Investigation Of A Circularly Polarized Laser Field For Mott Scattering Process Of Polarized Electrons

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Abstract: We present a study of Mott scattering of polarized electrons in the presence of a laser field with circular polarization using the helicity formalism and the introduction of the well known concept of non flip differential cross section as well as that of flip differential cross section. The results we have obtained in the presence of a laser field are coherent with those obtained in the absence of a laser field. We have compared our results with those obtained by B. Manaut *et al* [5] who described the process of Mott scattering of polarized electrons in the presence of a laser field with linear polarization. Some differences in the theoretical results obtained are reported.

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I. Introduction

This work deals simultaneously with three important topics in Atomic Physics, namely the spin of a Dirac or a Dirac-Volkov particle, the concept of a spin polarized relativistic particle and finally how the polarization of the laser field affects the process during the scattering of such particles. Many focus issues and international symposia had already been devoted to this subject. To begin with, the first book entirely devoted to the theory of laser-atom-interactions was due to M. H. Mittleman [1]. It dealt mainly with the non relativistic aspects of such processes. In 1997, a pioneering paper by C. Szymanowski *et al* [2] dealing with the process of Mott Scattering in a circularly polarized laser field, used the formalism of Quantum Electrodynamics and the system of atomic units to provide very interesting physical insights that paved the way for other works for the community of atomic physicists. In 1998, a focus issue of the Journal Optics Express [3] provided the state of the art reached at that time and was devoted to the relativistic effects in strong electromagnetic fields. Y. Attaourti *et al* [4] studied the relativistic electronic dressing of the elastic excitation of relativistic atomic hydrogen by electron impact in presence of a circularly polarized laser field as well as the effects of the polarization properties of the laser (elliptic, linear and circular) on such a process. B. Manaut *et al* [5] used a linearly polarized laser field and the concept of polarized electrons to study the process of Mott scattering. S. M. Li *et al* [6] devoted a nice article to the process of Mott scattering in the presence of a linearly polarized laser field of medium intensity. The article of B. Manaut *et al* [5] will serve as a guide to get some physical insights and differences between these two processes particularly how the laser polarization affects or gives different results and how we can shed some light on those. It is important, before presenting our investigation about laser-assisted Mott scattering of polarized electrons to sketch the principal steps of our treatment. For purpose of clarity and simplicity, we begin by the most basic results of Mott Scattering of polarized electrons in the absence of a laser field. Then, in the presence of a circularly polarized laser field, we give a detailed account of the formalism used and we compare the results with those obtained in the absence of a laser field. The organization of this work is as follows: in section 2, we present the laser assisted Mott scattering of polarized electrons in a laser field with circular polarization. In section 3, comparisons and differences of some of our results with those of B. Manaut *et al* [5] are investigated. Finally, we end by a brief conclusion in section 4. Throughout this work, we use atomic units and work with the metric tensor $g_{\mu\nu} = diag(1,-1,-1,-1)$. In many equations of this paper, the Feynman 'slash notation' is used. For

any 4-vector A, $\not{A} = A^{\mu}\gamma_{\mu}$ where the matrices γ are the well known Dirac matrices. See [13] for more details.

II. Theory

The 4-vector potential A^{μ} chosen is such that :

$$A^{\mu} = a_1{}^{\mu}\cos(\phi) + a_2{}^{\mu}\sin(\phi) \quad (1)$$

the polarization 4-vectors $a_1{}^{\mu}$ and $a_2{}^{\mu}$ are such

that :

$$a_1{}^{\mu} = |a| e_x^{\mu} = |a|(0,1,0,0)\ (a_1.a_1) = -|a|^2 = -|A|^2 \quad (2)$$

And

$$a_2{}^{\mu} = |a| e_x^{\mu} = |a|(0,0,1,0)\ (a_2.a_2) = -|a|^2 = -|A|^2 \quad (3)$$

In Eq (1), $\phi = (k.x) = k_{\mu}x^{\mu}$ with $k^{\mu} = (k^{\mu},0,0,k^3) = {}^{\omega}/_{c}\,(1,0,0,1)\ A^{\mu}$ satisfies the Lorentz condition :

$$k_{\mu}A^{\mu} = 0 \quad (4)$$

which implies $(k.a_1) = (k.a_2) = 0$ i.e, we choose k along the direction of the Oz axes. Now, in presence of a laser field, the Dirac wave functions describing the incident and scattered electrons have to be replaced by the Dirac-Volkov wave function [8] which are given by :

$$\psi_q(x) = R(q)\frac{u(p,s)}{\sqrt{2QV}}e^{iS(q,s)} \quad (5)$$

$$R(q) = R(p) = 1 + \frac{\not{k}\not{A}}{2c(k.q)} = 1 + \frac{\not{k}\not{A}}{2c(k.p)} \quad (6)$$

The function $S(q,s)$ appearing in Eq.(3) is such that :

$$S(q,s) = -(q.x) - i\frac{(p.a_1)}{c(k.p)}\sin(\phi) + i\frac{(p.a_2)}{c(k.p)}\cos(\phi) \quad (7)$$

One has for the transition matrix element:

$$S_{fi} = -\frac{i}{c}\int d^4x\bar{\psi}_{q_f}(x)\not{A}_{coul}(x)\psi_{q_i}(x) \quad (8)$$

One must be very cautious not to confuse the Coulomb 4-vector potential due to the charge of the nucleus with the 4-vector potential of the circularly polarized laser field. As before one has $\gamma^0/|x|$ and therefore:

$$S_{fi} = -\frac{i}{c}\int d^4x\bar{\psi}_{q_f}(x)\frac{\gamma^0}{|x|}\psi_{q_i}(x) \quad (9)$$

Now, we transform $e^{\{iS(q_i,x)-iS(q_f,x)\}}$, if we introduce :

$$z = \sqrt{\alpha_1^2 + \alpha_2^2} \quad (10)$$

with

$$\alpha_1 = \frac{(a_1.p_i)}{c(k.p_i)} - \frac{(a_1.p_f)}{c(k.p_f)}\ ;\ \alpha_2 = \frac{(a_2.p_i)}{c(k.p_i)} - \frac{(a_2.p_f)}{c(k.p_f)} \quad (11)$$

we get :

$$S(q_i,x) - S(q_f,x) = -(q_i - q_f).x - z\sin(\phi - \phi_0) \quad (12)$$

where :

$$\cos(\phi_0) = \frac{\alpha_1}{z}\quad or\quad \sin(\phi_0) = \frac{\alpha_2}{z} \quad (13)$$

Now, let us look closely at the quantities in Eq.(9). First, we have:

$$\bar{\psi}_{q_f}(x)\gamma^0\psi_{q_i}(x) = \bar{u}(p_f,s_f)\bar{R}(p_f)\gamma^0R(p_i)u(p_i,s_i) \quad (14)$$

In particular, we remark that:

$$\bar{R}(p_f)\gamma^0R(p_i) = \{1 + c(p_f)[\not{a}_1\not{k}\cos(\phi) + \not{a}_2\not{k}\sin(\phi)]\}$$
$$\times\gamma^0\{1 + c(p_i)[\not{k}\not{a}_1\cos(\phi) + \not{k}\not{a}_2\sin(\phi)]\}$$
$$= \{\gamma^0 + c(p_f)[\not{a}_1\not{k}\gamma^0\cos(\phi) + \not{a}_2\not{k}\gamma^0\sin(\phi)]\}$$
$$\times\gamma^0\{1 + c(p_i)[\not{k}\not{a}_1\cos(\phi) + \not{k}\not{a}_2\sin(\phi)]\} \quad (15)$$

Before invoking the well known relations involving ordinary Bessel functions, we have first to transform Eq.(15) in a more compact form. Eq.(15) contains nine terms, and after some algebraic calculations the final result for $\bar{R}(q_f)\gamma^0R(q_i) = \bar{R}(p_f)\gamma^0R(p_i)$

$$\bar{R}(q_f)\gamma^0R(q_i) = C_0 + C_1\cos(\phi) + C_2\sin(\phi) \quad (16)$$

With

$$C_0 = \gamma^0 - 2k_0 a^2 \not{k} c(p_i) c(p_f)$$
$$C_1 = c(p_i)\gamma^0 \not{k}\not{a}_1 + c(p_f)\not{a}_1 \not{k}\gamma^0 \quad (17)$$
$$C_2 = c(p_i)\gamma^0 \not{k}\not{a}_2 + c(p_f)\not{a}_2 \not{k}\gamma^0$$

Now we have

$$\left.\begin{matrix} 1 \\ \cos(\phi) \\ \sin(\phi) \end{matrix}\right\} e^{-iz\sin(\phi-\phi_0)} = \sum_{s=-\infty}^{\infty} \left\{\begin{matrix} B_s \\ B_{1s} \\ B_{2s} \end{matrix}\right. \quad (18)$$

With

$$\left\{\begin{matrix} B_s \\ B_{1s} \\ B_{2s} \end{matrix}\right. = \left\{\begin{matrix} J_s(z)e^{is\phi_0} \quad (19) \\ [J_{s+1}(z)e^{i(s+1)\phi_0} + J_{s-1}(z)e^{i(s-1)\phi_0}]/2 \\ [J_{s+1}(z)e^{i(s+1)\phi_0} - J_{s-1}(z)e^{i(s-1)\phi_0}]/2i \end{matrix}\right.$$

Therefore :

$$\bar{R}(p_f)\gamma^0 R(p_i) = \sum_{n=-\infty}^{+\infty} [C_0 B_n(z) + C_2 B_{1n}(z) + C_3 B_{2n}(z)]e^{\{-in\phi\}}$$

$$= \sum_{n=-\infty}^{+\infty} \Gamma_n e^{\{-in\phi\}} \quad (20)$$

where

$\Gamma_n = C_0 B_n(z) + C_2 B_{1n}(z) + C_3 B_{2n}(z)$ The transition matrix element becomes:

$$S_{fi} = iZ\,2\pi\delta(Q_i - Q_f + n\omega)\frac{4\pi}{|q|^2}\frac{\bar{u}(p_f,s_f)}{\sqrt{2Q_f V}}\Gamma_n \frac{u(p_i,s_i)}{\sqrt{2Q_i V}}$$

Using the standard procedures of QED, one obtains for the polarized DCS, we have :

$$\frac{d\sigma}{d\Omega_f} = \sum_{n=-\infty}^{+\infty} \frac{d\sigma^{(n)}}{d\Omega_f} \quad (22)$$

where :

$$\frac{d\sigma^{(n)}}{d\Omega_f} = \frac{|q_f|}{|q_i|}\frac{Z^2}{c^4}\frac{1}{|q|^4}\left|\bar{u}(p_f,s_f)\Gamma_{\{n\}}u(p_i,s_i)\right|^2 \quad (23)$$

evaluated for $Q_f = Q_i + n\,\omega$. The squared matrix element is given by:

$$\left|\bar{u}(p_f,s_f)\Gamma u(p_i,s_i)\right|^2 =$$
$$Tr\left\{\Gamma_n \frac{1+\lambda_f\gamma_5\not{s}_f}{2}(\not{p}_f c + c^2)\bar{\Gamma}_n \frac{1+\lambda_i\gamma_5\not{s}_i}{2}(\not{p}_i c + c^2)\right\} \quad (24)$$

With $$\bar{\Gamma}_n = \gamma^0\Gamma^\dagger\gamma^0 \quad (25)$$

III. Results and discussion

Before presenting the results and their physical interpretation, we would like to emphasize the fact that the REDUCE code we have written gave very long analytical expressions which were difficult to incorporate in the corresponding Word manuscript we wrote to extract figures and tables. From now on, the pair of indices n, f will stand for " non flip " and the index f alone will stand for " flip ". For the non flip DCS, let us simply write

$$\left.\frac{d\sigma}{d\Omega}\right|_{(n,f)} = \sum_{n=-\infty}^{\infty} \left.\frac{d\sigma^{(n)}}{d\Omega}\right|_{(n,f)} \quad (26)$$

where :

$$\frac{d\sigma^{(n)}}{d\Omega_f} = \frac{|q_f|}{|q_i|}\frac{Z^2}{c^4}\frac{1}{|q|^4}A(\lambda_i=\lambda_f=1)J_n^2(z) + B(\lambda_i=\lambda_f=1)[J_{n+1}^2(z)+J_{n-1}^2(z)]$$
$$+ C(\lambda_i=\lambda_f=1)J_{n+1}(z)J_{n-1}(z) + D(\lambda_i=\lambda_f=1)J_n(z)[J_{n+1}(z)+J_{n-1}(z)] \quad (27)$$

The four coefficients $A(\lambda_i(2\text{-}1)\lambda_f = 1)$, $B(\lambda_i = \lambda_f = 1)$, $C(\lambda_i = \lambda_f = 1)$ and $D(\lambda_i = \lambda_f = 1)$ are very long to write down so we prefer to focus on their global contents instead of giving tedious details and explanations. First, we noticed that in both coefficients $A(\lambda_i = \lambda_f = 1)$ and $B(\lambda_i = \lambda_f = 1)$, there is no occurrence of the various completely anti-symmetric tensors $\varepsilon_{\alpha\beta\gamma\delta}$ (where the indices are Lorentz ones and take integer values from 0 to 3). This clearly means that these tensors were totally contracted and we ended up with two tractable coefficients that where very easy to incorporate into the main FORTRAN code. Let us remind the reader that we used throughout this work, the convention :

$$\varepsilon_{0123} = 1$$

meaning that $\varepsilon_{\alpha\beta\gamma\delta} = 1$ for an even permutation of the Lorentz indices whereas $\varepsilon_{\alpha\beta\gamma\delta} = -1$ for for an odd permutation of the Lorentz indices and finally $\varepsilon_{\alpha\beta\gamma\delta} = 0$ otherwise. Second, the coefficients $B(\lambda_i = \lambda_f = 1)\ and\ D(\lambda_i = \lambda_f = 1)$ contained various non contracted tensors. For example in $B_{n,f}$, there are thirty one non contracted tensors involving $\varepsilon_{\alpha\beta\gamma\delta}$ whereas in $D_{n,f}$ there are sixty four. Particle physicists are very often dealing with these. As for us, we were confronted for the first time with coefficients like for example in $B_{n,f}$:

$$\varepsilon(a_1, a_2, k, p_i) = \varepsilon_{\alpha\beta\gamma\delta} a_1^\alpha a_2^\beta k^\gamma p_i^\delta \tag{28}$$

$$\varepsilon(a_1, a_2, k, v) = \varepsilon_{\alpha\beta\gamma\delta} a_1^\alpha a_2^\beta k^\gamma v^\delta \tag{29}$$

Such coefficients are, at first sight, very complicated to evaluate. But, when following the conventions of A.G Grozin [9], we found that the only non-vanishing contribution of a_1 corresponds to the Lorentz index $\alpha = 1$ while the only non-vanishing contribution of a_2 corresponds to the Lorentz index $\beta = 2$ These helpful values are due to the choice of the 4-potential A^μ. Also, it allows to deduce that the only non trivial indices remaining for $k^\gamma\ and\ p_i^\delta$ are the pairs ($\gamma = 0, \delta = 3\ and\ \delta = 3$ So, the non contracted tensor $\varepsilon(a_1, a_2, k, p_i)$ reduces to :

$$\varepsilon(a_1, a_2, k, p_i) = |a|^2[\varepsilon_{1203}k^0 p_i^3 + \varepsilon_{1230}k^3 p_i^0] = |a|^2 \frac{\omega}{c}\left[|p_i|\cos(\theta_i) - \frac{E_i}{c}\right] \tag{30}$$

where θ_i is the angle between the initial electron momentum and the Oz axis. The coefficient $\varepsilon(a_1, a_2, k, v)$ is easier to evaluate since the Lorentz index corresponding to $v = (1,0,0,0)$ is zero $\delta = 0$. So, the only choice left for k is $\gamma = 3$. Thus, we have. It is not necessary to give the whole set of the coefficients involving the non contracted tensors appearing in $B_{(n,f)}$ and $D_{(n;f)}$. It is sufficient to follow the rules concerning the tensor $\varepsilon_{\alpha\beta\gamma\delta}$ and the geometry chosen for a_1, a_2 and k bearing in mind that $a_1^\mu = |a| e_1^\mu = |a|(0,1,0,0)$, $a_2^\nu = |a| e_2^\nu = |a|(0,0,1,0)$, and finally $k^\sigma = \frac{\omega}{c}(1,0,0,1)$. The same holds for the coefficients $B(\lambda_i = -\lambda_f = 1)\ and\ D(\lambda_i = -\lambda_f = 1)$

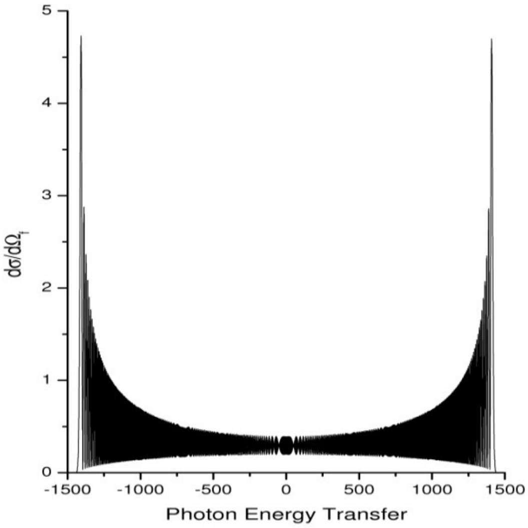

Fig 1: Envelope of the unpolarized relativistic DCS scaled in $10^{-10}\ a.u$ as a function of the energy transfer scaled in units of the laser photon energy Ω for an electrical field strength of $E = 0.01\ a.u$ and a relativistic parameter $\gamma = 2$.

The complete expressions of the coefficients , B, C and D in both cases (nf, f) can be found in [9]. It is important to remind that these coefficients occur only in presence of a laser field. If we put $A^\mu = (0,0,0,0)$, we easily recover the results of section 2. Now, let us turn to the discussion of the results concerning $\left.{d\sigma}/{d\Omega_f}\right|_{n,f}$ and $\left.{d\sigma}/{d\Omega_f}\right|_{f}$. The relativistic regime corresponds to a relativistic parameter $\gamma = 2$ and an electric field strength of medium intensity $E = 0.01\ a.u.$ We have to show numerically that the sum of $(DCS)_f$ and $(DCS)_{n,f}$ always gives the unpolarized DCS and second, the non relativistic description for the unpolarized DCS must give the unpolarized DCS we have found in the formalism we have developed. Once again, there is a weak asymmetry between the emission part of the envelope and the absorption part of the same envelope for the unpolarized DCSs because of the presence of the denominator $|q|^4$ in Eq.(23). In Fig.(1), we show the envelope of ${d\sigma}/{d\Omega_f}$ as a function of the net number of photons exchanged.

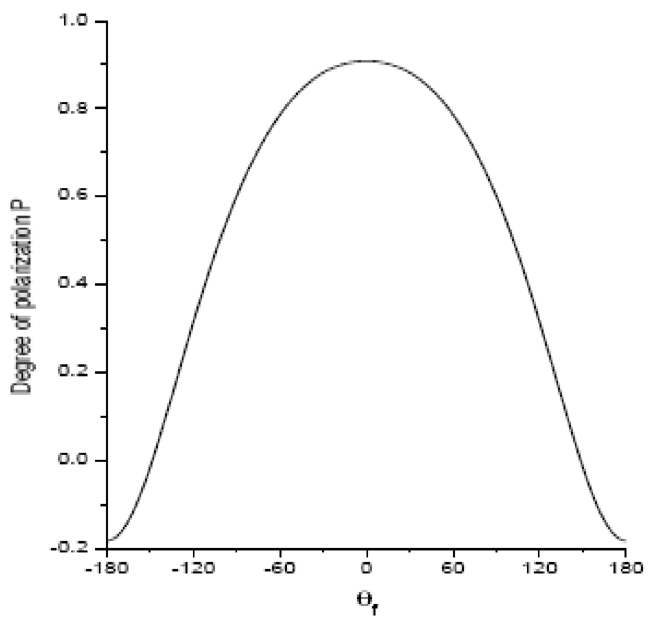

Fig 2: Degree of polarization P for an electrical field strength of $E = 0.01\ a.u$ and a relativistic parameter $\gamma = 2$.

The cutoffs are $n \simeq -1500$ photons for the negative part of the envelope and $n \simeq +1500$ photons for the positive part of the envelope. The geometry chosen for Fig.(1) is $\theta_i = 45°$, $\phi_i = 0°$,$\theta_f = 75°$ and $\phi_f = 90°$.

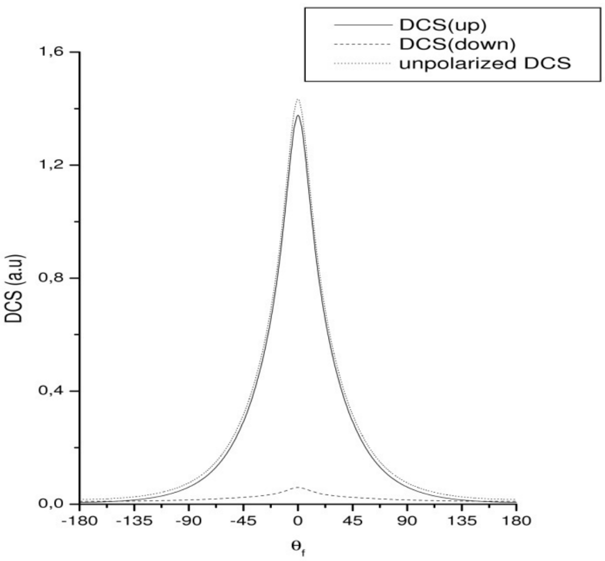

Fig 3: The various relativistic DCSs scaled in $10^{-7}\ a.u$ as a function of the angle θ_f in degrees for an electrical field strength of $E = 0.01\ a.u$ and a relativistic parameter $\gamma = 2$ for the circular polarization of the laser field. The corresponding number of photons exchanged is ± 200.

One many ask legitimately if such cutoffs are geometry-dependent. The answer is that indeed they are geometry dependent since for a different choice of the initial, final momentum angular coordinates p_i and p_f namely $\theta_i = 30°$, $\phi_i = 0°$., $\theta_f = 75°$ and $\phi_f = 90°$. the cutoffs obtained are $n \simeq -1600$ photons and $n \simeq +1600$photons. For the degree of polarization shown in Fig. (2), we obtained a qualitative result similar to that of a linearly polarized field. We reached the same conclusion as in [5] that the degree of polarization

$$P = 1 - 2\frac{\frac{d\sigma(\downarrow)}{d\Omega_f}}{\frac{d\sigma(\uparrow)}{d\Omega_f} + \frac{d\sigma(\downarrow)}{d\Omega_f}} \qquad (31)$$

is weakly dependent on the number of photons exchanged. This degree of polarization P varies as a function of the angle θ_f for the following geometry ($\theta_i = 45°$, $\phi_i = 0°$., $\phi_f = 90°$ and $0 \le \theta_f \le 180$. To begin with, we have made simulations concerning the various DCSs for a set of net number of photons exchanged. These sets

($\pm 100, \pm 200, \pm 300, \pm 400, \pm 500, \pm 1000$) showed that the order of magnitude of the non flip DCS is close to the unpolarized DCS but the contribution of the flip DCS is not completely negligible even if it is small compared to both. The behavior of the three DCSs when the number of photons exchanged increases has an influence over the numerical values of the differential cross sections but as we are limited in our computational capabilities, it is not possible to achieve numerical convergence but the most important result is that the sum of DCS(up) and DCS(down) always gives the unpolarized DCS as this is shown in Fig.(3). Now, the comparison between the results for the circular and the linear polarization of the laser field must be addressed. Fig.(4) shows that the various DCSs for circular polarization of the laser field are lower than those for linear polarization of the laser field. This property has been checked for various geometry and a first simple physical explanation is as follows: when embedded in a linearly polarized laser field, the electron travels in an electric and magnetic

fields that are perpendicular whereas in a circularly polarized laser field, it travels also in an electric and magnetic fields that are perpendicular but at the same time rotating in a circle whose radius corresponds to the magnitude of the field. Consequently, the probability that the electron scatters with the heavy nucleus is lower for the circularly polarized laser field than that corresponding to a linearly polarized laser field. As this probability is directly linked to the concept of differential cross section in scattering theory, the results shown in Fig.(4) are coherent with this explanation.

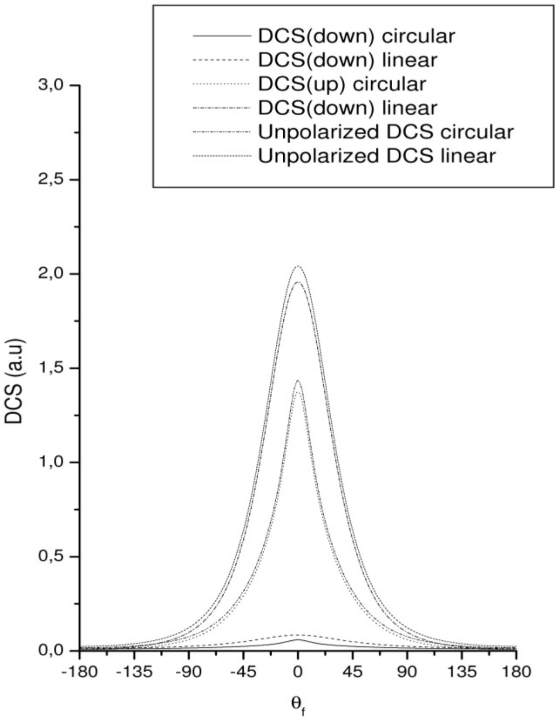

Fig 4: Comparison of the various relativistic DCSs scaled in 10^{-7} $a.u$ a.u as a function of the angle θ_f in degrees for an electrical field strength of $E = 0.01$ $a.u$ a.u and a relativistic parameter $\gamma = 2$. The corresponding number of photons exchanged is ± 200

IV. Conclusion

In this work, we have checked that results already known in the absence of a laser field are also valid in presence of a laser field, at least in first order of perturbation theory. The motivation of this study was to compare the order of magnitude for the various DCSs in the relativistic regime that we already found in [5] particularly in the case of a laser field with linear polarization and we have found (see Fig.(4)) that indeed when scaled in 10^{-7}, both DCSs are of similar order of magnitude and shape. Moreover, when focusing only on the formalism for polarized electrons, the main difference between the linear and the circular polarization of the laser field, is that a thorough analysis of the contents of the formal expression of DCS(up) and DCS(down) is more complicated in the case of the circular polarization than the linear polarization of the laser field and this is due to the fact that these two DCSs contain non contracted symbols that we had to deal with for the first time. We succeeded in the simulations of the main two consistency checks namely that the sum of DCS(up) and DCS(down) always gives the unpolarized DCS regardless of the number of net photons exchanged and also that in the non relativistic limit, both unpolarized DCSs (non relativistic and relativistic) give very close results. We also gave the envelopes of the two unpolarized DCSs in both regimes for the sake of illustration. The asymmetric deviation from the elastic peak is now weaker than in the case of an electric field strength of $E = 1$ $a.u$ since the intensity of the laser we choose is now a medium electric field $E = 0.01$ $a.u.$ The envelope for the relativistic regime was also obtained. Needless to say that working in simple precision, the results presented are nonetheless sound and coherent.

V. References

[1] M. H. Mittleman, Introduction to The Theory of Laser-Atom Interactions, Springer, 2ed, (1993).
[2] C. Szymanowski et al, Phys. Rev. A, **56** 3846, (1997).
[3] J. H. Eberly Editor, Focus Issue : Relativistic effects in strong electromagnetic fields, Optics Express, **2**, p 261-297, (1998).
[4] Y. Attaourti et al, Phys. Rev. A, **69**, 063407, (2004); Phys. Rev. A, **70**, 023404, (2004).
[5] B. Manaut et al, Phys. Rev. A **71**, 043401 (2005).
[6] S. M. Li et al, Phys. Rev. A **67**, 063409 (2003).
[7] W. Greiner, Relativistic Quantum Mechanics, 3ed, Springer,(2000); W. Greiner and J. Reinhardt Quantum Electrodynamics, 3ed, Springer,(2003).
[8] Landau and E. M Lifshitz, Quantum Mechanics, Non Relativistic Theory, 3ed, New York Pergamon, (1997).
[9] A. G. Grozin, Using Reduce in High Energy Physics, Cambridge University Press, Cambridge, (1997).