Diffusion of stochastic magnetic field lines in a confined plasma using the generalized mapping

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The evolution of stochastic magnetic field lines has been studied numerically. Also, we have studied the transition from partial stochasticity to global stochasticity of magnetic field lines. We have developed the analytical calculations of diffusion coefficient of magnetic field lines through magnetic surfaces using the Fourier paths technique. The magnetic field lines are described by the generalized mapping. A comparison between our analytical and numerical calculations has been done.

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I. INTRODUCTION

The magnetic confinement as applied in tokamaks is based on the existence of magnetic surfaces which maintain hot plasma far from the wall. The magnetic field perturbation, involve magnetic surfaces destruction and thus a magnetic field lines diffusion through the magnetic surfaces. To a lowest order approximation, electrons and ions of plasma follow the magnetic field lines [1]. Then, the magnetic field perturbation can contribute to anomalous transport by particles and energy loses.

In the present work, we study the magnetic field lines diffusion induced by a magnetic perturbation. Such studies were done by M. Hugon and al. [2], J. T. Mendonça [3] and A. Oualyoudine and al. [4]. In the references [2-3], it have been shown that the diffusion coefficient is close to quasi-linear limit for 1 γ 1.6 where γ is the overlapping parameter, and start to saturate for γ 1.8 . To describe the stochastic magnetic field lines behavior, the authors have used a Poincaré section obtained by the magnetic field line intersection with the plan $\zeta = 0$. J.T. Mendonça have derived to describe this poincaré section the standard mapping equations [3]. We are interested in the present paper to study the stochastic magnetic field lines behavior which are described by the generalized mapping to the second order [5]. We have calculated the magnetic field lines diffusion coefficient using the Fourier paths technique [6-8].

In the section 2, we give the magnetic field lines equations. From this equations we derive in the section 3 the discrete mapping equations and give then the generalized mappings to the second order which has been used to describe the stochastic magnetic field lines behavior. After in the section 4, we determine analytically using the Fourier paths technique the diffusion coefficient. In the section 5, we represent the Poincaré section that is described by the generalized mapping and give a comparison between our analytical and numerical calculations of the diffusion coefficient.

II. The magnetic field lines equations

Let ψ, θ and ζ be the magnetic coordinates. It is shown that an arbitrary magnetic field **B** can be written in a form [9]

$$\mathbf{B} = \nabla\psi \times \nabla\theta + \nabla\zeta \times \nabla\psi_p \qquad (1)$$

where the poloidal flux ψ_p is a function of ψ, θ and ζ. The magnetic field lines are defined by

$$\frac{d\psi}{d\zeta} = \frac{\mathbf{B}.\nabla\psi}{\mathbf{B}.\nabla\zeta},\ \frac{d\theta}{d\zeta} = \frac{\mathbf{B}.\nabla\theta}{\mathbf{B}.\nabla\zeta} \qquad (2)$$

which by Eq. (1), satisfy

$$\frac{\partial\theta}{\partial\zeta} = \frac{\partial\psi_p}{\partial\psi},\ \frac{\partial\psi}{\partial\zeta} = -\frac{\partial\psi_p}{\partial\theta} \qquad (3)$$

If there is a nearby field with good surfaces one can choose ψ_p so that

$$\psi_p(\psi,\theta,\zeta) = \psi_{p_0}(\psi) + \tilde{\psi}_p(\psi,\theta,\zeta) \qquad (4)$$

with $\left|\tilde{\psi}_p\right| << \left|\psi_{p_0}\right|$. This is a standard action (ψ)-, angle (θ)- and 'time' (ζ)-dependent Hamiltonian form.

When $\tilde{\psi}_p = 0$, the magnetic field lines equations (3) are integrable and magnetic lines are all tangents to regular magnetic surfaces defined by $\psi =$ constant.

For a perturbed magnetic field lines ($\tilde{\psi}_p \neq 0$), some magnetic surfaces around the rational surfaces will eventually be destroyed and replaced by stochastic regions. If these regions are enough to overlapped, significant magnetic field lines diffusion will take place.

III. Discrete mapping

In this section, we transform the magnetic field lines equations to a discrete mapping and give. In the case of a regular magnetic surfaces $\tilde{\psi}_p = 0$, the Poincaré section of magnetic field line in the plan $\zeta = 0$ (mod 2π) is described by

$$\begin{aligned} \psi_{k+1} &= \psi_k \\ \theta_{k+1} &= \theta_k + 2\pi t(\psi_{k+1}) \end{aligned} \qquad (5)$$

When the magnetic perturbed ($\tilde{\psi}_p \neq 0$), it is possible to describe the Poincaré section by the discrete mapping

$$\begin{aligned} \psi_{k+1} &= \psi_k + f(\psi_{k+1},\theta_k) \\ \theta_{k+1} &= \theta_k + 2\pi t(\psi_{k+1}) + g(\psi_{k+1},\theta_k) \end{aligned} \qquad (6)$$

where the functions f and g are determined by the following equations

$$f = -\frac{\partial \tilde{F}}{\partial \theta_k}\ ,\ g = \frac{\partial \tilde{F}}{\partial \psi_k} \tag{7}$$

$$\frac{\partial f}{\partial \psi_{k+1}} + \frac{\partial g}{\partial \theta_k} = 0 \tag{8}$$

To determine the generating function $\tilde{F}$ from the perturbed Hamiltonian, we consider the Fourier expansion of $\tilde{\psi}_p$

$$\tilde{\psi}_p(\psi,\theta,\zeta) = \sum_m \sum_n a_{nm}(\psi)\exp(im\theta + in\zeta) \tag{9}$$

We assume that $a_{nm} = f_{nm}\exp(i\phi_{nm})$ where f_{nm} and ϕ_{nm} are real, assuming further that $f_{nm} = f_m$ independent of the toroïdal mode number n, and that the phases ϕ_{nm} can be split as $\phi_{nm} = \phi_n + \phi_m$, we obtain

$$\tilde{\psi}_p = \sum_m f_m \cos(m\theta + \phi_m) \sum_n \cos(n\zeta + \phi_n) \tag{10}$$

where now m only takes positive entire values. In the next we consider the existence of an infinite number of Fourier components (extending the sum over n varies from $-\infty$ to $+\infty$) and assuming that $\phi_n = n\alpha$, where is a very large irrational $\alpha >> 1$. All this leads to the following expression for the Hamiltonian field perturbation

$$\tilde{\psi}_p(\psi,\theta,\zeta) = 2\pi\tilde{F}\sum_{n=-\infty}^{+\infty}\delta(\zeta + \alpha - 2\pi n) \tag{11}$$

where

$$\tilde{F} = \sum_m f_m \cos(m\theta + \phi_m) \tag{12}$$

Integration of the field lines Eqts. (3), with Eq. (12) taken into account, over an interval $\Delta\zeta = 2\pi$, lead then to an explicit form of the discrete mapping

$$\begin{aligned} \psi_{k+1} &= \psi_k + \sum_m (2\pi f_m)\sin(m\theta_k + \phi_m) \\ \theta_{k+1} &= \theta_k + 2\pi t(\psi_{k+1}) \end{aligned} \tag{13}$$

If we neglected the phase differences $\phi_m = 0$ and we assume that $t(\psi_{k+1}) = \psi_{k+1}$, we obtain

$$\begin{aligned} I_{k+1} &= I_k + \sum_m K_m \sin(m\theta_k) \\ \theta_{k+1} &= \theta_k + I_{k+1} \end{aligned} \tag{14}$$

where

$$I_k = 2\pi\psi_k\ ,\ K_m = (2\pi)^2 m f_m \tag{15}$$

For $m = 1$, we find the standard mapping [3,6,10].
The generalized mapping that we have used in this work correspond to $m = 2$ writes as

$$\begin{aligned} I_{k+1} &= I_k + K_1\sin(\theta_k) + K_2\sin(2\theta_k) \\ \theta_{k+1} &= \theta_k + I_{k+1} \end{aligned} \tag{16}$$

IV- Diffusion coefficient

In this section, we are going to calculate the diffusion coefficient of the stochastic magnetic field lines described by the generalized mapping Eqts. (16). The diffusion coefficient of the magnetic field lines is expressed in term of the conditional probability density W as [6,8]

$$\begin{aligned} D_n &= \frac{(\Delta I)^2}{2n} \\ &= \frac{1}{2n}\int W(I,\theta,n/I_0,\theta_0,0)\,(I - I_0)^2\, dI\, d\theta \end{aligned} \tag{17}$$

where n is the number of iterations to do from (I_0,θ_0) to (I,θ). We are interested in long-time behaviour (i.e. n is large) so the value of I_0 is irrelevant. Then, the diffusion coefficient can be simplified to

$$D_n = \frac{1}{2n}\int W(I,\theta,n/I_0,\theta_0,0)\, I^2\, dI\, d\theta \tag{18}$$

Substituting W by the expansion

$$W(I,\theta,n/I_0,\theta_0,0) = \sum_m \int dq\, \exp(im\theta + iIq)\, a_n(m,q) \tag{19}$$

in the equation (18) we obtain (see reference 6)

$$D_n = -\frac{(2\pi)^2}{2n}\left.\frac{\partial^2 a_n(0,q)}{\partial q^2}\right|_{q=0} \tag{20}$$

To explicit the diffusion coefficient expression, we need to determine the Fourier coefficient a_n. From the W expansion we have

$$\begin{aligned} a_n(m,q) = \frac{1}{(2\pi)^2}\int dI\, d\theta\, \exp(-im\theta - iqI) \\ \times W(I,\theta,n/I_0,\theta_0,0) \end{aligned} \tag{21}$$

The first Fourier coefficient a_0 which correspond to n=0 is

$$a_0(m,q) = \frac{1}{(2\pi)^2}\exp(-im\theta_0 - iqI_0). \tag{22}$$

For n > 0, introducing

$$\begin{aligned} W(I,\theta,n/I_0,\theta_0,0) = \int dI'\, d\theta'\, W(I,\theta,n/I',\theta',n-1) \\ \times W(I',\theta',n-1/I_0,\theta_0,0) \end{aligned} \tag{23}$$

where

$$W(I,\theta,n/I',\theta',n-1)=\delta(I-I'-K_1\sin\theta' \\ -K_2\sin 2\theta')\times\delta(\theta-\theta'-I'-K_1\sin\theta'-K_2\sin 2\theta')$$

in the equation (21) we can write

$$a_n(m,q)=\frac{1}{(2\pi)^2}\quad dI\,d\theta\,\exp(-im\theta-iqI) \\ \times\ dI'd\theta'\delta(I-I'-K_1\sin\theta'-K_2\sin 2\theta') \\ \times\ \delta(\theta-\theta'-I'-K_1\sin\theta'-K_2\sin 2\theta') \\ \times\ dq' \underset{m'}{\ }\exp(im'\theta'+iq'I')\,a_{n-1}(m',q'). \tag{24}$$

The θ - and I' -integration gives

$$a_n(m,q)=\frac{1}{2\pi}\sum_{m'}\ d\theta'\,a_{n-1}(m',q') \\ \times\exp[i(m'-m)\theta'-iq'(K_1\sin\theta'+K_2\sin 2\theta')]. \tag{25}$$

with $q'=q+m$.

We have used the following developments

$$\exp(iq'K_1\sin\theta')=\underset{l=-\infty}{\overset{+\infty}{\ }}J_l(K_1|q'|)\exp(il\theta'\mathrm{sgn}q'),$$

$$\exp(iq'K_2\sin 2\theta')=\underset{l'=-\infty}{\overset{+\infty}{\ }}J_{l'}(K_2|q'|)\exp(i2l'\theta'\,\mathrm{sgn}\,q'). \tag{26}$$

where $J_{l,\,l'}$ are the ordinary Bessel functions. After integrating over θ', we find

$$a_n(m_n,q_n)=\underset{l_n}{\ }J_{l_n}(K_1|q_{n-1}|) \\ \times\underset{l'_n}{\ }J_{l_n'}(K_2|q_{n-1}|)a_{n-1}(m_{n-1},q_{n-1}), \tag{27}$$

with

$$m_n=m_{n-1}-(l_n+2l_n')\ \mathrm{sgn}\,q_{n-1} \\ q_n=q_{n-1}-m_n. \tag{28}$$

Applying n times this recursion relation, we obtain a_n in term of a_0

$$a_n(m_n,q_n)=\underset{l_n\ldots\ldots l_1}{\ }J_{l_n}(K_1|q_{n-1}|)\ldots\ldots J_{l_1}(K_1|q_0|) \\ \times\underset{l_n'\ldots\ldots l_1'}{\ }J_{l_n'}(K_2|q_{n-1}|)\ldots\ldots J_{l_1'}(K_2|q_0|)\ a_0(m_0,q_0). \tag{29}$$

If we put $L_n=l_n+2\,l_n'$, the relations (28) become

$$m_n=m_{n-1}-L_n\ \mathrm{sgn}\,q_{n-1} \\ q_n=q_{n-1}-m_n. \tag{30}$$

The same relations were found for the standard mapping and we can then use the Fourier paths technique to calculate the summation in the equation (29). In this case, the paths in the Fourier space (m,q) are defined through (30) by the set of the n integers $\{L_n\ldots\ldots L_1\}$. From (20), they must end at the origin $m_n=0$ and $q_n=0$.

The simplest path is one which never leaves the origin. This correspond to all are L_i are equal to zero. Only, the case where l_i=0 and l_i'=0 gives a contribution to a_n. This contribution is

$$a_n(0,q)=\frac{1}{(2\pi)^2}[J_0(K_1q)]^n[J_0(K_2q)]^n\exp(-iqI_0). \tag{31}$$

Substituting a_n by this expression in the equation (20), we obtain

$$D_n=\frac{K_1^2}{4}+\frac{K_2^2}{4}+\frac{I_0^2}{2n} \tag{32}$$

For n large $D_n\to D_{QL}=\dfrac{K_1^2+K_2^2}{4}$. We Note that we obtain the same result from the following relation

$$D_{QL}=\frac{D}{2}=\frac{1}{4\pi}\ \overset{2\pi}{\underset{0}{\ }}\ (\Delta I_1^{\,2})\,d\theta_0 \tag{33}$$

where $\Delta I_1=K_1\sin(\theta_0)+K_2\sin(2\theta_0)$.

Now, to find the correction to the quasi-linear result, suppose that the path leave the origin at the step r, make a three-step excursion and return to the origin at the step r+3. we must have m_3=0 and $q_{r+3}=q_r$. We put $m_1=m$ (m must be different to zero otherwise, we will have a path that remains at the origin for n steps). Then, $m_2=-m$.

Through the equation (30), we deduce the values of L_{r+1}, L_{r+2} and L_{r+3} which are

$$L_{r+1}=-m \\ L_{r+2}=2m\,\mathrm{sgn}(q_r-m) \\ L_{r+3}=-m \tag{34}$$

The other L_i take the value 0. This implies as in the previous case that $l_i=l_i'=0$. a_n writes then as

$$a_n=\frac{1}{(2\pi)^2}[J_0(K_1q)]^{n-3}[J_0(K_2q)]^{n-3} \\ \times\underset{l_{r+1}',\,l_{r+3}'}{\ }J_{-m-2l_{r+1}'}(K_1q)J_{l_{r+1}'}(K_2q) \\ \times J_{-m-2l_{r+3}'}(K_1q)J_{l_{r+3}'}(K_2q) \\ \times\sum_{l_{r+2}'}J_{2m\,\mathrm{sgn}(q-m)-2l_{r+2}'}(K_1|q-m|)$$

$$\times J_{l_{r+2}'}(K_2|q-m|)\exp(-iqI_0) \qquad (35)$$

The second derivative of a_n for $q=0$ is

$$\frac{\partial^2}{\partial q^2}a_n\bigg|_{q=0} = -\frac{1}{(2\pi)^2}[\frac{n-3}{2}(K_1^2+K_2^2)+I_0^2]A\Big|_{q=0}$$
$$-\frac{2iI_0}{(2\pi)^2}\frac{\partial A}{\partial q}\bigg|_{q=0}+\frac{1}{(2\pi)^2}\frac{\partial^2 A}{\partial q^2}\bigg|_{q=0} \qquad (36)$$

where

$$A = {}_{l_{r+1}',\, l_{r+3}'} J_{-m-2l_{r+1}'}(K_1 q) J_{l_{r+1}'}(K_2 q)$$
$$\times J_{-m-2l_{r+3}'}(K_1 q) J_{l_{r+3}'}(K_2 q)$$
$$\times {}_{l_{r+2}'} J_{2m\,\mathrm{sgn}(q-m)-2l_{r+2}'}(K_1|q-m|) J_{l_{r+2}'}(K_2|q-m|)$$

The first and the second terms are null. Then, we obtain

$$\frac{\partial^2}{\partial q^2}a_n\bigg|_{q=0} = \frac{1}{(2\pi)^2}\frac{\partial^2 A}{\partial q^2}\bigg|_{q=0} \qquad (37)$$

We have shown also that

$$\frac{\partial^2 A}{\partial q^2}\bigg|_{q=0} = {}_{l_{r+1}',\, l_{r+3}'}\frac{\partial^2}{\partial q^2}(J_{-m-2l_{r+1}'}(K_1 q)$$
$$\times J_{l_{r+1}'}(K_2 q) J_{-m-2l_{r+3}'}(K_1 q) J_{l_{r+3}'}(K_2 q))\Big|_{q=0}$$
$$\times {}_{l_{r+2}'} J_{2m\,\mathrm{sgn}(-m)-2l_{r+2}'}(K_1|m|) J_{l_{r+2}'}(K_2|m|) \qquad (38)$$

This expression is simplified to

$$\frac{\partial^2 A}{\partial q^2}\bigg|_{q=0} = {}_{\lfloor l_{r+1}',\, l_{r+3}'}\frac{K_1^2}{2}(J_{-m-2l_{r+1}'-1}(K_1 q)$$
$$\times J_{l_{r+1}'}(K_2 q) J_{-m-2l_{r+3}'-1}(K_1 q) J_{l_{r+3}'}(K_2 q))$$
$$+\frac{K_1^2}{2}(J_{-m-2l_{r+1}'+1}(K_1 q)$$
$$\times J_{l_{r+1}'}(K_2 q) J_{-m-2l_{r+3}'+1}(K_1 q) J_{l_{r+3}'}(K_2 q))$$
$$+\frac{K_2^2}{2}(J_{-m-2l_{r+1}'}(K_1 q)$$
$$\times J_{l_{r+1}'-1}(K_2 q) J_{-m-2l_{r+3}'}(K_1 q) J_{l_{r+3}'-1}(K_2 q))$$
$$+\frac{K_2^2}{2}(J_{-m-2l_{r+1}'}(K_1 q)$$
$$\times J_{l_{r+1}'+1}(K_2 q) J_{-m-2l_{r+3}'}(K_1 q) J_{l_{r+3}'+1}(K_2 q))\}\Big|_{q=0}$$
$$\times {}_{l_{r+2}'} J_{2m\,\mathrm{sgn}(-m)-2l_{r+2}'}(K_1|m|) J_{l_{r+2}'}(K_2|m|) \qquad (39)$$

The first term in the summation is different to 0 when $q \to 0$, if we have

$$l_{r+1}' = l_{r+3}' = 0 \text{ and } m = -1$$

Paths corresponding to above conditions have the form (see fig. 1)

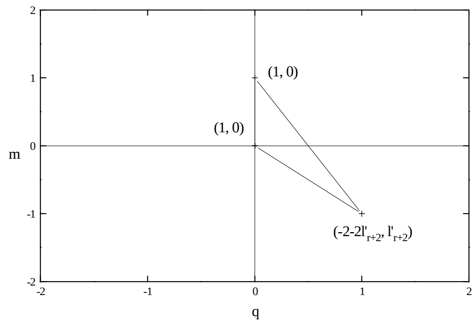

Fig. 1. A three path in the Fourier space which correspond to m = -1.

The contribution of these paths to the diffusion coefficient is

$$D_n = -\frac{K_1^2}{4n}\,{}_{l=-\infty}^{+\infty} J_{-2-2l}(K_1) J_1(K_2) \qquad (40)$$

The other terms give these contribution

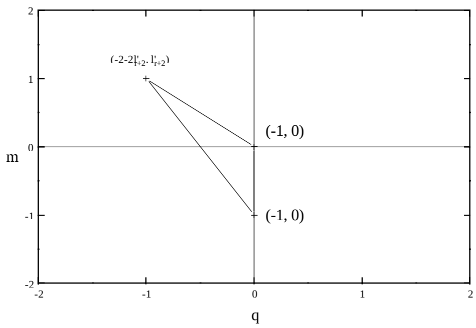

Fig. 2.

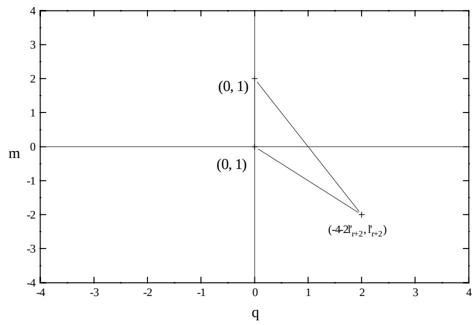

Fig . 3

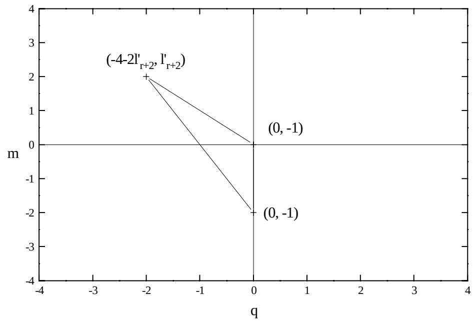

Fig. 4.
Figures 2, 3 and 4 represent the three paths which contribute to diffusion and which correspond respectively to m = 1, m = -2 and m =2.

$$D_n = -\frac{K_1^2}{4n}\sum_{l=-\infty}^{+\infty} J_{-2-2l}(K_1)\,J_l(K_2) - \frac{K_2^2}{2n}\sum_{l=-\infty}^{+\infty} J_{-4-2l}(2K_1)\,J_l(2K_2) \qquad (41)$$

Thus, the coefficient of diffusion resulting from all of this paths is

$$D_n = -\frac{K_1^2}{2n}\sum_{l=-\infty}^{+\infty} J_{-2-2l}(K_1)\,J_l(K_2) - \frac{K_2^2}{2n}\sum_{l=-\infty}^{+\infty} J_{-4-2l}(2K_1)\,J_l(2K_2) \qquad (42)$$

We note finally that this calculation has been done for any r which varies from 1 to n-2. Then for n large, the diffusion coefficient is

$$D_n = \frac{K_1^2 + K_2^2}{4} - \frac{K_1^2}{2}\sum_{l=-\infty}^{+\infty} J_{-2-2l}(K_1)\,J_l(K_2) - \frac{K_2^2}{2}\sum_{l=-\infty}^{+\infty} J_{-4-2l}(2K_1)\,J_l(2K_2) \qquad (43)$$

V. Simulations

A. Poincaré section of magnetic field lines

To study the stochastic magnetic field lines we have represented the Poincaré section described by the generalized mapping Eqts. (16) in the (ψ,θ) and (x,y) for different values of (K_1, K_2). In the figures 5 to 8 the Poincaré section is represented in (ψ,θ) plan, whereas figures 9 to 12 in the (x,y) plan with $x = \psi\cos\theta$ and $y = \psi\sin\theta$.

Figures 5-6 and 9-10, correspond to the case of weak islands overlap (partial stochasticity). There exist many integrable trajectory (KAM tori) separating different stochastic regions. On the other hand, in plots 7-8 and 11-12, the islands are large enough to overlap (global stochasticity). The stochastic regions cover all spaces (there are no KAM tori). We note also that in this case ($K_2 \neq 0$) the last KAM tori is destroyed for values of K_1 different to K_c (K_c =0.97 is the Chirikov constant). The transition points (K_1,K_2) from partial to global stochasticity have been calculated numerically in the reference 4.

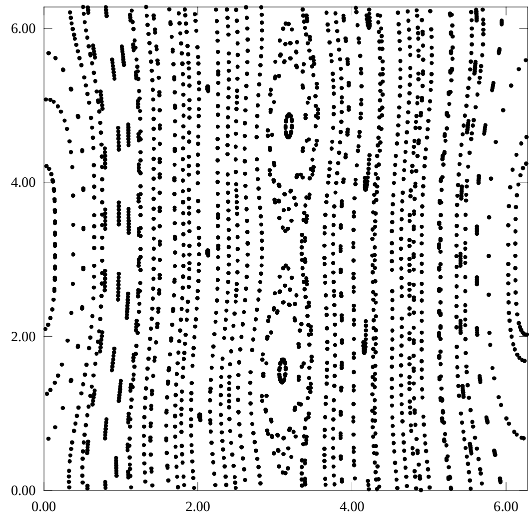

Fig. 5 . Poincaré section of the magnetic field lines for (K_1, K_2) = (0.08, 0.04)

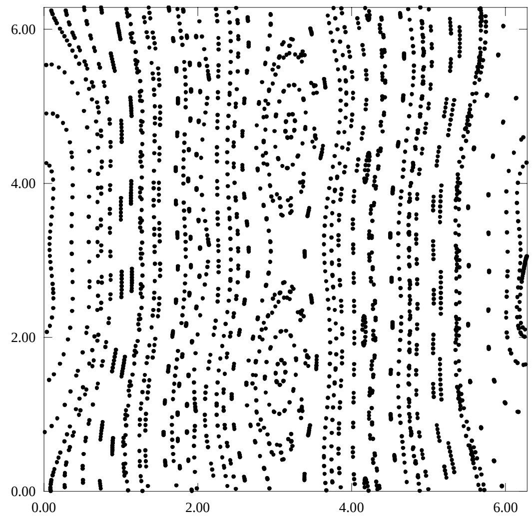

Fig. 6 . Poincaré section of the magnetic field lines for (K_1, K_2) = (0.12, 0.08)

B. Diffusion of magnetic field lines

In this work, we have calculated numerically the diffusion coefficient of magnetic field lines which are described by the generalized mapping Eqts. (16). To this end, we have used expression

$$D_n = \frac{1}{2.n_p.n}\sum_{i=1}^{i=n_p}(I_n^i - I_0^i)^2 \qquad (44)$$

where n=50 is the number of iterations and n_p=3000 is the field lines number that we have considered. We have plot

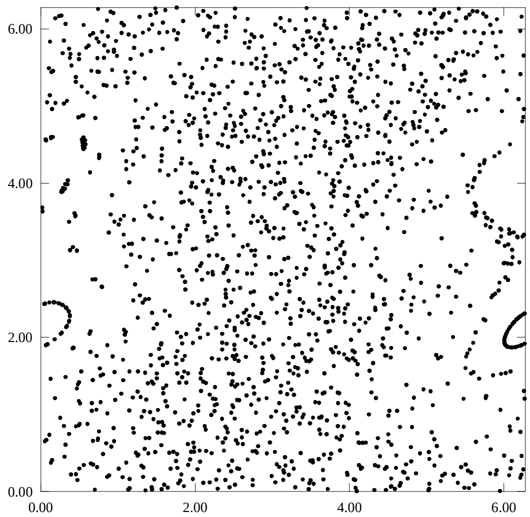

Fig. 7 . Poincaré section of the magnetic field lines for $(K_1, K_2) = (0.9, 0.9)$

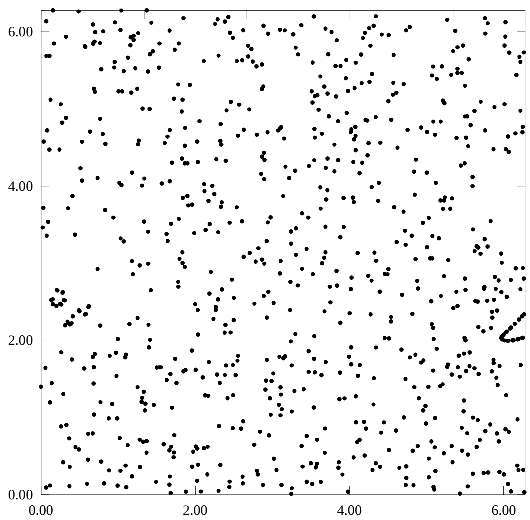

Fig. 8 . Poincaré section of the magnetic field lines for $(K_1, K_2) = (1.8, 1.6)$

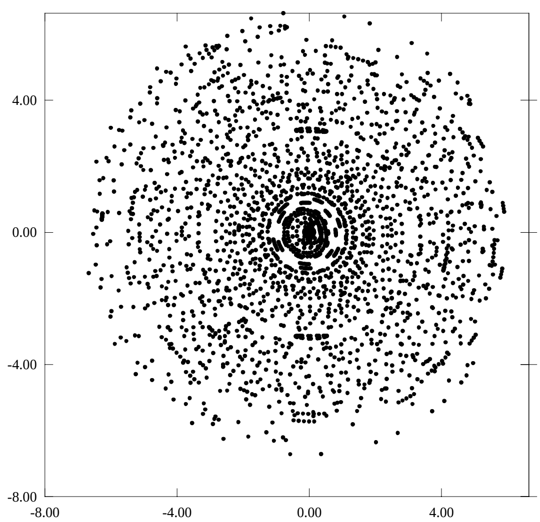

Fig. 9.- Evolution of stochastic magnetic field lines in the plane (x, y) for values of $(K_1, K_2) = (0.08, 0.04)$.

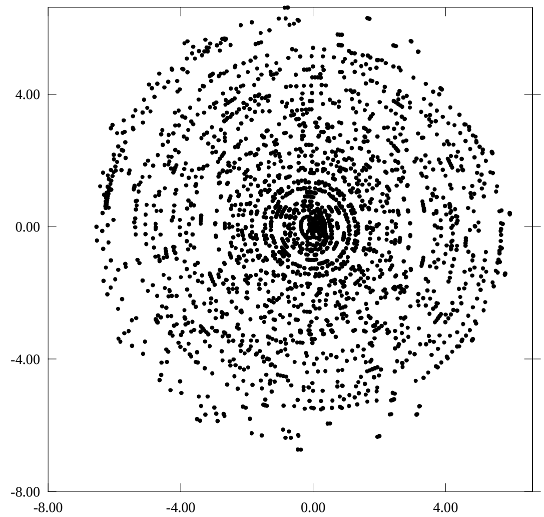

Fig. 10.- Evolution of stochastic magnetic field lines in the plane (x, y) for values of $(K_1, K_2) = (0.12, 0.08)$.

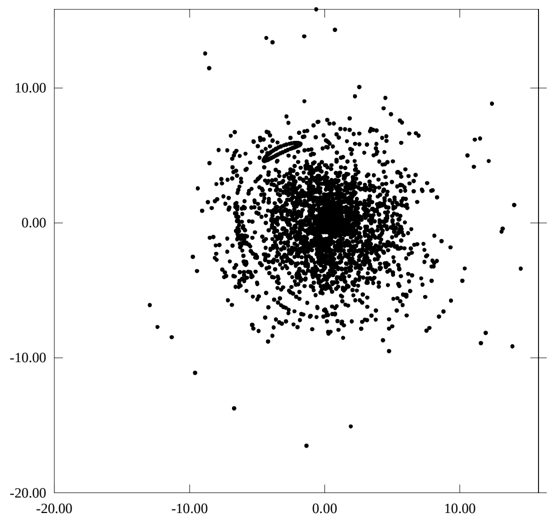

Fig. 11.- Evolution of stochastic magnetic field lines in the plane (x, y) for values of $(K_1, K_2) = (0.9, 0.9)$.

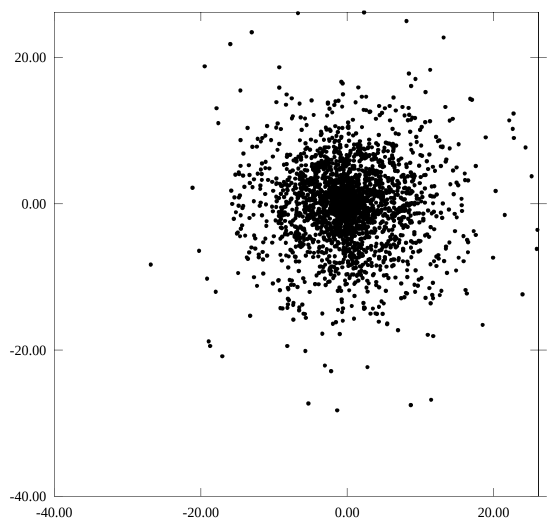

Fig. 12.- Evolution of stochastic magnetic field lines in the plane (x, y) for values of $(K_1, K_2) = (1.8, 1.6)$.

(Figures. 13-17) the ratio D/D_{QL} versus K_1 for different values of K_2 (K_2=0,1,5,10 and 15). The solid lines correspond to the analytical calculation Eq. (43) while the points represent the numerical results. We see that the analytical results agree with our numerical calculations.

When K_2=0, the stochastic magnetic field lines are described by the standard mapping. In this case the ratio D/D_{QL} oscillate around the value 1 (Fig. 13). This ratio tend to 1 when K_1 increases. For K_2 is different to zero, the oscillations become more and more close to 1 when K_2 increases.

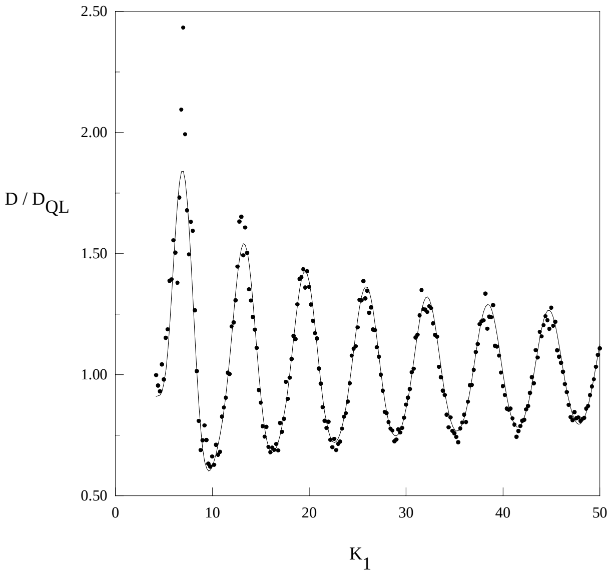

Fig. 13. Plot of the ratio D/D_{QL} versus K_1 for the mapping standard (K_2 = 0).

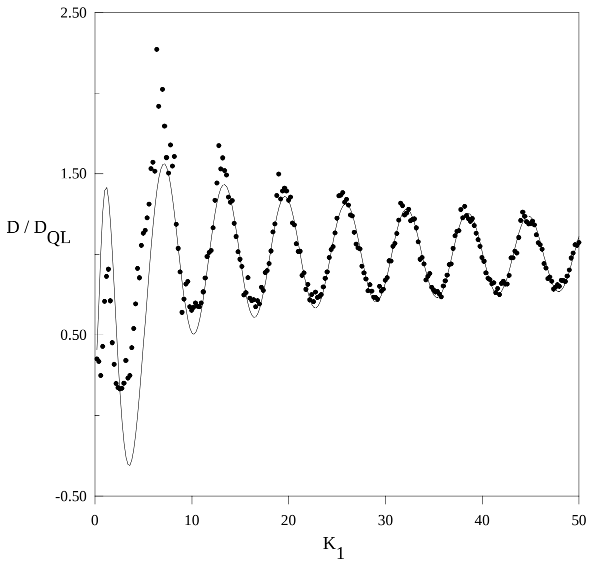

Fig. 14. Plot of the ratio D/D_{QL} versus K_1 for K_2 = 1.

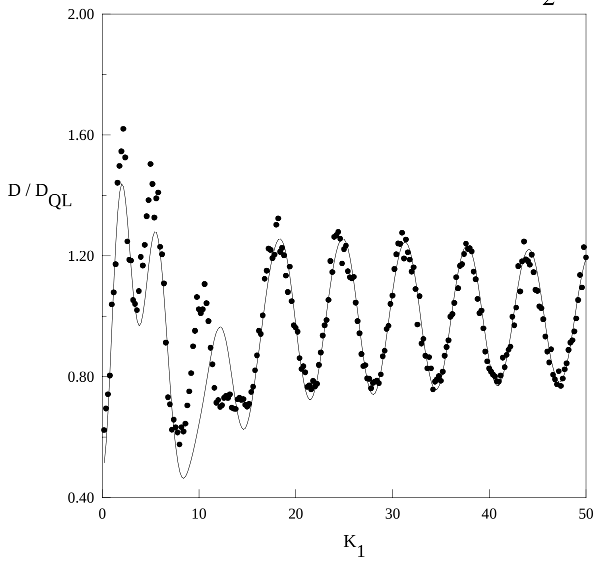

Fig. 15. Plot of the ratio D/D_{QL} versus K_1 for K_2 =5.

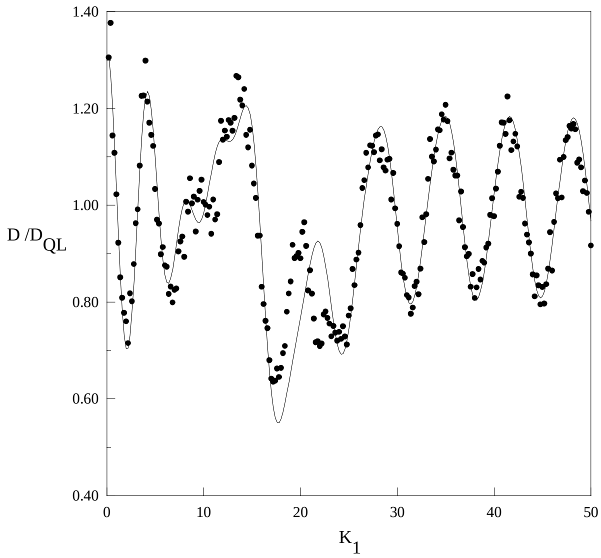

Fig. 16. Plot of the ratio D/D_{QL} versus K_1 for K_2 =10.

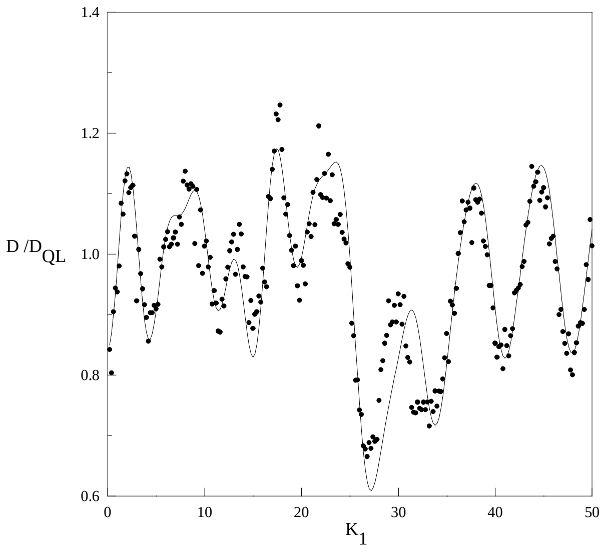

Fig. 17. Plot of the ratio D/D_{QL} versus K_1 for K_2 = 15.

VI- Conclusion

In the first part of this work, we have shown that transition from partial stochasticity to global stochasticity of magnetic field lines is produced rapidly with the generalized mapping compared to results obtained for the standard mapping.
in the second part, we have elaborate an analytical calculation of the diffusion coefficient using the Fourier paths technique. The simulations of the diffusion coefficient in the framework of the generalized mapping agree with results of the analytic calculation. Our results are in good qualitative agreement with these of authors [3,4,5,10,11].

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