Phase diagram of the mixed spin Ising model with four-spin interaction and next-nearest neighbor couplings

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Abstract

The phase diagrams of the ferromagnetic two-dimensional mixed spin-1 and spin-1/2 Ising model with four-spin interaction J_4 and next-nearest couplings J' are investigated by the use of the finite cluster approximation based on single-site cluster theory. The state equations are derived for the two-dimensional square lattice. The phase diagram is qualitatively and quantitatively different from that obtained when the interaction J' is ignored. In fact, the system exhibits a variety of interesting features resulting from the introduction of the next-nearest neighbor interaction (NNN). It undergoes two kinds of behavior according to the negative and the positive value of J'. In particular, for J_4/J_2=-4, the system keeps the coexistence of the two ground state up to J'-dependent finite temperature.

Keywords: mixed spin Ising model, four-spin interactions, next-nearest neighbor, phase diagram.

I. Introduction

There has recently been an increase interest both theoretically and experimentally in two-dimensional Ising systems with interactions beyond first neighbors. These models are interesting because they found their theoretical explanation in the theories of super exchange interaction, the magnetoelastic effect and the spin-phonon coupling [1,2]. Moreover, it was pointed out that the models with the higher-order exchange interactions may exhibit rich phase diagrams and can describe phase transition in some physical systems. Additionally, they show physical behavior not detected in the usual spin systems. For example, the nonuniversal critical phenomena [3,4], and deviation from $T^{3/2}$ Block law at low temperature [5,6].

From the theoretical point of view, the monoatomic Ising models with multispin interactions have been investigated in detail within different methods, such as mean field approximation [7,8], effective field theory [9,10], some more accurate treatments such as series expansion [11–13], renormalization group methods [14–16], Monte Carlo simulations [17–19], and also exact calculations [20–24]. Experimentally, an interesting fact for the models with multispin interactions has been reported. Indeed, it can be used to describe various physical systems such as classical fluid [25], solid ^{3}He [26], lipid bilayers [27], and rare gases [28]. Moreover for some materials it has been shown that the multispin interactions play a significant role; and they are comparable or even much important than the bilinear ones. The models with pair and quartet interaction have been applied successfully to study and explain the existence of first order phase transition in squaric acid crystal $H_2C_2O_4$ [29,30]. Such models have been also used to describe thermodynamical properties of hydrogen-bonded ferroelectric $PbHPO_4$, $PbDPO_4$ [31], some copolymers [32] and optical conductivity [33] observed in cuprate ladder $La_xCa_{14-X}Cu_{24}O_{41}$. On the other hand, some experimental studies on $La_6Ca_8Cu_{24}O_{41}$ [34,35] and $La_4Sr_{10}Cu_{24}O_{41}$ [36] reveal that they could be explained by the introduction of the four-spin interaction. It is worthy to note here that this later plays an important role in the two dimensional antiferromagnet La_2CuO_2 [37], the parent material of high-T_C superconductors.

Intense interest has been directed to study the magnetic properties of two-sublattices mixed spin Ising system. They have less translational symmetry than their single spin counterparts, and are well adapted to study a certain type of ferrimagnetism [38]. Experimentally, it has been shown that the MnNi(EDTA)-6H_2O complex [39] is a good example of a mixed system. The mixed Ising model consisting of spin-1/2 and spin-1 with only two-bilinear interaction has been studied by the renormalization group techniques [40,41], by high temperature series expansion [42], by free fermion approximation [43] and by finite cluster approximation [44]. The introduction of the next nearest neighbor (N.N.N.) coupling has been studied using numerical transfer matrix techniques [45] and Monte Carlo simulation [46,47]. Attention has been devoted to study the ground-state and the influence of the NNN on the transition temperature.

In a very recent work [48], we have studied the phase diagram and the thermal dependence of the magnetizations of the mixed spin Ising model with four-

spin and NNN interactions on square lattice, using Monte Carlo simulation. For instance, it has been shown that the phase diagram displays two kinds of behaviors according to the negative and positive value of *J'*.

The purpose of this paper is to investigate the phase diagram of the ferromagnetic mixed spin Ising model with four-spin and NNN interactions on square lattice using finite cluster approximation (FCA) and, in particular, compare our phase diagram with that obtained very recently by the Monte Carlo treatment.. Such system can be described by the following Hamiltonian:

$$H = -J_2 \sum_{\langle ij \rangle} \sigma_i S_j - J_4 \sum_{\{i,j,k,l\}} \sigma_i \sigma_k S_j S_l - J' \sum_{\langle ik \rangle} \sigma_i \sigma_k \qquad (1)$$

The underlying lattice is composed of two interpenetrating sublattices. One occupied by spins with spin moment $\sigma=\pm1/2$ and the other one is occupied by spins with moment $S= 0,\pm1$. The first summation carried out only over nearest-neighbor pair of spins. The second and the third summations represent the four-spin and NNN interactions, respectively, where the summations concern all alternate squares shaded in Fig.1. To this end, we use the finite cluster approximation [49,50] within the framework of a single site cluster.

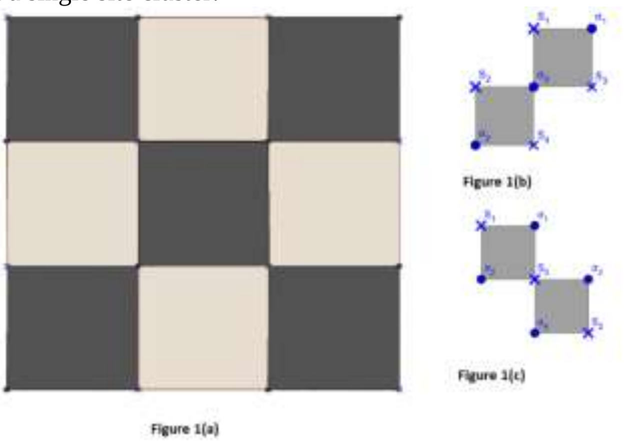

Fig.1: (a) Part of the square lattice. ⬤ and× correspond to σ and S-sublattice sites, respectively.
(b) Neighbors of spin σ_0 with which directly interacts.
(c) Neighbors of spin S_0 with which directly interacts.

The layout of this work is as follows. In section 2, we briefly outline the theoretical framework and calculate the state equations. The results and discussions are presented in section 3.

II. Theoretical framework

The theoretical framework that we adopt in the study of the mixed spin-1/2 and spin-1 Ising model with four-spin and NNN interactions, described by the Hamiltonian (1), is the finite cluster approximation (FCA) [49,50] based on a single-site cluster theory. We have to mention that this method has been successfully applied to a number of interesting pure and disordered spin Ising systems [51–54]. It has also been used for transverse Ising models [55–59] and semi-infinite Ising systems [60–64]. In all these applications, it was shown that the FCA improves qualitatively and quantitatively the results obtained in the frame of the mean-field theory. In this approach, attention is focused on a cluster comprising just a single selected spin $\sigma_o(S_o)$ and its neighbour spins $\{\sigma_1,\sigma_2,S_1,S_2,S_3,S_4\}$ $(\{S_1,S_2,\sigma_1,\sigma_2,\sigma_3,\sigma_4\})$ with which it directly interacts (see Figs. 1-(a) & (b)).

We split the total Hamiltonian (1) into two parts, $H=H_0+H'$ where H_o includes all parts of H associated with the lattice site 0. In the present system, H_o takes the form

$$H_{0\sigma} = -\left[J_2 \sum_{i=1}^{4} S_i + J_4 (S_1 S_2 \sigma_1 + S_3 S_4 \sigma_2) + J' (\sigma_1 + \sigma_2) \right] \qquad (2)$$

$$H_{0S} = -\left[J_2 \sum_{i=1}^{4} \sigma_i + J_4 (\sigma_1 \sigma_2 S_1 + \sigma_3 \sigma_4 S_2) \right] \qquad (3)$$

whether the lattice site 0 belongs to *σ* or *S*-sublattice, respectively.

The problem consists in evaluating the sublattice magnetizations and the quadrupolar moment. To this end, we denote by $<\sigma_o>_c$ and, $<S_o^n>_c$ (n=1, 2), respectively the mean value of σ_o and S_o^n for a given configuration c of all other spins (i.e. when all other spin σ_i and S_j (i, j≠0) are kept fixed). $<\sigma_o>_c$ and $<S_o^n>_c$ are given by

$$\langle \sigma_0 \rangle_C = \frac{\underset{\sigma_0}{Tr} \sigma_0 exp(-\beta H_{0\sigma})}{\underset{\sigma_0}{Tr} exp(-\beta H_{0\sigma})} \qquad (4)$$

$$\langle S_0^n \rangle_C = \frac{\underset{S_0}{Tr} S_0^n exp(-\beta H_{0S})}{\underset{S_0}{Tr} exp(-\beta H_{0S})} \qquad (5)$$

where $Tr_{\sigma o}$ (or Tr_{So}) means the trace performed over σ_o (or S_o) only. As usual β=1/T where T is the absolute temperature. The sublattice magnetizations μ, m and the quadrupolar moment q are then given by

$$\mu \equiv \langle \langle \sigma_0 \rangle_C \rangle = \langle \frac{\underset{\sigma_0}{Tr} \sigma_0 exp(-\beta H_{0\sigma})}{\underset{\sigma_0}{Tr} exp(-\beta H_{0\sigma})} \rangle \qquad (6)$$

$$m \equiv \langle \langle S_0 \rangle_C \rangle = \langle \frac{\underset{S_0}{Tr} S_0 exp(-\beta H_{0S})}{\underset{S_0}{Tr} exp(-\beta H_{0S})} \rangle \qquad (7)$$

$$q \equiv \langle \langle S_0^n \rangle_C \rangle = \langle \frac{\underset{S_0}{Tr} S_0^n exp(-\beta H_{0S})}{\underset{S_0}{Tr} exp(-\beta H_{0S})} \rangle \qquad (8)$$

where <⋯> denotes the average over all spin configurations. Performing the inner traces in (6), (7) and (8) over the states of the selected spin σ_o (S_o), we obtain the following exact relations

$$\mu = \langle\frac{1}{2}\tanh\left[\frac{K}{2}\{(S_1+S_2+S_3+S_4)+\alpha(S_1S_2\sigma_1+S_3S_4\sigma_2) + \gamma(\sigma_1+\sigma_2)\}\right]\rangle \quad (9)$$

$$m = \langle\frac{\sinh[K\{(\sigma_1+\sigma_2+\sigma_3+\sigma_4)+\alpha(\sigma_1\sigma_2S_1+\sigma_3\sigma_4S_2)\}]}{1+\cosh[K\{(\sigma_1+\sigma_2+\sigma_3+\sigma_4)+\alpha(\sigma_1\sigma_2S_1+\sigma_3\sigma_4S_2)\}]}\rangle \quad (10)$$

$$q = \langle\frac{\cosh[K\{(\sigma_1+\sigma_2+\sigma_3+\sigma_4)+\alpha(\sigma_1\sigma_2S_1+\sigma_3\sigma_4S_2)\}]}{1+\cosh[K\{(\sigma_1+\sigma_2+\sigma_3+\sigma_4)+\alpha(\sigma_1\sigma_2S_1+\sigma_3\sigma_4S_2)\}]}\rangle \quad (11)$$

where $K=\beta J_2$, $\alpha=J_4/J_2$ and $\gamma=J'/J_2$.

It is a formidable task to calculate the average on the right-hand sides of Eqs.(9) to (11) over all spin configurations. We can easily observe that any function such as $f(\sigma,S)$ of σ and S can be written as the linear superposition

$$f(\sigma,S) = f_1 + f_2\sigma + f_3S + f_4S^2 + f_5\sigma S + f_6\sigma S^2 \quad (12)$$

which appropriate coefficients f_i (i=1,….,6) . After applying this to all spins σ_i and S_j in expressions between brackets in equations (9) to (11), we average over all spin configurations. In this paper we use the simplest approximation in which we treat all spin self-correlations exactly while still neglecting correlations between quantities pertaining to different sites. This leads to the following coupled equations.

$$\begin{aligned}\mu = {} & \mu[2A_1+4(A_2+A_3)q+(2A_4+8A_5+2A_6)q^2 \\ & +4(A_7+A_8)q^3+2A_9q^4]+m[4A_{10}+(4A_{11}+8A_{12})q \\ & +(4A_{13}+8A_{14})q^2+4A_{15}q^3]+m\mu^2[4A_{16}+[4A_{17} \\ & +8A_{18})q+(4A_{19}+8A_{20})q^2+4A_{21}q^3]+\mu m^2[2A_{22} \\ & +8A_{23}+2A_{24}+(4A_{25}+8A_{26}+4A_{27}+8A_{28})q \\ & +(2A_{29}+8A_{30}+2A_{31})q^2]+m^3[4A_{32}+4A_{33}q] \\ & +\mu m^4[2A_{34}]+\mu^2m^3[4A_{35}+4A_{36}q] \end{aligned} \quad (13)$$

$$\begin{aligned} m = {} & \mu[4B_1+8B_2q+4B_3q^2]+m[2B_4+2B_5q]+\mu^3[4B_6 \\ & +4B_7q+4B_8q^2]+m\mu^2[2B_9+2B_{10}q]+\mu m^2[4B_{11}] \\ & +m\mu^4[2B_{12}+2B_{13}q]+m^2\mu^3[4B_{14}] \end{aligned} \quad (14)$$

$$\begin{aligned} q = {} & [C_1+2C_2q+C_3q^2]+\mu^2[6C_4+12C_5q+6C_6q^2] \\ & +m^2[C_7]+\mu m[4C_8+4C_9q]+\mu^4[C_{10}+2C_{11}q+C_{12}q^2] \\ & +m\mu^3[4C_{13}+4C_{14}q]+\mu^2m^2[6C_{15}]+m^2\mu^4[C_{16}] \end{aligned} \quad (15)$$

The non-zero coefficients quoted in Eq. (13) are listed in the Appendix while those quoted in Eqs. (14) and (15) can be taken from Appendix in [65], where only the definition of the functions g(x) and h(x) must be changed, respectively, to

$$g(x) \equiv \frac{2\sinh(Kx)}{1+2\cosh(Kx)} \quad , \quad h(x) \equiv \frac{2\cosh(Kx)}{1+2\cosh(Kx)}$$

If we replace m and q in (13) by their expressions taken from (14) and (15), we obtain an equation for μ of the form

$$\mu = a\mu + b\mu^3 + \cdots \quad (16)$$

In order to obtain the second-order transition temperature, we neglect higher- order terms in the magnetizations in Eqs. (13)-(15). Therefore, the critical temperature is analytically obtained through a determinantal equation, i.e.

$$\begin{aligned} 1 = {} & 2A_1+4(A_2+A_3)q_0+(2A_4+8A_5+2A_6)q_0^2 \\ & +4(A_7+A_8)q_0^3+2A_9q_0^4+\frac{4B_1+8B_2q_0+4B_3q_0^2}{1-(2B_4+2B_5q_0)}[4A_{10} \\ & +(4A_{11}+8A_{12})q_0+(4A_{13}+8A_{14})q_0^2++4A_{15}q_0^3] \end{aligned} \quad (17)$$

where q_0 is the solution of

$$q_0 = C_1 + 2C_2q_0 + C_3q_0^2 \quad (18)$$

Equation (17) means that its right-hand side corresponds to the coefficient a in (16).

In the vicinity of the second-order transition, the sublattice magnetization μ is given by

$$\mu^2 = \frac{1-a}{b} \quad (19)$$

The right-hand side of (19) is positive since we are in the long-ranged ordered regime. This means that the signs of *1-a* and *b* are the same.

When b changes sign (*1-a* keeping its sign), μ^2 becomes negative. It means that we are not in the vicinity of a second-order transition line. So, the transition is of the first order. Therefore the point at which

$$a(K,\alpha,\gamma) = 1 \text{ and } b(K,\alpha,\gamma) = 0 \quad (20)$$

characterizes the tricritical points.

To obtain the expression for b, one has to solve (13)-(15) for small μ and m. The solution is of the form

$$q = q_0 + q_1\mu^2 + q_2m^2 + q_3\mu m \quad (21)$$

where q_1, q_2 and q_3 are given by

$$q_1 = \frac{6C_4+12C_5q_0+6C_6q_0^2}{1-2C_2-2C_3q_0}, \quad q_2 = \frac{C_7}{1-2C_2-2C_3q_0},$$

$$q_3 = \frac{4C_8+4C_9q_0}{1-2C_2-2C_3q_0} \quad .$$

After some algebraic manipulations, (14) and (21) can be written in the following forms

$$m = \frac{A}{B}\mu + (\frac{8B_2q_4}{B} + \frac{4B_3q_0q_4}{B} + \frac{C}{B} + \frac{DA}{B^2} + \frac{2B_5q_4A}{B^2}$$

$$+\frac{4B_{11}A^2}{B^3})\mu^3+\cdots \qquad (22)$$

$$q=q_0+q_4\mu^2 \qquad (23)$$

where

$A=4B_1+8B_2q_0+4B_3q_0^2, B=1-(2B_4+2B_5q_0),$
$C=4B_6+4B_7q_0+4B_8q_0^2$
$D=2B_9+2B_{10}q_0 \quad , \qquad q_4=q_1+\left(\frac{A}{B}\right)^2q_2+\left(\frac{A}{B}\right)q_3$

By substituting m and q in Eq. (13), with their expressions taken from Eqs. (22) and (23), we obtain the Eq. (16), where b is given by

$$\begin{aligned}b=&[4(A_2+A_3)q_4+2(2A_4+8A_5+2A_6)q_0q_4\\&+3(4A_7+4A_8)q_0^2q_4+8A_9q_0^3q_4]+\frac{A}{B}[(4A_{11}+8A_{12})q_4\\&+2(4A_{13}+8A_{14})q_0q_4+12A_{15}q_0^2q_4]+\frac{A}{B}[4A_{16}\\&+4(A_{17}+8A_{18})q_0+(4A_{19}+8A_{20})q_0^2+4A_{21}q_0^3]\\&+\frac{EA^2}{B^2}+\frac{FA^3}{B^3}+[\frac{8B_2q_4+8B_3q_0q_4+C}{B}+\frac{AD+2B_5q_4A}{B^2}\\&+\frac{4B_{11}A^2}{B^3}][4A_{10}+(4A_{11}+8A_{12})q_0+(4A_{13}+8A_{14})q_0^2\\&+4A_{15}q_0^3] \qquad (24)\end{aligned}$$

with

$$E=2A_{22}+8A_{23}+2A_{24}+(4A_{25}+8A_{26}+4A_{27}+8A_{28})q_0+(2A_{29}+8A_{30}+2A_{31})q_0^2$$

and

$$F=4A_{32}+4A_{33}q_0$$

III. Results and discussions

Let us first consider the system with only bilinear interaction (J_4=J'=0). We recover the two dimensional mixed spin Ising model on the square lattice. The system undergoes a transition at the critical temperature 1.298, which is to be compared with the renormalizations group result 1.15 [40] and Monte Carlo simulation one 0.980 [48]. In the absence of the next-nearest neighbour interaction (J'=0), the system reduces to two-sublattice mixed spin-1/2 and spin-1 Ising model with four-spin interaction. This latter has been studied by one of us (N.B) [65]. It has been found that its phase diagram presents a second-transition line which ends in a tricritical point.

Secondly, we investigate the effects of the next-nearest neighbour coupling on the phase diagram obtained for J'=0. The resulting phase diagrams of the system under investigation are depicted in figures 2(a) and (b) summarizing the results of the influence of the NNN coupling. They present the dependence of the critical line and tricritical behaviour as a function of the four-spin interaction for both positive and negative value of NNN interaction J'. There curves give the sections of the critical

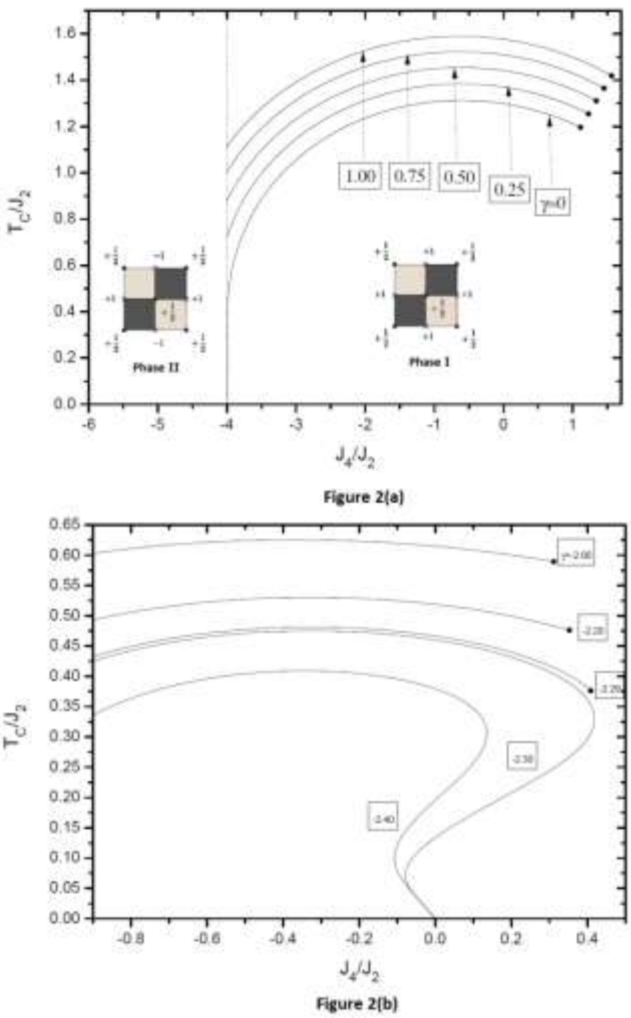

Fig.2: Variation of T_C as a function of α=J_4/J_2 with fixed (a) positive and (b) negative values of γ=J'/J_2.

surface $T_C(J_4,J')$ with planes of fixed values of NNN coupling. They show, as previously stated in [48], that the critical line $J'=0$ separates two different qualitatively behaviour corresponding to $\gamma>0$ and $\gamma<0$. Indeed, in Fig.2(a), we present the variation of the critical temperature T_C with α=J_4/J_2 for selected positive values of the NNN interactions. In this range of J' ($\gamma>0$) the system keeps its tricritical behaviour. The T-component of the tricritical point increases with the four-spin interaction. We note that a positive value of J' strengthens the order at low temperature. Therefore, the long-range ferromagnetic order domain becomes wider with increasing values of J' which is clearly shown in Fig.2(a). We have to mention that, at zero-temperature, the line ($\gamma>-4$, $\alpha=-4$) separates the two ground states I and II, shown in the figure, which correspond to $\alpha>-4$ and $\alpha<-4$, respectively. This means that at each point of this line, the phase I and phase II coexist. What is interesting is that for any given $\gamma>0$ (as seen in figure 2(b)), the system at $\alpha=-4$ keeps this coexistence up to

a J'-dependent finite temperature T^*/J_2. In Fig. 3, we plot the variation of the T^*/J_2 with J'/J_2, which delimits the stability

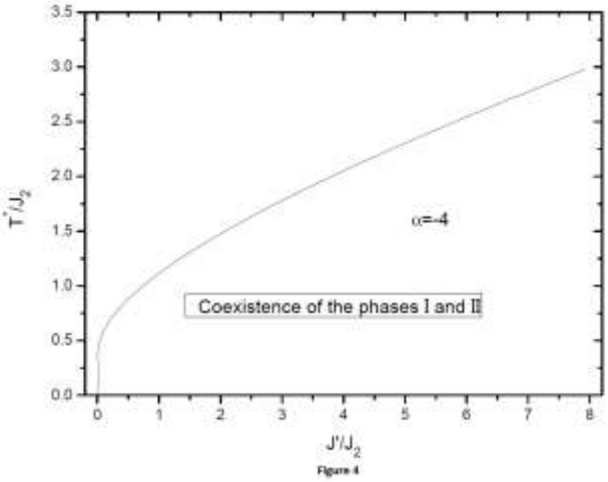

Fig.4: The dependence of the transition temperature *T*/J2* with *J'> 0* when *α=-4*.

domain of the above coexistence for any positive strength of the NNN coupling. It is worth mentioning that there results are consistent qualitatively with those using MC simulation [48]. On the other hand, the phase diagram when the NNN coupling belongs to the tange $-4<\gamma<0$, it acts against the order and reduce the ferromagnetic domain. As seen in Fig. 2(a), the system keeps its tricritical behaviour, namely in the range $-2.295\leq\gamma$. The remaining part of the phase diagram $-2.295>\gamma>-4$ is plotted in Fig. 2(b). It shows a qualitatively different behaviour from the previous range of γ. Thus the tricritical behaviour disappears and all transitions are always of second order. As shown in the figure, at low temperature the curves shows bulges, suggesting a re-entrant phenomenon. This re-entrance curve explained by a competition of energy and entropy S in the free energy $F=E-TS$; which is due in fact to the competition between pair interaction, quartet interaction and next-nearest coupling when these latters take appropriate signs.

Appendix

The coefficients appearing in Eq.(13) are given by:
For abbreviation we define new function:

$$f(x) = \frac{1}{2}\tanh\left(\frac{kx}{2}\right)$$

$$A_1 = f(\gamma)$$

$$A_2 = A_3 = -f(\gamma) + \frac{1}{2}\{f(-1+\gamma) + f(1+\gamma)\}$$

$$A_4 = \{f(\gamma) - f(-1+\gamma) - f(1+\gamma)\} - \frac{1}{2}\{f\left(\frac{\alpha}{2}\right) + f\left(\frac{\alpha}{2}-\gamma\right)\} + \frac{1}{4}\{f\left(2+\frac{\alpha}{2}+\gamma\right) + f\left(2+\frac{\alpha}{2}\right) + f\left(-2+\frac{\alpha}{2}+\gamma\right) + f\left(-2+\frac{\alpha}{2}\right)\}$$

$$A_5 = \frac{3}{2}f(\alpha') - \{f(-1+\gamma) + f(1+\gamma)\} + \frac{1}{4}\{f(2+\gamma) + f(-2+\gamma)\}$$

$$A_6 = \frac{1}{2}\left\{f\left(\frac{\alpha}{2}\right) - f\left(\frac{\alpha}{2}-\gamma\right)\right\} + \{f(\gamma) - f(-1+\gamma) - f(1+\gamma)\} + \frac{1}{4}\{f\left(2+\frac{\alpha}{2}+\gamma\right) - f\left(2+\frac{\alpha}{2}\right) + f\left(-2+\frac{\alpha}{2}+\gamma\right) - f\left(-2+\frac{\alpha}{2}\right)\}$$

$$A_7 = -2f(\alpha') + \frac{3}{2}\{f(1+\gamma) + f(-1-\gamma)\} + \frac{1}{2}\{f\left(\frac{\alpha}{2}\right) + f\left(\frac{\alpha}{2}-\gamma\right) - f(2+\gamma) - f(-2+\gamma)\} - \frac{1}{4}\{f\left(-2+\frac{\alpha}{2}\right) + f\left(2+\frac{\alpha}{2}\right) + f\left(-2+\frac{\alpha}{2}+\gamma\right) + f\left(2+\frac{\alpha}{2}+\gamma\right) + f\left(1+\frac{\alpha}{2}-\gamma\right) + f\left(-1+\frac{\alpha}{2}-\gamma\right)\} + \frac{1}{8}\{f\left(-3+\frac{\alpha}{2}+\gamma\right) - f\left(1+\frac{\alpha}{2}\right) - f\left(-1+\frac{\alpha}{2}\right) + f\left(-1+\frac{\alpha}{2}+\gamma\right) + f\left(3+\frac{\alpha}{2}\right) + f\left(3+\frac{\alpha}{2}+\gamma\right) + f\left(-3+\frac{\alpha}{2}\right) + f\left(1+\frac{\alpha}{2}+\gamma\right)\}$$

$$A_8 = -2f(\alpha') + \frac{3}{2}\{f(1+\gamma) + f(-1+\gamma)\} + \frac{1}{2}\{-f\left(\frac{\alpha}{2}\right) + f\left(\frac{\alpha}{2}-\gamma\right) - f(2+\gamma) - f(-2+\gamma)\} + \frac{1}{4}\{f\left(2+\frac{\alpha}{2}\right) + f\left(-2+\frac{\alpha}{2}\right) - f\left(-2+\frac{\alpha}{2}+\gamma\right) - f\left(2+\frac{\alpha}{2}+\gamma\right) - f\left(1+\frac{\alpha}{2}-\gamma\right) - f\left(-1+\frac{\alpha}{2}-\gamma\right)\} + \frac{1}{8}\{f\left(1+\frac{\alpha}{2}\right) + f\left(-1+\frac{\alpha}{2}\right) + f\left(-3+\frac{\alpha}{2}+\gamma\right) + f\left(-1+\frac{\alpha}{2}+\gamma\right) + f\left(1+\frac{\alpha}{2}+\gamma\right) + f\left(3+\frac{\alpha}{2}+\gamma\right) - f\left(-3+\frac{\alpha}{2}\right) - f\left(3+\frac{\alpha}{2}\right)\}$$

$$A_9 = 3f(\alpha') - 2f(1+\gamma) - 2f(-1+\gamma) + \frac{1}{8}f(\alpha+\gamma) + \left\{f\left(1+\frac{\alpha}{2}-\gamma\right) + f\left(-1+\frac{\alpha}{2}-\gamma\right) - f\left(\frac{\alpha}{2}-\gamma\right)\right\} + \frac{1}{16}\{f(4+\alpha+\gamma) + f(-4+\alpha+\gamma)\} + \frac{1}{4}\{5f(2+\gamma) + 5f(-2+\gamma) - f(\alpha-\gamma)\} + \frac{1}{2}\{f\left(-2+\frac{\alpha}{2}+\gamma\right) + f\left(2+\frac{\alpha}{2}+\gamma\right) - f\left(-3+\frac{\alpha}{2}+\gamma\right) - f\left(-1+\frac{\alpha}{2}+\gamma\right) - f\left(1+\frac{\alpha}{2}+\gamma\right) - f\left(3+\frac{\alpha}{2}+\gamma\right)\}$$

$$A_{10} = \frac{1}{2}f(1) + \frac{1}{4}\{f(1+\gamma) - f(-1+\gamma)\}$$

$$A_{11} = -\frac{1}{2}f(1) + \frac{1}{4}\{f(-1+\gamma) - f(1+\gamma)\} + \frac{1}{8}\{f\left(2+\frac{\alpha}{2}\right)$$

$$-f\left(-2+\frac{\alpha}{2}\right)+f\left(2+\frac{\alpha}{2}+\gamma\right)-f\left(-2+\frac{\alpha}{2}+\gamma\right)\}$$

$$A_{12}=\frac{1}{4}\{f(2)-f(1)\}+\frac{1}{8}\{f(2+\gamma)+3f(-1+\gamma) -f(-2+\gamma)-3f(1+\gamma)\}$$

$$A_{13}=\frac{1}{2}\{f(1+\gamma)-f(-1+\gamma)-f(2)\}+\frac{1}{4}\{f(-2+\gamma) -f(2+\gamma)\}+\frac{1}{8}\{f\left(1+\frac{\alpha}{2}-\gamma\right)-f\left(-1+\frac{\alpha}{2}-\gamma\right)\} +\frac{1}{16}\{f\left(1+\frac{\alpha}{2}\right)-f\left(-1+\frac{\alpha}{2}\right)-f\left(-3+\frac{\alpha}{2}+\gamma\right) +f\left(-1+\frac{\alpha}{2}+\gamma\right)-f\left(1+\frac{\alpha}{2}+\gamma\right)+f\left(3+\frac{\alpha}{2}+\gamma\right) +f\left(3+\frac{\alpha}{2}\right)-f\left(-3+\frac{\alpha}{2}\right)\}$$

$$A_{14}=\frac{1}{4}\{f(1)-f(2)\}+\frac{1}{8}\{3f(1+\gamma)-3f(-1+\gamma) +f\left(-2+\frac{\alpha}{2}\right)-f\left(2+\frac{\alpha}{2}\right)+f\left(-2+\frac{\alpha}{2}+\gamma\right) -f\left(2+\frac{\alpha}{2}+\gamma\right)-f(2+\gamma)+f(-2+\gamma)\} +\frac{1}{16}\{f\left(1+\frac{\alpha}{2}\right)-f\left(-1+\frac{\alpha}{2}\right)-f\left(-3+\frac{\alpha}{2}+\gamma\right) -f\left(-1+\frac{\alpha}{2}+\gamma\right)+f\left(1+\frac{\alpha}{2}+\gamma\right)+f\left(3+\frac{\alpha}{2}+\gamma\right) -f\left(-3+\frac{\alpha}{2}\right)+f\left(3+\frac{\alpha}{2}\right)\}$$

$$A_{15}=\frac{1}{32}\{f(4+\alpha+\gamma)-f(-4+\alpha+\gamma)\}+\frac{1}{2}\{f(2) +f(-1+\gamma)-f(1+\gamma)\}+\frac{1}{16}\{f(2+\alpha)+f(4) -f(-2+\alpha)+f\left(-1+\frac{\alpha}{2}+\gamma\right)-f\left(1+\frac{\alpha}{2}+\gamma\right) -3f\left(1+\frac{\alpha}{2}\right)+3f\left(-1+\frac{\alpha}{2}\right)-3f\left(3+\frac{\alpha}{2}+\gamma\right) +3f\left(-3+\frac{\alpha}{2}\right)-3f\left(3+\frac{\alpha}{2}\right)+3f\left(-3+\frac{\alpha}{2}+\gamma\right) +5f(2+\gamma)-5f(-2+\gamma)\}+\frac{1}{8}\{f\left(-1+\frac{\alpha}{2}-\gamma\right) -f\left(1+\frac{\alpha}{2}-\gamma\right)-f\left(-2+\frac{\alpha}{2}\right)+f\left(2+\frac{\alpha}{2}\right) -f\left(-2+\frac{\alpha}{2}+\gamma\right)+f\left(2+\frac{\alpha}{2}+\gamma\right)\}$$

$$A_{16}=\{f(1+\gamma)-f(-1+\gamma)\}-2f(1)$$

$$A_{17}=2f(1)+\{f(-1+\gamma)-f(1+\gamma)\}+\frac{1}{2}\{f\left(2+\frac{\alpha}{2}+\gamma\right) -f\left(-2+\frac{\alpha}{2}+\gamma\right)-f\left(2+\frac{\alpha}{2}\right)+f\left(-2+\frac{\alpha}{2}\right)\}$$

$$A_{18}=\{f(1)-f(2)\}+\frac{1}{2}\{f(-1+\gamma)-f(-2+\gamma) -f(1+\gamma)+f(2+\gamma)\}$$

$$A_{19}=2f(2)+\{f(-2+\gamma)-f(2+\gamma)\}+\frac{1}{2}\{f\left(1+\frac{\alpha}{2}-\gamma\right) -f\left(-1+\frac{\alpha}{2}-\gamma\right)\}+\frac{1}{4}\{f\left(-1+\frac{\alpha}{2}\right)-f\left(1+\frac{\alpha}{2}\right) +f\left(3+\frac{\alpha}{2}+\gamma\right)-f\left(-3+\frac{\alpha}{2}+\gamma\right)+f\left(-1+\frac{\alpha}{2}+\gamma\right) -f\left(1+\frac{\alpha}{2}+\gamma\right)+f\left(-3+\frac{\alpha}{2}\right)-f\left(3+\frac{\alpha}{2}\right)\}$$

$$A_{20}=\{f(2)-f(1)\}+\frac{1}{2}\{f(1+\gamma)-f(-1+\gamma)-f(2+\gamma) +f\left(2+\frac{\alpha}{2}\right)-f\left(-2+\frac{\alpha}{2}\right)+f\left(-2+\frac{\alpha}{2}+\gamma\right) -f\left(2+\frac{\alpha}{2}+\gamma\right)+f(-2+\gamma)\}+\frac{1}{4}\{f\left(-1+\frac{\alpha}{2}\right) -f\left(1+\frac{\alpha}{2}\right)-f\left(-3+\frac{\alpha}{2}+\gamma\right)-f\left(-1+\frac{\alpha}{2}+\gamma\right) +f\left(1+\frac{\alpha}{2}+\gamma\right)+f\left(3+\frac{\alpha}{2}+\gamma\right)+f\left(-3+\frac{\alpha}{2}\right) -f\left(3+\frac{\alpha}{2}\right)\}$$

$$A_{21}=-2f(2)+\frac{1}{2}\{f\left(2+\frac{\alpha}{2}+\gamma\right)-f\left(-2+\frac{\alpha}{2}+\gamma\right) -f\left(1+\frac{\alpha}{2}-\gamma\right)+f\left(-1+\frac{\alpha}{2}-\gamma\right)+f\left(-2+\frac{\alpha}{2}\right)\} +\frac{1}{8}\{f(4+\alpha+\gamma)-f(-4+\alpha+\gamma)\}+\frac{1}{4}\{f(-2+\alpha) -f(2+\alpha)-f(4)+f\left(-1+\frac{\alpha}{2}+\gamma\right)-f\left(1+\frac{\alpha}{2}+\gamma\right) +3f\left(1+\frac{\alpha}{2}\right)-3f\left(-1+\frac{\alpha}{2}\right)+3f\left(-3+\frac{\alpha}{2}+\gamma\right) -3f\left(3+\frac{\alpha}{2}+\gamma\right)-3f\left(-3+\frac{\alpha}{2}\right)+3f\left(3+\frac{\alpha}{2}\right) +5f(2+\gamma)-5f(-2+\gamma)\}$$

$$A_{22}=\frac{1}{4}\{f\left(2+\frac{\alpha}{2}\right)+f\left(-2+\frac{\alpha}{2}+\gamma\right)+f\left(-2+\frac{\alpha}{2}\right) +f\left(2+\frac{\alpha}{2}+\gamma\right)\}+\frac{1}{4}\{f\left(\frac{\alpha}{2}\right)+f\left(\frac{\alpha}{2}-\gamma\right)\}$$

$$A_{23}=-\frac{1}{2}f(\gamma)+\frac{1}{4}\{f(2+\gamma)+f(-2+\gamma)\}$$

$$A_{24}=\frac{1}{2}\{f\left(\frac{\alpha}{2}-\gamma\right)-f\left(\frac{\alpha}{2}\right)\}+\frac{1}{4}\{f\left(2+\frac{\alpha}{2}+\gamma\right)-f\left(2+\frac{\alpha}{2}\right) +f\left(-2+\frac{\alpha}{2}+\gamma\right)-f\left(-2+\frac{\alpha}{2}\right)\}$$

$$A_{25}=-\frac{1}{2}\{f\left(\frac{\alpha}{2}\right)+f\left(\frac{\alpha}{2}-\gamma\right)\}+\frac{1}{4}\{f\left(1+\frac{\alpha}{2}-\gamma\right) -f\left(2+\frac{\alpha}{2}\right)+f\left(-1+\frac{\alpha}{2}-\gamma\right)-f\left(-2+\frac{\alpha}{2}+\gamma\right) -f\left(2+\frac{\alpha}{2}+\gamma\right)-f\left(-2+\frac{\alpha}{2}\right)\}+\frac{1}{8}\{3f\left(1+\frac{\alpha}{2}\right) +3f\left(-1+\frac{\alpha}{2}\right)+f\left(-3+\frac{\alpha}{2}+\gamma\right)+f\left(-1+\frac{\alpha}{2}+\gamma\right) +f\left(1+\frac{\alpha}{2}+\gamma\right)+f\left(3+\frac{\alpha}{2}+\gamma\right)+f\left(-3+\frac{\alpha}{2}\right) +f\left(3+\frac{\alpha}{2}\right)\}$$

$$A_{26}=\frac{1}{2}f(\alpha')-\frac{1}{4}\{f(2+\gamma)+f(-2+\gamma)\}+\frac{1}{8}\{f\left(1+\frac{\alpha}{2}\right)+f\left(3+\frac{\alpha}{2}+\gamma\right)+f\left(-3+\frac{\alpha}{2}+\gamma\right)-f\left(-1+\frac{\alpha}{2}+\gamma\right)-f\left(1+\frac{\alpha}{2}+\gamma\right)-f\left(-3+\frac{\alpha}{2}\right)-f\left(3+\frac{\alpha}{2}\right)+f\left(-1+\frac{\alpha}{2}\right)\}$$

$$A_{27}=\frac{1}{2}\{f\left(\frac{\alpha}{2}\right)-f\left(\frac{\alpha}{2}-\gamma\right)\}+\frac{1}{4}\{f\left(-2+\frac{\alpha}{2}\right)+f\left(2+\frac{\alpha}{2}\right)-f\left(-2+\frac{\alpha}{2}+\gamma\right)-f\left(2+\frac{\alpha}{2}+\gamma\right)+f\left(1+\frac{\alpha}{2}-\gamma\right)+f\left(-1+\frac{\alpha}{2}-\gamma\right)\}+\frac{1}{8}\{f\left(-3+\frac{\alpha}{2}+\gamma\right)-f\left(3+\frac{\alpha}{2}\right)+f\left(-1+\frac{\alpha}{2}+\gamma\right)+f\left(1+\frac{\alpha}{2}+\gamma\right)+f\left(3+\frac{\alpha}{2}+\gamma\right)-f\left(-3+\frac{\alpha}{2}\right)-3f\left(1+\frac{\alpha}{2}\right)-3f\left(-1+\frac{\alpha}{2}\right)\}$$

$$A_{28}=\frac{1}{2}f(\gamma)-\frac{1}{4}\{f(2+\gamma)+f(-2+\gamma)\}+\frac{1}{8}\{-f\left(1+\frac{\alpha}{2}\right)+f\left(3+\frac{\alpha}{2}+\gamma\right)+f\left(-3+\frac{\alpha}{2}+\gamma\right)-f\left(-1+\frac{\alpha}{2}+\gamma\right)-f\left(1+\frac{\alpha}{2}+\gamma\right)+f\left(-3+\frac{\alpha}{2}\right)+f\left(3+\frac{\alpha}{2}\right)-f\left(-1+\frac{\alpha}{2}\right)\}$$

$$A_{29}=\frac{1}{8}f(\alpha+\gamma)+\frac{1}{16}\{f(4+\alpha+\gamma)+f(-4+\alpha+\gamma)\}+\frac{1}{2}\{f\left(\frac{\alpha}{2}-\gamma\right)-f\left(1+\frac{\alpha}{2}-\gamma\right)-f\left(-1+\frac{\alpha}{2}-\gamma\right)+f\left(\frac{\alpha}{2}\right)\}+\frac{1}{4}\{f(2+\alpha)+f(\alpha-\gamma)+f(-2+\alpha)-f\left(-3+\frac{\alpha}{2}+\gamma\right)-f\left(-1+\frac{\alpha}{2}+\gamma\right)-f\left(1+\frac{\alpha}{2}+\gamma\right)-f\left(3+\frac{\alpha}{2}+\gamma\right)-f\left(-3+\frac{\alpha}{2}\right)-f\left(3+\frac{\alpha}{2}\right)+f\left(2+\frac{\alpha}{2}\right)+f\left(-2+\frac{\alpha}{2}\right)+f\left(-2+\frac{\alpha}{2}+\gamma\right)+f\left(2+\frac{\alpha}{2}+\gamma\right)-3f\left(1+\frac{\alpha}{2}\right)-3f\left(-1+\frac{\alpha}{2}\right)\}$$

$$A_{30}=-\frac{1}{2}f(\gamma)-\frac{1}{8}f(\alpha+\gamma)+\frac{1}{16}\{f(-4+\alpha+\gamma)+f(4+\alpha+\gamma)\}+\frac{1}{4}\{f\left(-1+\frac{\alpha}{2}+\gamma\right)-f\left(-3+\frac{\alpha}{2}+\gamma\right)+f\left(1+\frac{\alpha}{2}+\gamma\right)-f\left(3+\frac{\alpha}{2}+\gamma\right)+f(2+\gamma)+f(-2+\gamma)\}$$

$$A_{31}=\frac{1}{8}f(\alpha+\gamma)+\frac{1}{16}\{f(4+\alpha+\gamma)+f(-4+\alpha+\gamma)\}+\frac{1}{2}\{f\left(\frac{\alpha}{2}-\gamma\right)-f\left(1+\frac{\alpha}{2}-\gamma\right)-f\left(-1+\frac{\alpha}{2}-\gamma\right)-f\left(\frac{\alpha}{2}\right)\}+\frac{1}{4}\{f\left(3+\frac{\alpha}{2}\right)-f(2+\alpha)+f(\alpha-\gamma)-f(-2+\alpha)-f\left(-2+\frac{\alpha}{2}\right)-f\left(2+\frac{\alpha}{2}\right)+f\left(-3+\frac{\alpha}{2}\right)+f\left(-2+\frac{\alpha}{2}+\gamma\right)+f\left(2+\frac{\alpha}{2}+\gamma\right)-f\left(-3+\frac{\alpha}{2}+\gamma\right)-f\left(-1+\frac{\alpha}{2}+\gamma\right)-f\left(1+\frac{\alpha}{2}+\gamma\right)-f\left(3+\frac{\alpha}{2}+\gamma\right)+3f\left(1+\frac{\alpha}{2}\right)+3f\left(-1+\frac{\alpha}{2}\right)\}$$

$$A_{32}=\frac{1}{16}\{3f\left(-1+\frac{\alpha}{2}\right)-3f\left(1+\frac{\alpha}{2}\right)+f\left(3+\frac{\alpha}{2}+\gamma\right)-f\left(-3+\frac{\alpha}{2}+\gamma\right)+f\left(-1+\frac{\alpha}{2}+\gamma\right)-f\left(1+\frac{\alpha}{2}+\gamma\right)-f\left(-3+\frac{\alpha}{2}\right)+f\left(3+\frac{\alpha}{2}\right)\}+\frac{1}{8}\{f\left(-1+\frac{\alpha}{2}-\gamma\right)-f\left(1+\frac{\alpha}{2}-\gamma\right)\}$$

$$A_{33}=\frac{1}{32}\{f(4+\alpha+\gamma)-f(-4+\alpha+\gamma)\}+\frac{1}{16}\{f(4)-f(2+\alpha)+f(-2+\alpha)+f\left(-3+\frac{\alpha}{2}+\gamma\right)-f(2+\gamma)-f(2-\gamma)-f\left(3+\frac{\alpha}{2}+\gamma\right)+f\left(-3+\frac{\alpha}{2}\right)-f\left(3+\frac{\alpha}{2}\right)+3f\left(1+\frac{\alpha}{2}\right)-f\left(-1+\frac{\alpha}{2}+\gamma\right)-3f\left(-1+\frac{\alpha}{2}\right)+f\left(1+\frac{\alpha}{2}+\gamma\right)\}+\frac{1}{8}\{f\left(1+\frac{\alpha}{2}-\gamma\right)-f\left(-1+\frac{\alpha}{2}-\gamma\right)\}$$

$$A_{34}=\frac{1}{8}f(\alpha+\gamma)+\frac{1}{16}\{f(4+\alpha+\gamma)+f(-4+\alpha+\gamma)\}+\frac{1}{4}\{f(2-\gamma)-f(2+\gamma)-f(\alpha+\gamma)\}$$

$$A_{35}=\frac{1}{2}\left\{f\left(-1+\frac{\alpha}{2}-\gamma\right)-f\left(1+\frac{\alpha}{2}-\gamma\right)\right\}+\frac{1}{4}\{3f\left(1+\frac{\alpha}{2}\right)-3f\left(-1+\frac{\alpha}{2}\right)-f\left(-3+\frac{\alpha}{2}+\gamma\right)+f\left(3+\frac{\alpha}{2}+\gamma\right)+f\left(-1+\frac{\alpha}{2}+\gamma\right)-f\left(1+\frac{\alpha}{2}+\gamma\right)+f\left(-3+\frac{\alpha}{2}\right)-f\left(3+\frac{\alpha}{2}\right)\}$$

$$A_{36}=\frac{1}{8}\{f(4+\alpha+\gamma)-f(-4+\alpha+\gamma)\}+\frac{1}{4}\{f(2+\alpha)-f(-2+\alpha)-f(4)+f\left(-3+\frac{\alpha}{2}+\gamma\right)-f\left(3+\frac{\alpha}{2}+\gamma\right)-f\left(-1+\frac{\alpha}{2}+\gamma\right)+f\left(1+\frac{\alpha}{2}+\gamma\right)-f\left(-3+\frac{\alpha}{2}\right)+f\left(3+\frac{\alpha}{2}\right)-f(2+\gamma)-f(2-\gamma)-3f\left(1+\frac{\alpha}{2}\right)+3f\left(-1+\frac{\alpha}{2}\right)\}+\frac{1}{2}\left\{f\left(1+\frac{\alpha}{2}-\gamma\right)-f\left(-1+\frac{\alpha}{2}-\gamma\right)\right\}$$

References

[1] T. Iwashita, N. Uryu, J. Phys. C:. 17 (1984) 855–868.

[2] P. Gluck, O. Wohlman-entin, Phys. Status Solidi. 52 (1972) 323–333.

[3] F. Wu, Phys. Rev. B. 4 (1971) 2312–2314.

[4] L. Kadanoff, F. Wegner, Phys. Rev. B. 4 (1971) 3989–3993.

[5] U. Köbler, R. Mueller, L. Smardz, D. Maier, K. Fischer, B. Olefs, W. Zinn, Z. Phys. B:. 100 (1996) 497–506.

[6] E. Müller-Hartmann, U. Köbler, L. Smardz, J. Magn. Magn. Mater. 173 (1997) 133–140.
[7] C.L. Wang, Z.K. Qin, D.L. Lin, J. Magn. Magn. Mater. 88 (1990) 87–92.
[8] K.G. Chakraborty, J. Magn. Magn. Mater. 114 (1992) 155–160.
[9] T. Kaneyoshi, T. Aoyama, J. Magn. Magn. Mater. 96 (1991) 67–76.
[10] B. Laaboudi, M. Kerouad, Physica A. 241 (1997) 729–736.
[11] D.W. Wood, H.P. Griffiths, J. Phys. C:. 7 (1974) L54–L58.
[12] H.P. Griffiths, D.W. Wood, J. Phys. C Solid State Phys. 7 (1974) 4021–4036.
[13] J. Oitmaa, R.W. Gibberd, J. Phys. C:. 6 (1973) 2077–2088.
[14] F. Lee, H. Chen, F. Wu, Phys. Rev. B. 40 (1989) 4871–4876.
[15] M. Gitterman, M. Mikulinsky, J. Phys. C:. 10 (1977) 4073–4078.
[16] M.P. Nightingale, Phys. Lett. A. 59 (1977) 486–488.
[17] T. Iwashita, K. Uragami, A. Shimizu, A. Nagaki, T. Kasama, T. Idogaki, J. Magn. Magn. Mater. 310 (2007) e435–e437.
[18] T. Iwashita, K. Uragami, K. Goto, M. Arao, T. Kasama, T. Idogaki, J. Magn. Magn. Mater. 272-276 (2004) 672–673.
[19] G.-M. Zhang, C.-Z. Yang, Phys. Status Solidi. 175 (1993) 459–463.
[20] S. Lacková, T. Horiguchi, Physica A. 319 (2003) 311–318.
[21] F. Y. Wu, Phys. Lett. A. 38 (1972) 77–78.
[22] F. Wang, M. Suzuki, Physica A. 230 (1996) 639–650.
[23] A. Lipowski, Physica A. 248 (1998) 207–212.
[24] L. Gálisová, Phys. Status Solidi. 250 (2013) 187–195.
[25] M. Grimsditch, P. Loubeyre, A. Polian, Phys. Rev. B. 33 (1986) 7192–7200.
[26] M. Roger, J. Hetherington, J. Delrieu, Rev. Mod. Phys. 55 (1983) 1–64.
[27] H. Scott, Phys. Rev. A. 37 (1988) 263–268.
[28] J. Barker, Phys. Rev. Lett. 57 (1986) 230–233.
[29] C.L. Wang, Z.K. Qin, D.L. Lin, Solid State Commun. 71 (1989) 45–48.
[30] C. Wang, Z. Qin, D. Lin, Phys. Rev. B. 40 (1989) 680–685.
[31] W. Chunlei, Q. Zikai, Z. Jingbo, Ferroelectrics. 77 (1988) 21–29.
[32] P.R. Silva, B.V. Costa, R.L. Moreira, copolymers, Polymer (Guildf). 34 (1993) 3107–3108.
[33] T.S. Nunner, P. Brune, T. Kopp, M. Windt, M. Grueninger, Acta Phys. Pol. B. 34 (2003) 1545-1548.
[34] S. Brehmer, H.-J. Mikeska, M. Müller, N. Nagaosa, S. Uchida, Phys. Rev. B. 60 (1999) 329–334.
[35] M. Matsuda, K. Katsumata, R. Eccleston, S. Brehmer, H.-J. Mikeska, Phys. Rev. B. 62 (2000) 8903–8908.
[36] S. Notbohm, P. Ribeiro, B. Lake, D. A. Tennant, K. Schmidt, G. S. Uhrig, C. Hess, R. Klingeler, G. Behr, B. Buchner, M. Reehuis, R. I. Bewley, C. D. Frost, P. Manuel, R. S. Eccleston , Phys. Rev. Lett. 98 (2007) 027403.
[37] R. Coldea, S.M. Hayden, G. Aeppli, T.G. Perring, C.D. Frost, T.E. Mason, S.-W. Cheong, Z. Fisk, Phys. Rev. Lett. 86 (2001) 5377–5380.
[38] L. Néel, Ann. L'institut Fourier. 1 (n.d.) 163–183.
[39] M. Drillon, E. Coronado, D. Beltran, R. Georges, Chem. Phys. 79 (1983) 449–453.
[40] N. Benayad, Z. Phys. B:. 81 (1990) 99–105.
[41] S.L. Schofield, R.G. Bowers, J. Phys. A:. 13 (1980) 3697–3706.
[42] B.Y. Yousif, R.G. Bowers, J. Phys. A:. 17 (1984) 3389–3394.
[43] K.-F. Tang, J. Phys. A:. 21 (1988) L1097–L1098.
[44] N. Benayad, A. Klümper, J. Zittartz, A. Benyoussef, Z. Phys. B:. 77 (1989) 333–338.
[45] G.M. Buendía, M.A. Novotny, J. Phys. Condens. Matter. 9 (1997) 5951–5964.
[46] W. Selke, C. Ekiz, J. Phys. Condens. Matter. 23 (2011) 496002.
[47] G.M. Buendia, E. Machado, M.A. Novotny, MRS Proc. 517 (2011) 361-366.
[48] M. Azhari, N. Benayad, M. Mouhib, Superlattices Microstruct. 79 (2015) 96–107.
[49] N. Boccara, Phys. Lett. A. 94 (1983) 185–187.
[50] A. Benyoussef, N. Boccara, J. Phys. 44 (1983) 1143–1147.
[51] N. Benayad, A. Klümper, J. Zittartz, A. Benyoussef, Z. Phys. B:. 77 (1989) 339–341.
[52] N. Benayad, A. Dakhama, A. Klümper, J. Zittartz, Ann. Phys. 508 (1996) 387–398.
[53] N. Benayad, L. Khaya, A. Fathi, J. Phys: Condens. Matter. 14 (2002) 9667–9685.
[54] N. Benayad, A. Fathi, L. Khaya, Physica A. 300 (2001) 225–244.
[55] M. Ghliyem, N. Benayad, M. Azhari, Physica A. 402 (2014) 14–29.
[56] N. Benayad, A. Fathi, L. Khaya, J. Magn. Magn. Mater. 278 (2004) 407–429.
[57] N. Benayad, A. Fathi, R. Zerhouni, J. Magn. Magn. Mater. 222 (2000) 355–367.
[58] N. Benayad, R. Zerhouni, A. Klümper, Eur. Phys. J. B. 5 (1998) 687–695.
[59] A. Benyoussef, H. Ez-Zahraouy, J. Phys: Condens. Matter. 6 (1994) 3411–3420.
[60] T. Lahcini, N. Benayad, Phase Transitions 82 (2009) 197–210.
[61] L. Khaya, N. Benayad, A. Dakhama, Phys. Status Solidi. 241 (2004) 1078–1087.
[62] N. Benayad, A. Dakhama, J. Magn. Magn. Mater. 168 (1997) 105–120.
[63] N. Benayad, A. Dakhama, Phys. Rev. B. 55 (1997) 12276–12289.

[64] A. Benyoussef, N. Boccara, M. Saber, J. Phys. C:. 18 (1985) 4275–4289.
[65] N. Benayad, M. Ghliyem, Physica B. 407 (2012) 6–13.
[66] N. Benayad, M. Ghliyem, J. Magn. Magn. Mater. 343 (2013) 99–107.

