases, and reentrant phenomenon occurs. However, in their works [5-6] they predicted within the RS solution that, the transition line between flat and wrinkled phases is only second order. Moreover, by considering small fluctuations of the order parameter around the symmetric solution we derive, analytically, a line of instability. This line is analogous to the Almeida-Thouless (AT) line in spin glass systems [41], above which only a solution involving broken replica symmetry is stable. On the other hand, in presence of random field, we restrict ourselves to the replica symmetric solution. We present a study of the effect of a local quenched random field on the RS phase diagram of randomly frustrated membrane. A phase diagram for various values of random local width is given. We find that the random field plays the same role as that of randomness in preferred metric.

This paper is a review of previous works published in references [42] and [43]. We have focus on the effect of quenched disorder in the magnetic and biological model systems, using the replica formalism.

II. MODEL AND METHOD

We consider the model describing characterized by the following Hamiltonian

$$H = -\frac{K}{2N}\sum_{i\neq j}\vec{S}_i\vec{S}_j - \frac{1}{2}\sum_{i\neq j}\vec{D}_{ij}(\vec{S}_i \wedge \vec{S}_j) - \sum_i h_i S_{i1} \quad (1)$$

where $\sum_{\mu=1}^{l} S_{i\mu}S_{i\mu} = 1$, with i=1,....,N and the sum $\sum_{i\neq j}$ extends over all pairs of pseudospin $\vec{S_i}$, and K is the exchange interaction. $\vec{D_{ij}}$ is a random vector, and h_i chosen in $\mu = 1$ direction is a random local field. The $\vec{D_{ij}}$ and h_i are quenched random variables distributed according to their respective Gaussian probability densities:

$$P(\vec{D_{ij}}) = \left(\frac{2\pi J^2}{N}\right)^{-\frac{3}{2}} \exp\left(-\frac{N}{2}\left(\frac{\vec{D_{ij}}}{J}\right)^2\right) \quad (2)$$

$$P(h_i) = (2\pi h^2)^{-\frac{1}{2}} \exp\left(-\frac{1}{2}\left(\frac{h_i}{h}\right)^2\right) \quad (3)$$

The N dependence in the probability distribution $P(\vec{D_{ij}})$ is imposed, as usual for infinite range problems, such as to ensure a sensible thermodynamic limit. For J=0 this model (Kh-Model) is a generalization of that proposed by Schneider and Pytte. However for l=3 and $J \neq 0$, the model can be used to describe the phase diagram of randomly polymerized membrane (KJh-Model).

A. Kh-Model

In the first case We consider l-classical spins interacting through an infinite ranged exchange interaction K, and in the absence of random DM interaction. These spins are coupled to a random field

with a Gaussian distribution, defined in Eq.(3). The components are continuous variables subject to the conditions $\overset{l}{\underset{\mu=1}{}} S_{i\mu} S_{i\mu} = l$

Since a quenched system is considered, it is appropriate in the thermodynamics to average free energy per spin, F, over the distribution of disorder, in Eq.(3). The most celebrated method for the calculation of $<f>_h$ is the replica trick [44] which allows this average to be taken by analitically continuing the n-fold replicated disorder averaged partition function from positive integer n to n=0. The disorder averaged free energy per spin is then obtained as:

$$-\beta F = \underset{N\to\infty}{\mathrm{Lim}}\ \underset{n\to 0}{\mathrm{Lim}}\left[\frac{< Z^n >_h - 1}{n}\right] \tag{4}$$

where $<...>_h$ denote the average over the disorder. The replica Hamiltonian, H_n, is given by

$$-\beta H_n = \frac{\beta K}{2N} \underset{\alpha,\mu\, i\neq j}{} S^{\alpha}_{i\mu} S^{\alpha}_{j\mu} + \underset{i}{} \beta h_i S^{\alpha}_{i1} \tag{5}$$

where α =1,..., n denote different replicas and label cartesian directions in l-space. The replica partition is:

$$Z^n = \ D(s) \exp(-\beta H_n) \tag{6}$$

the integral $D(s)$ is performed over spins at all sites and in all replicas subject to the condition. The disorder variables occur linearly in the exponent of Eq.(6). So it is easy to perform the disorder average, which yields:

$$\beta F = \frac{\beta K}{2n} \sum_{\alpha,\mu} (m^2_{\mu})^2 - \frac{1}{n} \mathrm{Log}(\ D(s) \exp(Q)) \tag{7}$$

with

$$Q = \beta K \underset{\alpha,\mu}{} S^{\alpha}_{\mu} m^{\alpha}_{\mu} + \frac{(\beta h)^2}{2} (\underset{\alpha}{} S^{\alpha}_{i})^2 \tag{8}$$

where $m^{\alpha}_{\mu} =< S^{\alpha}_{\mu} >$ is the order parameter and the integral $D(s)$ is now taken over the spins in all replicas at one site. If we assume that at the saddele-point all are equal and we use the Hubbard-Stratonovich transformation, the disorder averaged free energy per spin can be expressed as:

$$\beta F = \frac{\beta K}{2} \sum_{\mu} m^2_{\mu} - \ \frac{dw}{\sqrt{2\pi}} \exp(-\frac{1}{2} w^2) \mathrm{Log}(Z_0) \tag{9}$$

where

$$Z_0 = \underset{|S|=\sqrt{l}}{} dS \exp(\sum_{\mu} t_{\mu} S_{\mu}) \tag{10}$$

and

$$t_{\mu} = \beta K m_{\mu} + \beta h w \delta_{\mu,1} \tag{11}$$

In order to discuss the thermodynamic properties of this model, we introduce the independent spin order parameter [40]:

$$r_{\mu} =<< S_{i\mu} >^2>_h \tag{12}$$

and the quadrupolar parameter

$$1 + (l\delta_{\mu,1} - 1) q =<< S^2_{i\mu} >>_h \tag{13}$$

So, the order parameters which characterizes the transitions [16] may be occurring in our model are given by:

$$\begin{aligned} m_{\mu} &= \ \frac{dw}{\sqrt{2\pi}} \exp(-\frac{1}{2} w^2)(\frac{1}{Z_0} \frac{\partial Z_0}{\partial t_{\mu}}) \\ r_{\mu} &= \ \frac{dw}{\sqrt{2\pi}} \exp(-\frac{1}{2} w^2)(\frac{1}{Z_0} \frac{\partial Z_0}{\partial t_{\mu}})^2 \\ 1 + (l\delta_{\mu,1} - 1) &= \ \frac{dw}{\sqrt{2\pi}} \exp(-\frac{1}{2} w^2)(\frac{1}{Z_0} \frac{\partial^2 Z_0}{\partial t^2_{\mu}}) \end{aligned} \tag{14}$$

and the generating function in Eq.(10) can be expressed as

$$Z_0 = 2\sqrt{2\pi} (\frac{\sqrt{l}t}{2})^{1-l/2} I_{l/2-1}(\sqrt{l}t) \tag{15}$$

and

$$t = \sqrt{\underset{\mu}{} t^2_{\mu}} \tag{16}$$

where $I_{\lambda}(x)$ is a modified Bessel function.

At this point we can pass to the limit. We have

$$\underset{l\to 1}{\mathrm{Lim}}\, Z_0 = 2 \cosh(\beta K m + \beta h w) \tag{17}$$

and q=0. It follows that we find the same expressions for the averaged disordered free energy and equations of state as Schneider and Pytte [40].

B. KJh-Model

Now, we consider the model describing a membrane with quenched random spontaneous curvature [5,6] from the magnetic point of view, to which we added a local quenched random field. Within the mean field approximation [12], the model of a membrane with a randomness, is characterized by the hamiltonian given in Eq.(1) with l=3 for simplicity.

The same procedure as in the case of the simple Kh-model, the free energy averaged over the joint probability distribution $P(\overrightarrow{D_{ij}})$ and $P(h_i)$ can be obtained via the well-known replica formalism. The standard procedure leads to the final expression of the averaged free energy density per spin :

$$\beta f = -\frac{3}{2} \beta^2_{\gamma} - \underset{n\to 0}{\mathrm{Lim}}\, g(m^{\alpha}, r^{\alpha\beta}, \Delta^{\alpha\beta}, q^{\alpha}) \tag{18}$$

where

$$\begin{aligned} g \equiv & -\frac{\beta_{\gamma} k_{\gamma}}{2} \underset{\alpha}{} m^2_{\alpha} + \frac{3}{2} \beta^2_{\gamma} \underset{\alpha}{} q^{\alpha}(1 + q^{\alpha}) \\ & - \beta^2_{\gamma} \underset{(\alpha\beta)}{} T^{\alpha\beta}_{x} (T^{\alpha\beta}_{y} - \frac{T^{\alpha\beta}_{x}}{4}) + \mathrm{Log}\mathrm{Tr}_n \exp(Q_n) \end{aligned} \tag{19}$$

and

$$\begin{aligned} Q_n \equiv & \beta_{\gamma} k_{\gamma} \ m_{\alpha} S^{\alpha}_{1} + \frac{1}{2} \underset{\alpha}{} ((\beta h)^2 - 3\beta^2_{\gamma} q^{\alpha})(S^{\alpha}_{1})^2 \\ & + \underset{(\alpha\beta)\ \upsilon}{} S^{\alpha}_{\upsilon} S^{\beta}_{\upsilon} (\beta^2_{\gamma} T^{\alpha\beta}_{\upsilon} + (\beta h)^2 \delta_{\upsilon,1}) \end{aligned} \tag{20}$$

above, α and β are replica labels and $_{(\alpha\beta)}$ denote sums over distinct pairs of replica. The notations, $\beta_\gamma = \beta J$, $k_\gamma = K/J$, $T_\mu^{\alpha\beta} = 2r^{\alpha\beta} - (3\delta_{\mu,1} - 1)\Delta_{\alpha\beta}$ has been used.

The extreme of the functional $g(m^\alpha, r^{\alpha\beta}, \Delta^{\alpha\beta}, q^\alpha)$ give us the coupled self-consistent equations :

$$
\begin{aligned}
m^\alpha &= < S_1^\alpha >, \forall\alpha \\
r^{\alpha\beta} &= \frac{1}{3}\,{}_{\mu=1}^{3} < S_\mu^\alpha S_\mu^\beta >, \forall(\alpha\beta) \\
\Delta^{\alpha\beta} &= \frac{1}{6}\,{}_{\mu=1}^{3} (3-\delta_{\mu,1}) < S_\mu^\alpha S_\mu^\beta >, \forall(\alpha\beta) \\
1 + 2q^\alpha &= < (S_1^\alpha)^2 >, \forall\alpha
\end{aligned}
\tag{21}
$$

where <...> indicate thermal averages with respect to the effective hamiltonian. Q_n The parameters m^α, $r^{\alpha\beta}$, $\Delta^{\alpha\beta}$ and q^α are the magnetization, isotropic, anisotropic part of spin glass and
quadrupolar order parameters, respectively. As in other anisotropic vector models [45] the order parameters q^α and m^α are strictly α-independent both in replica symmetric state and in the case of replica symmetric breaking (RSB).

The RS state is obtained by setting $r^{\alpha\beta} = r$, $\Delta^{\alpha\beta} = \Delta$ and $T_\mu^{\alpha\beta} = T_\mu$ in Eq.(19) and Eq.(20). Then, the analytic continuation $n \to 0$ in Eq.(18) may be easily performed. With this choice, the free energy per pseudospin is given by:

$$
\begin{aligned}
\beta f = &\frac{\beta_\gamma k_\gamma}{2} m^2 - \frac{3}{2}\beta_\gamma^2 (1 + q + q^2) - \frac{\beta_\gamma^2}{2} T_1 (T_2 - \frac{T_1}{4}) \\
&+ \frac{3}{2}\beta_\gamma^2 T_1 - \prod_{\mu=1}^{3} \frac{dt_\mu}{\sqrt{2\pi}} \exp(-\frac{1}{2}t_\mu^2) \log(Z)
\end{aligned}
\tag{22}
$$

with

$$
Z = \underset{|\vec{S}|=\sqrt{3}}{} d\vec{S} \exp({}_{\mu=1} a_\mu S_\mu + \frac{b}{2} S_1^2) \tag{23}
$$

where

$$
a_\mu = t_\mu (\beta_\gamma^2 T_\mu + (\beta h)^2 \delta_{\mu,1})^{1/2} + \beta_\gamma k_\gamma m \delta_{\mu,1} \tag{24}
$$

and

$$
b = 3\beta_\gamma^2 (\Delta - q) \tag{25}
$$

By linearizing the quadratic forms in S_1, the trace in Eq.(23) can be evaluated as an integral over the 3-dimensional solid angle. We finally have:

$$
Z = 3 \ \frac{dw}{\sqrt{2\pi}} \exp(-\frac{1}{2}w^2) \frac{\sinh(\sqrt{3}A)}{\sqrt{3}A} \tag{26}
$$

With

$$
A = ({}_{\mu=1}^{3} (a_\mu + \delta_{\mu,1}\sqrt{b}w)^2)^{1/2}
$$

Thus, the set of coupled self consistent equations now yields:

$$
\begin{aligned}
m &= \prod_\mu \frac{dt_\mu}{\sqrt{2\pi}} \exp(-\frac{1}{2}t_\mu^2)(\frac{1}{Z}\frac{\partial Z}{\partial a_1}) \\
r &= \frac{1}{3} \prod_\mu \frac{dt_\mu}{\sqrt{2\pi}} \exp(-\frac{1}{2}t_\mu^2) \sum_{\mu=1}^{3} (\frac{1}{Z}\frac{\partial Z}{\partial a_\mu})^2 \\
\Delta &= \frac{1}{6} \prod_\mu \frac{dt_\mu}{\sqrt{2\pi}} \exp(-\frac{1}{2}t_\mu^2) \sum_{\mu=1}^{3} (3 - \delta_{\mu,1}) (\frac{1}{Z}\frac{\partial Z}{\partial a_\mu})^2 \\
1 + 2q &= \prod_\mu \frac{dt_\mu}{\sqrt{2\pi}} \exp(-\frac{1}{2}t_\mu^2)(\frac{1}{Z}\frac{\partial^2 Z}{\partial^2 a_1})
\end{aligned}
\tag{28}
$$

We could have also obtained the set of Eq.(28) directly from Eq.(21), since within the RS approximation any disorder averaged product of thermodynamic averages is simply related to an average in replica space [46].

III RESULTS AND DISCUSSIONS

A. Phase diagrams of the Kh-Model

We will study at length the case l=3, which corresponds to the Gaussian random field Heisenberg model (GRFHM), and we give only the phase diagram (T/K,h/K) for the case l=1 corresponding to Ising model (GRFIM). For the first case (GRFHM), the disorder averaged free energy is given by:

$$
\beta F = \frac{\beta K}{2} m^2 - \frac{dw}{\sqrt{2\pi}} \exp(-\frac{1}{2}w^2) \mathrm{Log}(Z_0) \tag{29}
$$

and

$$
Z_0 = 4\pi \frac{\sinh(\sqrt{3}t_1)}{\sqrt{3}t_1} \tag{30}
$$

with

$$
t_1 = \beta K m + \beta h w \tag{31}
$$

and the self-consistence equations can be expressed as

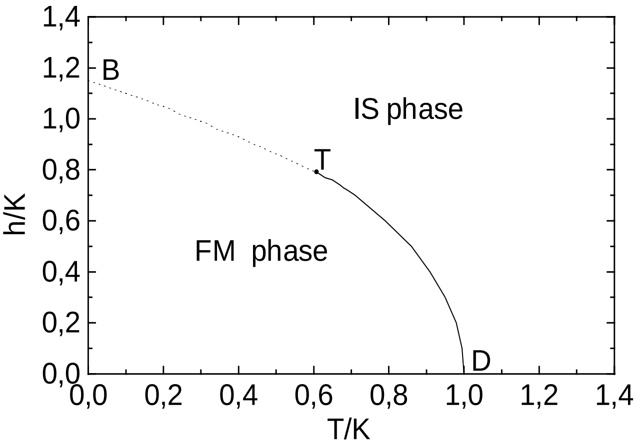

FIG.1. Phase diagram in the (T/K,h/K) plane for the Heisenberg model (l=3). Solid and dashed lines indicate seconde and first-order transition lines, respectively. The solid dot denote the tricritical point. (IS) and (FM) denote independent spins and ferromagnetic phase, respectively.

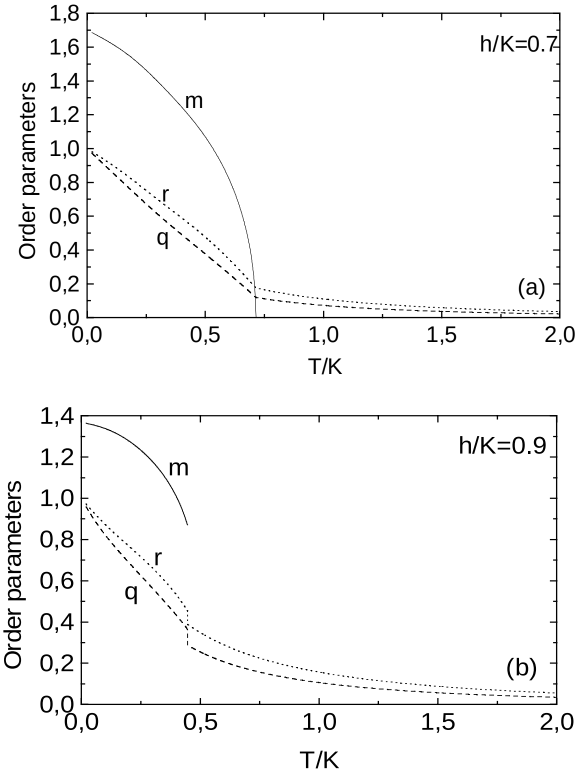

FIG.2. Temperature dependences of m, r and q for the system (l=3) in both cases (a) h/K=0.7 and (b) h/K=0.9.

$$
\begin{aligned}
m &= \frac{dw}{\sqrt{2\pi}} \exp(-\frac{1}{2} w^2) \phi(w, m) \\
r &= \frac{dw}{\sqrt{2\pi}} \exp(-\frac{1}{2} w^2) \phi^2(w, m) \qquad (32) \\
q &= 1 - \frac{dw}{\sqrt{2\pi}} \exp(-\frac{1}{2} w^2) \frac{\phi(w, m)}{t_1}
\end{aligned}
$$

where

$$\phi(w, m) = \sqrt{3} \coth(\sqrt{3} t_1) - \frac{1}{t_1} \qquad (33)$$

To investigate the instability of ferromagnetic phase and order of transition we will consider the transition region from independent spin (m=0, $r \neq 0$, $q \neq 0$) phase to ferromagnetic ($m \neq 0$, $r \neq 0$, $q \neq 0$) order, which is our main interest here. The ferromagnetic (FM)-independent spin (IS) phase transition occurs by softening to zero of magnetization parameter m. So, if the transition is continuous one can find the transition line by expanding Eq.(32) around m=0. This yields,

$$m = am - bm^3 - cm^5 - O(m^7) \qquad (34)$$

Then, a second order line is given by a=1 and b>0. For a=1 and b=0 there exist a tricritical point which is found at T/K=0.615 and h/K=0.785, and for b<0 and c>0 the system undergoes a first-order transition.

On the other hand, we have perform a numerical analysis to solve the nonlinear Eq.(32) directly. The corresponding phase diagram is depicted in fig.(1) in terms of the dimensionless parameters T/K and h/K. As is seen, there is a (FM) phase separated from a (IS) phase by a second-order transition line (DT) in a higher temperature and by a first-order curve (TB) in a lower temperature.

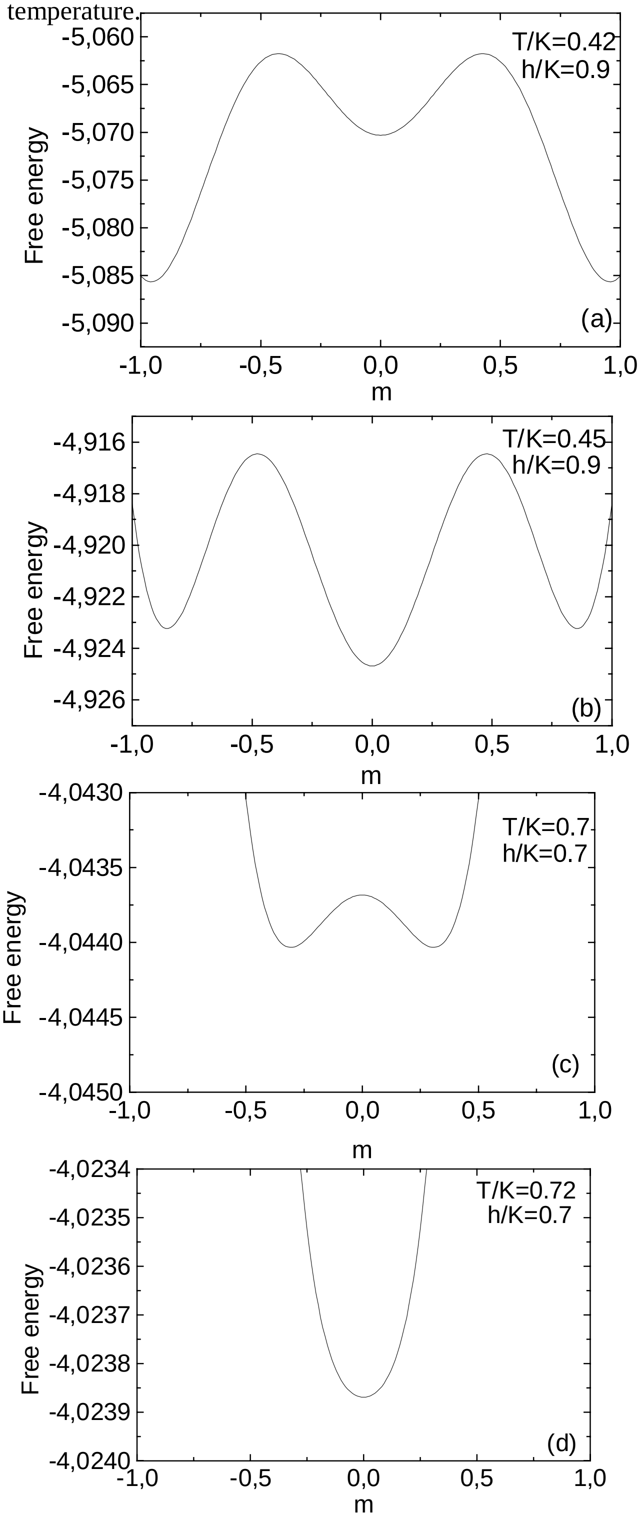

FIG.4. Phase diagram in the (T/K,h/K) plane in the limit l=1; solid and dashed lines indicate second and first order transition lines, respectively.

Fig.(2-a) and fig.(2-b) show the temperature dependences of m, r and q for the system when the value of h/K=0.7 and h/K=0.9, respectively. In the first case,

the order parameter m decreases as function of temperature and vanishes continuously at critical temperature. However in second case, the parameters m, r and q undergoes a discontinuity at critical temperature. In both figures, r and q follows a smooth curve approaching a saturation value at high temperature, i.e, the paramagnetic phase (m=0 and r=0 and q=0) is obtained only in the limit of infinite temperature where r and q behaves as $(\beta h)^2$.

In fig.(3), the free energy is plotted in terms of magnetization m for selected values of and, namely: h/K=0.9 and T/K=0.42 (fig.(3-a)), h/K=0.9 and T/K=0.45 (fig.(3-b)), h/K=0.7 and T/K=0.7 (fig.(3-c)), h/K=0.7 and T/K=0.72 (fig.(3-d)). We can show in fig.(3-a) that for $T/K < T_c/K$, the free energy has two minima. One of them corresponds to an unstable phase (IS) and the second to a stable phase (FM). However in up neighboring of the transition temperature, on the line (TB), the free energy has also two minimas (fig.(3-b)), but the (IS) phase is the stable one. Consequently, In the line (TB) we have the coexistence of (IS) and (FM) phases. Whereas, such behavior of the free energy is not observed in the second order transition line (DT) (See, fig.(3-c) and fig.(3-d)).

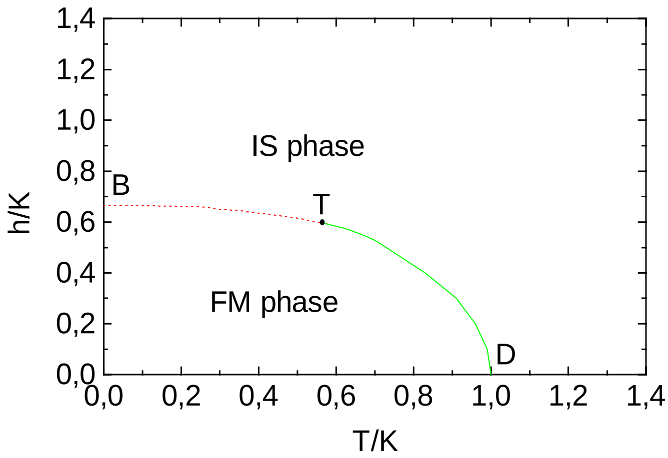

FIG.3 Free energy for the Heisenberg model (l=3) plotted as a function of magnetization m for selected values of h/K and T/K : (a) h/K=0.9 and T/K=0.42, (b) h/K=0.9 and T/K=0.45, (c) h/K=0.7 and T/K=0.7, (d) h/K=0.7 and T/K=0.72.

Now, we return to the case l=1. In such limit of our model, we obtain the disorder averaged free energy

$$\beta F = \frac{\beta K}{2} m^2 - \frac{dw}{\sqrt{2\pi}} \exp(-\frac{1}{2} w^2) \mathrm{Log}(Z_0) \tag{35}$$

where

$$Z_0 \equiv 2\cosh(\beta K m + \beta h w) \tag{36}$$

and equations of states

$$m = \frac{dw}{\sqrt{2\pi}} \exp(-\frac{1}{2} w^2) \tanh(\beta K m + \beta h w)$$
$$r = \frac{dw}{\sqrt{2\pi}} \exp(-\frac{1}{2} w^2) \tanh^2(\beta K m + \beta h w) \tag{37}$$
$$q = 0$$

which are the same expressions that were derived by Schneider and Pytte [40]. They claim that the transition between (IS) and (FM) phases is always of second-order. However, the same procedure and arguments applied in the case l=3, lead to the analogous phase diagram. Our result concerns the correct phase diagram of GRFIM is plotted in fig.(4).

B. Phase diagram of the KJh-model in zero random field

In this section we present numerical solutions to the set of coupled consistent equations, Eq.(28) for zero width of random field. First, it is well-known that in the absence of an homogeneous field, the quadrupolar parameter q is zero, while the anisotropic part of the spin glass order parameter vanishes in spin glass and paramagnetic phases [47] (see below, Fig.(6)). Consequently, the phase diagram can be determined with. $\Delta = q = 0$. The resulting equations are given by:

$$m = \prod_{\mu} \frac{dt_\mu}{\sqrt{2\pi}} \exp(-\frac{1}{2} t_\mu^2)[\frac{\sqrt{3}a_1}{|a|} g(|a|) - \frac{a_1}{|a|^2}]$$
$$r = \frac{1}{3} \prod_{\mu} \frac{dt_\mu}{\sqrt{2\pi}} \exp(-\frac{1}{2} t_\mu^2)[\sqrt{3} g(|a|) - \frac{1}{|a|}]^2 \tag{38}$$

where

$$|a| \equiv (\sum_{\mu=1}^{3} a_\mu^2)^{1/2}, g(|a|) \equiv \coth(\sqrt{3}|a|) \tag{39}$$

and

$$a_\mu = t_\mu \beta_\gamma \sqrt{2r} + \beta_\gamma k_\gamma m \delta_{\mu,1}. \tag{40}$$

Hence, only three phases are possible [47] in this case, namely, the paramagnetic (r=0,m=0), the isotropic spin glass ($r \neq 0$,m=0) and ferromagnetic ($r \neq 0$, $m \neq 0$), which correspond to, crumpled, wrinkled and flat phases of membrane, respectively.

At this level, we can examine the Eq.(38) analytically. If the transition line from wrinkled phase to flat phase is continuous, we can expand the Eq.(38) for small m. Then the second-order transition line is given by:

$$k_\gamma^{-1} = -\frac{1}{6\beta_\gamma(1-(\beta_\gamma k_\gamma)^{-1})} + \frac{1}{\sqrt{3\pi}(1-(\beta_\gamma k_\gamma)^{-1})^{1/2}}$$
$$\int_0^\infty dt \exp(-\frac{1}{2} t^2) t^3 \Phi(t, \beta_\gamma k_\gamma) \tag{41}$$

Where

$$\Phi(t, \beta_\gamma k_\gamma) \equiv \coth(\sqrt{6}\beta_\gamma (1-(\beta_\gamma k_\gamma)^{-1})^{1/2} t) \tag{42}$$

the critical value of disorder at T=0 for which the flat phase becomes unstable is $J_c/K = \sqrt{4/3\pi}$, which is far from the value $2/\sqrt{18\pi}$, as quoted by previous authors [5,6].

Moreover, by performing an expansion in powers of a_μ up to eighth order, which is equivalent to high temperature expansion for βf up to seventh order in β_γ, we find from Eq.(38):

$$\begin{aligned}
m &= m_d - 2\beta_\gamma^2 m_d r - \frac{1}{5}m_d^3 + 8\beta_\gamma^4 m_d r^2 \\
&+ \frac{8}{5}\beta_\gamma^2 m_d^3 r + \frac{2}{35}m_d^5 - \frac{216}{5}\beta_\gamma^6 m_d r^3 \\
&- \frac{324}{25}\beta_\gamma^4 m_d^3 r^2 - \frac{162}{175}\beta_\gamma^2 m_d^5 r - \frac{1}{175}m_d^7 \\
r &= 2\beta_\gamma^2 r + \frac{1}{3}m_d^2 - 8\beta_\gamma^4 r^2 - \frac{8}{3}\beta_\gamma^2 m_d^2 r \\
&- \frac{2}{15}m_d^4 + \frac{216}{5}\beta_\gamma^6 r^3 + \frac{108}{5}\beta_\gamma^6 m_d^2 r^2 \\
&+ \frac{54}{25}\beta_\gamma^2 m_d^4 r + \frac{2}{175}m_d^6
\end{aligned} \tag{43}$$

Where

$$m_d \equiv \beta_\gamma k_\gamma m \tag{44}$$

Eq.(43) indicate that the model exhibits a second order transition between flat and crumpled phases at T/K for J/K $< \sqrt{2}/2$, and between crumpled and wrinkled phases at J/K= $\sqrt{2}/2$ for T/K>1. Indeed, this equation provide information only for high temperature and is not able to reproduce the correct phase diagram at low temperature. However, the RS phase diagram can be obtained completely by investigating the detailed numerical solutions of Eq.(28) (with h=0) each time with a minimization of the free energy given by Eq.(22) to locate the correct transition point.

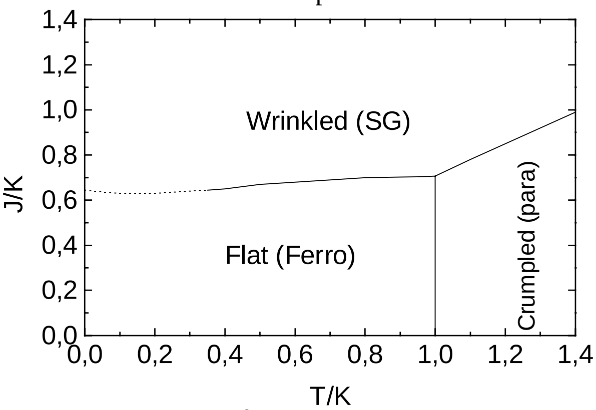

FIG.5 Phase diagram of Hamiltonian (1) with zero random local field h/K. The continuous line is the second order transition and the dashed line is the first order transition

The result is shown in figure (5) where we plot the RS phase diagram supported by the behavior of order parameters m, r, Δ and q as a function of temperature. The existence of crumpled (m=0, r=0), wrinkled (m=0, $r \neq 0$) and flat (m≠0, $r \neq 0$) phases were previously recognized [5,6]. The novel features of the phase diagram is the reentrance phenomenon which appears at low temperature, and the existence of first and second order transition between wrinkled and flat phases.

As is seen in figure(6), one observes that for J/K=0.4 and J/K=0.9, there are continuous transitions respectively, between flat and wrinkled phases (Fig.6a), and between wrinkled and crumpled phases (Fig.6b) (see also Eq.(43)). Whereas, in Fig.6c, we show that the model exhibits a

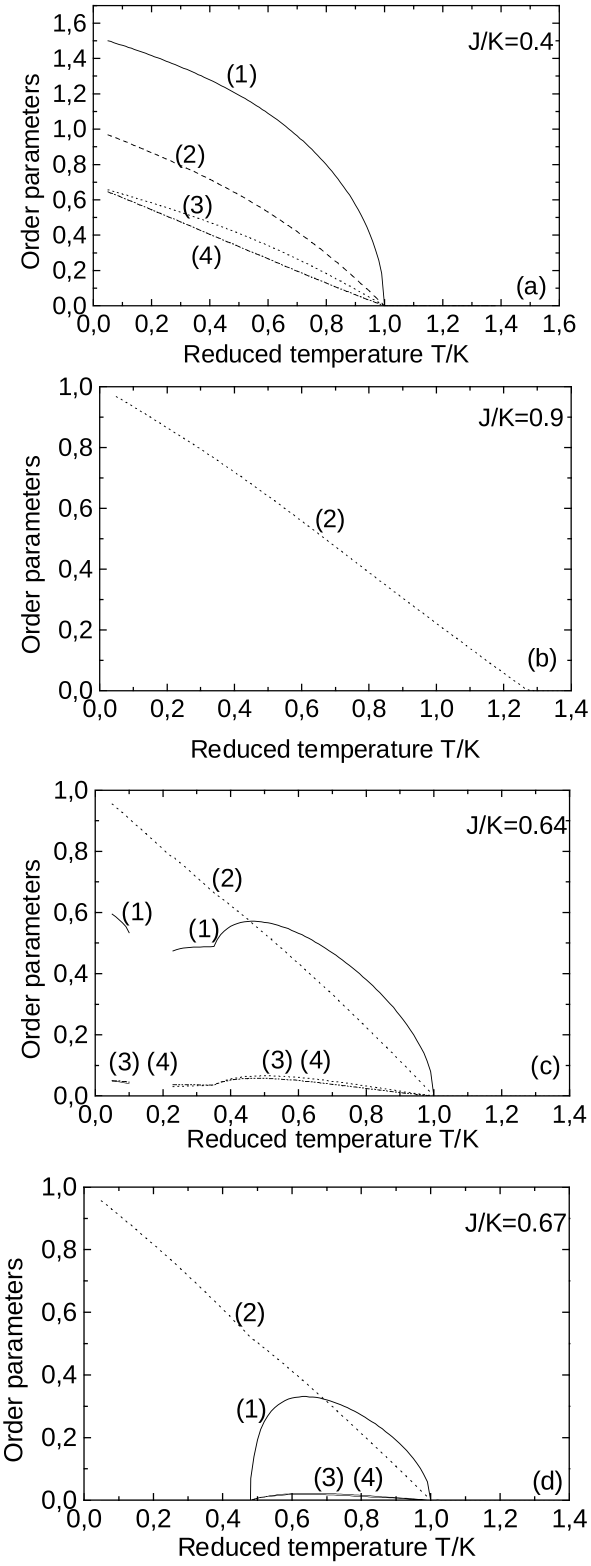

FIG.6 Order parameters as a function of reduced temperature for h/K=0 and various values of J/K. The label (1), (2), (3) and (4) correspond respectively to m, r, Δ and q.

reentrant transition (flat-wrinkled-flat) at low temperature for J/K=0.64, by a first order transition from a flat to wrinkled phase and from a wrinkled to flat phase; while for J/K=0.67 this transition is of second order (Fig.6d).

Note that in figure(6), the variation of order parameters q and Δ as function of temperature is very small in vicinity of the flat-wrinkled phase transition, then our assumption Δ=q=0 is reasonable. Consequently, the second order boundary between flat and wrinkled phases can be obtained analytically by Eq.(19).

C. Stability limit of RS solution of the kJh-model in zero random field

It is well known that the RS solution is generally unstable against RSB [41,48]. In presence or absence of an homogeneous external field, a borderline separating the regions of stable and unstable RS solution can be drawn, known as the AT line [41]. Thus the spin glass transition should correspond to a change from a single RS spin glass order parameter at $T>T_g$ (where T_g is the temperature of freezing) to a RSB form at $T<T_g$ represented by the Parisi function r(x) [49], 0<x<1. Following Almeida and Thouless the stability of given solution is ensured by positive definite Hessian matrix associated with the functional $g(m^{\alpha}, r^{\alpha\beta}, \Delta^{\alpha\beta}, q^{\alpha})$ given by Eq.(19).

As discussed in detail in ref [41,48], the problem of stability reduces to the requirement that all eigenvalues of this Hessian matrix must be negative. In our case, it can be shown that the RS solution is stable if:

$$\beta_{\gamma}^{-2} > \frac{2}{3} \prod_{\mu} \frac{dt_{\mu}}{\sqrt{2\pi}} \exp(-\frac{1}{2} t_{\mu}^{2}) \Phi(m, r) \tag{45}$$

Where

$$\Phi(m,r) \equiv 9\coth^{4}(\sqrt{3}|a|) + 9(1-2\coth^{2}(\sqrt{3}|a|)) - \frac{4\sqrt{3}}{|a|^{3}}\coth(\sqrt{3}|a|) + \frac{3}{|a|^{2}}(2+\frac{1}{|a|^{2}}) \tag{46}$$

the order parameter m and r are given by Eq.(38).

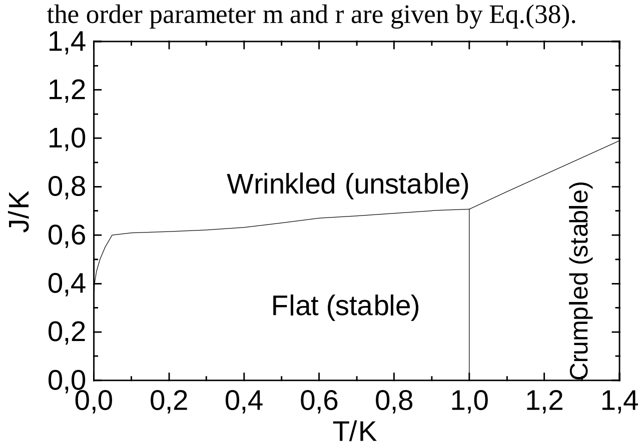

FIG.7 Phase diagram showing the limits of unstability of replica symmetry solution.

The line of instability obtained by numerical evaluation of simultaneous solution of Eq.(38) and the equality in Eq.(45) is shown in figure (7). Above that line only a solution with broken replica symmetry provides a correct description of wrinkled state. It should be noted that approximate analytic expression for this line can be derived for the region where T and J go to zero. The result we have in this region is:

$$\frac{T}{J} \approx \alpha_0 \exp(-\frac{3}{4}(\frac{K}{J})^2) \tag{47}$$

where α_0=0.65416. Notice that the AT line of this model has the same behavior at low temperature as the AT line of a SK model. This result is not surprising because of the strong coupling between the spin components in this model [47].

D. Effect of the random field on the flat phase of the KJh-model

Recently it has been shown that randomness in the metric destabilizes a flat membrane towards a wrinkled phase [10,48], whereas random curvature yields a new T=0 flat phase [5,6]. Here we discuss the effect of random local field on the flat phase. Within the replica symmetric solution, the free energy per pseudospin and the equilibrium equations are given by Eq.(22) and Eq.(28), respectively. As is seen in Eq.(22), the variance of the random field h acts as an effective ordering field for the wrinkled state ($r \neq 0$, m=0), the corresponding order parameters r and Δ are non zero at all temperatures. That behavior which have already been presented in the literature [50,51], can be seen from the high temperature expansion of the Eq.(28) which gives at relevant order to:

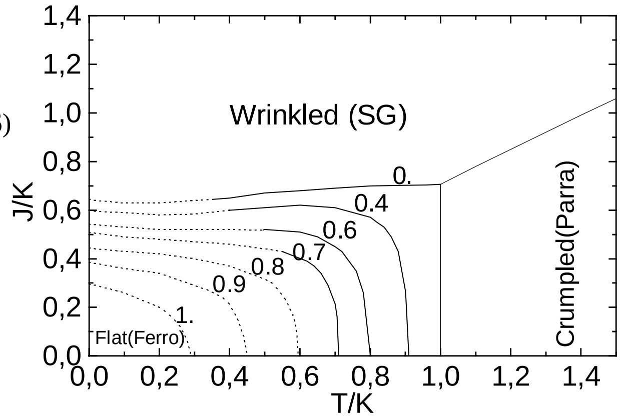

FIG.8 Mean field phase diagram in presence of a random local field of with h/K. The number accompanying each curve correspond to the selected values of h/K. The dashed lines define the first order transition and the solid line denote the second order transition.

$$\begin{aligned}
m &= m_d(1-\frac{6}{5}(\beta h)^2) - \frac{1}{5}m_d^3(1-\frac{40}{7}(\beta h)^2) + \ldots \\
r &= \frac{2}{3}(\beta h)^2 + 2\beta_{\gamma}^2 r(1-\frac{8}{3}(\beta h)^2) + \ldots \\
\Delta &= \frac{2}{3}(\beta h)^2 - 2\beta_{\gamma}^2 \Delta(1-\frac{68}{15}(\beta h)^2) + \ldots \\
q &= \frac{2}{5}(\beta h)^2 - \frac{3}{5}\beta_{\gamma}^2 q(1+\frac{4}{7}(\beta h)^2) + \ldots
\end{aligned} \tag{48}$$

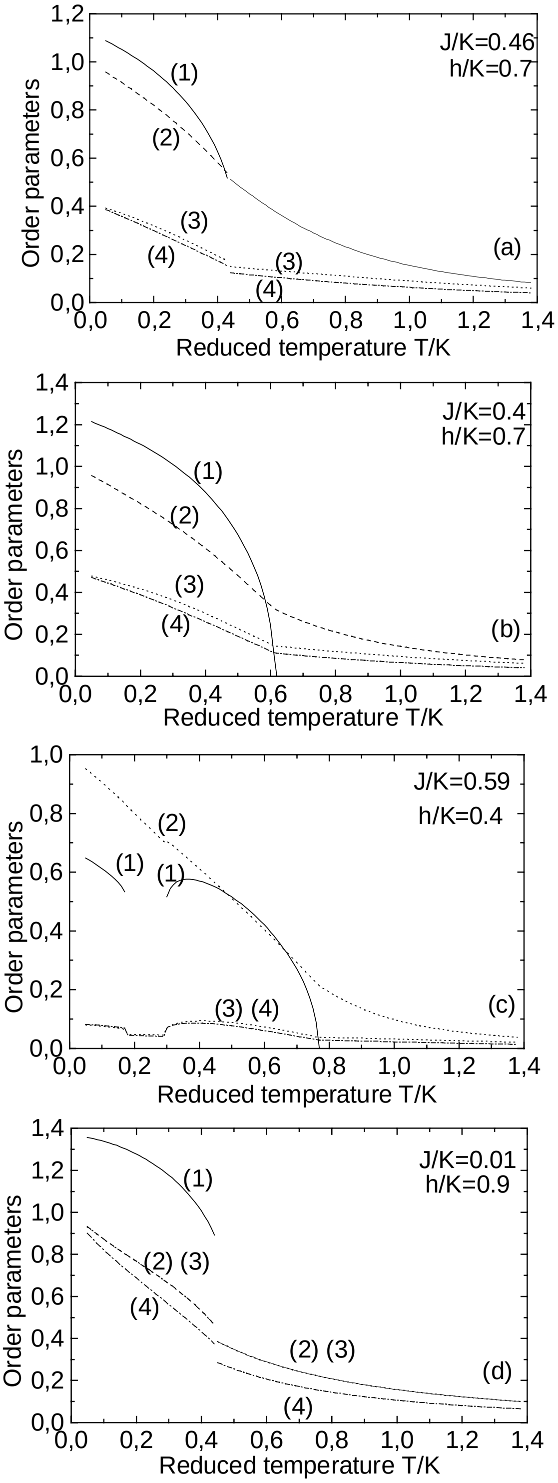

FIG.9 Order parameters as a function of reduced temperature for selected values of h/K and J/K. The label (1), (2), (3) and (4) correspond respectively to m, r, Δ and q.

Thus, a conventional phase transition, where r would change from zero to a non zero value, is here remarkably absent (see figure(9)).

In order to draw a phase diagram for different values of h/K, we have studied the detailed numerical solutions of Eq.(28) with a minimization of the free energy given by Eq.(22). The surface of second order phase transitions can be calculated analytically by linearizing The parameter m in Eq.(28) around m=0, which gives:

$$\frac{T}{K} = [1 + 2q - (r + 2\Delta)] \tag{49}$$

where r, Δ and q are determined via Eq.(28). The resulting phase diagram is displayed in figure(8). From which, one can observes that there exists a tricritical line separating the surfaces of second and first order transitions.

The dependence of m, r, Δ and q as function of the temperature for some relevant values of h/K and J/K, is shown in figure (9). From these figures it is clear that the magnetization undergoes a discontinuity at the first order transition (Fig.9a), while its vanishes
continuously at the second order transition (Fig.9b). For h/K=0.4 and J/K=0.59, the magnetization vanishes twice (Fig.9c); this is a feature of a reentrant phenomena in agreement with the phase diagram in figure (8). However, when increasing the variance of random field h/K, the reentrant phenomena is canceled. This result is in agreement with previous works related to spin glass model [51]. Moreover, by increasing the variance of random field, the flat phase is unstable and the wrinkled phase takes over this result is consistent with what has been concluded in Ref [10] and Ref [52]. Furthermore, when h/K become large then h/K=0.712 (see Fig.9d), the random field makes the transition between flat and wrinkled phase first order.

IV. CONCLUSION

In the first case, we have investigated the l-component spin vector model with random field, which is considered as a generalization of Schneider and Pytte model [40]. We show that, the GRFIM and GRFHM exhibits a second-order and first-order transition separated by a tricritical point. All these transition are between independent spin and ferromagnetic phases. Our result concerns the phase diagram in the case l=1 (GRFIM) is in disagreement with the previous work [40]; This inconsistencies is clearly related to an insufficient numerical analysis of the thermodynamic properties of the random field model. We have also investigated the case l large (not given in the text), and we have find the same topological phase diagram as in GRFIM (l=1) and GRFHM (l=3).

In the second case, We have studied the randomly frustrated membrane model in presence of a Gaussian random field with variance h. This type of randomness could be caused by a global fluctuations of surface normal to the membrane . Within replica symmetry and $h=0$, we have take up again the investigation of randomly disordered membrane model which has been introduced previously [5,6]. We find that, the model exhibits a second and first order transition between flat (ferromagnetic) and wrinkled (spin glass) phases, and the reentrance phenomenon occurs at low temperature. These main results which are not mentioned in refs [5,6]. A related model has been studied by Rubinstein et al [52] who also pointed out the existence of a reentrant

transition from a ferromagnetic (flat) ordered phase to a spin glass (wrinkled) phase. However, by analogy to spin glass problem [52], the reentrance of wrinkled phase is shown to be linked to the negative entropy at T=0 which makes the RS solution to be a poor representation of the wrinkled phase. Thus, it would be very useful to find a replica symmetry breaking solution of a such model with $h=0$. We note that a Parisi RSB formalism for this model has been derived by Attal et al [6]. However, in our analysis of RSB we have follow the same route as Almeida and Thouless in their investigation of instability limit of RS solution for SK model [41]. We have find the analytical equations of the equivalent AT line of that model. Therefore, as in ref [6], within the RSB scheme we can concluded that (see Eq.(47) the model exhibits a wrinkled-flat phase as T go to zero even for infinitesimal disorder. On other hand, we have also investigate the randomly frustrated membrane by adding a Gaussian random field, within the RS solution. We find that the random field width makes the flat phase unstable and the wrinkled phase takes over. Moreover, the reentrant effect is completely removed when h increases and the transition between wrinkled and flat phase is first order.

ACKNOWLEDGEMENT

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