Coulomb Scattering Of An Electron In Strong Laser Fields

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Abstract : In this work, we review and correct the first Born differential cross section for the process of Mott scattering of a Dirac-Volkov electron, namely, the expression (26) derived by Szymanowski et al [Physical Review A56, 3846 (1997)]. In particular, we disagree with the expression of $d\sigma/d\Omega$ they obtained and we give the exact coefficients multiplying the various Bessel functions appearing in the scattering differential cross section. Comparison of our numerical calculations with those of Szymanowski et al. shows qualitative and quantitative differences when the incoming total electron energy and the electric field strength are increased particularly in the direction of the laser propagation. Such corrections are very important since the relativistic electronic dressing of any Dirac-Volkov charged particle gives rise to these coefficients that multiply the various Bessel functions and the relativistic study of other processes (such as excitation, ionization, etc....) depends strongly of the correctness and reliability of the calculations for this process of Mott Scattering in presence of a laser field. Our work has been accepted [Y. Attaourti, B. Manaut, Physical Review A68, 067401 (2003)] but only as a comment. In this paper, we give the full details of the calculations as well as the clear explanation of the large discrepancies that their results could cause when working in the ultra relativistic regime and using a very strong laser field corresponding to an electric field $\varepsilon = 5.89$ in atomic units.

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I. Introduction

In a pioneering and very often cited paper, Szymanowski et al. [1] have studied the Mott scattering process in a strong laser field. The main purpose of their work was to show that the modifications of the Mott scattering differential cross section for the scattering of an electron by the Coulomb potential of a nucleus in the presence of a strong laser field can yield interesting importance and the signatures of the relativistic effects. Their spin dependent relativistic description of Mott scattering permits to distinguish between kinematics and spin-orbit coupling effects. They have compared the results of a calculation of the first Born differential cross section for the Coulomb scattering of the Dirac-Volkov electrons dressed by a circularly polarized laser field to the first Born cross section for the Coulomb scattering of spinless Klein-Gordon particles and also to the non relativistic Schrödinger-Volkov treatment. The aim of our work is to provide the correct expression for the first-Born differential cross sections corresponding to the Coulomb scattering of the Dirac-Volkov electrons. On the one hand, we show that the terms proportional to $sin(2\varphi_0)$ are missing in [1], where φ_0 is the phase stemming from the expression of the circularly polarized electromagnetic field. The claim of [1] that they vanish is not true. These terms do not depend on the chosen description of the circular polarization in cartesian components. On the other hand, we perform the calculations with some details and throughout this work, we use atomic units $\hbar = e = m = 1$ where m denotes the electron mass. The abbreviation DCS stands for the differential cross section.

The organization of this paper is as follows: in Section 2, we give the expression of the S matrix transition amplitude as well as the formal expression of scattering DCS. In section 3, we give some estimates of the numerical significance of our corrections. In particular, we compare numerically the Dirac-Volkov DCS we have obtained with the corresponding DCS of [1]. We end by a brief conclusion in Section 4.

II. Theory

The interaction of the dressed electrons with the central Coulomb field

$$A^{\mu} = (-\frac{Z}{|x|}, 0, 0, 0) \qquad (1)$$

is considered as a first-order perturbation. This is well justified if $Z\alpha \ll 1$, where Z is the nuclear charge of the nucleus considered and α is the fine-structure constant. We evaluate the transition matrix element for the transition ($i \to f$)

$$S_{fi} = \frac{iZ}{c} \int d^4x \, \bar{\Psi}_{qf}(x) \frac{\gamma^0}{|x|} \psi_{qi}(x). \qquad (2)$$

We consider the quantity

$$\bar{\Psi}_{qf}(x) \frac{\gamma^0}{|x|} \psi_{qi}(x) = \frac{1}{\sqrt{2Q_i V}} \frac{1}{\sqrt{2Q_f V}}$$

$$\times \, \bar{u}(p_f, s_f) \bar{R}(p_f) \frac{\gamma^0}{|x|} R(P_i) u(p_i, s_i) \times e^{-i(S(q_f,x)-S(q_i,x))} \quad (3)$$

With $R(p) = e^{kA/2c(k.p)} = 1 + \frac{kA}{2c(k.p)}$

We have

$$e^{-i(S(q_f,x)-S(q_i,x))} = e^{[i(q_f-q_i)x - iz\sin(\phi-\phi_0)]} \quad (4)$$

Where z is such that

$$z = \sqrt{\alpha_1^2 + \alpha_2^2} \qquad (5)$$

Whereas the quantities α_1 and α_2 are given by

$$\alpha_1 = \frac{(a_1.p_i)}{c(k.p_i)} - \frac{(a_1.p_f)}{c(k.p_f)}$$

$$\alpha_2 = \frac{(a_2.p_i)}{c(k.p_i)} - \frac{(a_2.p_f)}{c(k.p_f)} \qquad (6)$$

and the phase ϕ_0 is such that

$$\phi_0 = arccos\left(\frac{\alpha_1}{z}\right) = arcsin\left(\frac{\alpha_2}{z}\right) = \arctan\left(\frac{\alpha_2}{\alpha_1}\right)$$

With $c(p) = 1/2c(k.p)$ and $k^0 = k_0 = \omega/c$. Therefore, the transition matrix element becomes

$$S_{fi} = \frac{iZ}{c} \int d^4x \frac{1}{\sqrt{2Q_i V}} \frac{1}{\sqrt{2Q_f V}} \bar{u}(p_f, s_f)[C_0 + C_1\cos(\phi) + C_2\sin(\phi)] u(p_i, s_i)$$
$$\times \, e^{[i(q_f-q_i)x - iz\sin(\phi-\phi_0)]} \quad (7)$$

where the three coefficients C_0 , C_1 and C_2 are respectively given by

$$C_0 = \gamma^0 - 2k_0 a^2 \not{k} c(p_i) c(p_f)$$
$$C_1 = c(p_i)\gamma^0 \not{k} \not{a}_1 + c(p_f) \not{a}_1 \not{k} \gamma^0 \quad (8)$$
$$C_2 = c(p_i)\gamma^0 \not{k} \not{a}_2 + c(p_f) \not{a}_2 \not{k} \gamma^0$$

We now invoke the well-known identities involving ordinary Bessel functions $J_s(z)$

$$\left.\begin{matrix} 1 \\ \cos(\phi) \\ \sin(\phi) \end{matrix}\right\} e^{-iz\sin(\phi-\phi_0)} = \sum_{s=-\infty}^{\infty} \left\{\begin{matrix} B_s \\ B_{1s} \\ B_{2s} \end{matrix}\right. \qquad (9)$$

With

$$\left\{\begin{matrix} B_s \\ B_{1s} \\ B_{2s} \end{matrix}\right. = \left\{\begin{matrix} J_s(z) e^{is\phi_0} \\ [J_{s+1}(z) e^{i(s+1)\phi_0} + J_{s-1}(z) e^{i(s-1)\phi_0}]/2 \\ [J_{s+1}(z) e^{i(s+1)\phi_0} - J_{s-1}(z) e^{i(s-1)\phi_0}]/2i \end{matrix}\right. \quad (10)$$

After some algebraic calculations, and using textbook by A. G. Grozin [6] which is full of worked examples in various fields of physics particularly in QED. We give the final result for the unpolarized DCS for the Mott scattering of a Dirac-Volkov electron:

$$\frac{d\bar{\sigma}}{d\Omega_f} = \sum_{s=-\infty}^{\infty} \left.\frac{d\sigma^{(s)}}{d\Omega_f}\right|_{Q_f+s\omega}$$

$$= \sum_{s=-\infty}^{\infty} \frac{Z^2}{c^2} \frac{|q_f|}{|q_i|} \frac{1}{|q_f - q_i - sk|^4} \frac{2}{c^2} \{J_s{}^2 A +$$
$$(J_{s+1}{}^2 + J_{s-1}{}^2) B + (J_{s+1} J_{s-1}) C + J_s (J_{s+1} +$$
$$J_{s-1}) D\}_{Q_f+s\omega}$$

(11)

The coefficients A, B, C and D are respectively given by

$$A = c^4 - (q_f.q_i)c^2 + 2Q_f Q_i - \frac{a^2}{2}\left(\frac{(k.q_f)}{(k.q_i)} +\right.$$
$$\left.\frac{(k.q_i)}{(k.q_f)}\right) + \frac{a^2\omega^2}{c^2(k.q_f)(k.q_i)}\left((q_f.q_i) - c^2\right) +$$
$$\frac{(a^2)^2\omega^2}{c^4(k.q_f)(k.q_i)} + \frac{a^2\omega}{c^2}(Q_f - Q_i)\left(\frac{1}{(k.q_i)} + \frac{1}{(k.q_f)}\right)$$

(12)

$$B = \frac{(a^2)^2\omega^2}{2c^4(k.q_f)(k.q_i)} + \frac{\omega^2}{2c^2}\left(\frac{(a_1.q_f)(a_1.q_i)}{(k.q_f)(k.q_i)} + \frac{(a_2.q_f)(a_2.q_i)}{(k.q_f)(k.q_i)}\right) - \frac{a^2}{2} + \frac{a^2}{4}\left(\frac{(k.q_f)}{(k.q_i)} + \frac{(k.q_i)}{(k.q_f)}\right) - \frac{a^2\omega^2}{2c^2(k.q_f)(k.q_i)}\left((q_f.q_i) - c^2\right) + \frac{a^2\omega}{2c^2}(Q_f - Q_i)\left(\frac{1}{(k.q_f)} + \frac{1}{(k.q_i)}\right) \tag{13}$$

$$C = \frac{\omega^2}{c^2(k.q_f)(k.q_i)}\left(\cos(2\phi_0)\{(a_1.q_f)(a_1.q_i) - (a_2.q_f)(a_2.q_i)\} + \sin(2\phi_0)\{(a_1.q_f)(a_2.q_i) - (a_2.q_f)(a_1.q_i)\}\right) \tag{14}$$

$$D = \frac{c}{2}(\dot{A}.q_f)(\dot{A}.q_i) - \frac{c}{2}\left\{\frac{(k.q_f)}{(k.q_i)}(\dot{A}.q_i) + \frac{(k.q_i)}{(k.q_f)}(\dot{A}.q_i)\right\} + \frac{\omega}{c}\left\{\frac{(A.q_f)}{(k.q_f)}Q_i + \frac{(A.q_i)}{(k.q_i)}Q_f\right\} \tag{15}$$

Where

$$\dot{A} = a_1\cos(\phi_0) + a_2\sin(\phi_0)$$

III. Results and discussion

For the description of the scattering geometry, we work in a coordinate system in which $k||e_z$. This means that the direction of the laser propagation is along the Oz axis. To avoid any confusion, we will compare the Dirac-Volkov DCS (26) of [1] with the corresponding DCS (11) we have obtained in the same coordinate system.

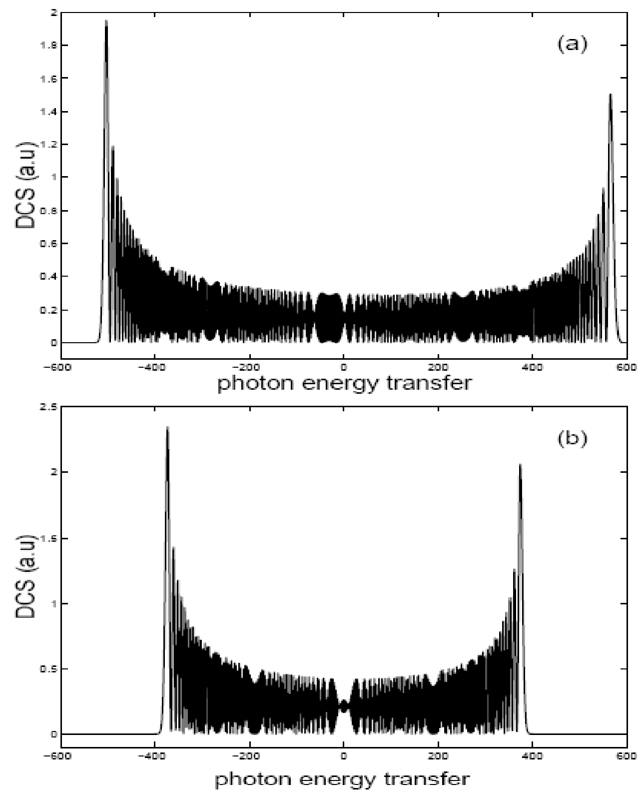


Fig 1 (a): Envelope of the non relativistic differential cross-section $\frac{d\sigma}{d\Omega}$ scaled in unit of 10^{-7} $a.u$ as a function of energy transfer $Q_f - Q_i$ scaled in units of the laser photon energy w for an electric field strength of 0.05 $a.u.$ and a relativistic parameter $\gamma = 1.0053$, (b). Envelope of the relativistic differential cross-section $d\sigma/d\Omega_{DV}$ scaled in unit of 10^{-7} $a.u$ as a function of the laser photon energy ω for the same parameters. The envelope for $d\sigma/d\Omega_{DV}$[1] and $d\sigma/d\Omega_{KG}$ are almost identical.

We have compared our Dirac-Volkov DCS and the Dirac-Volkov DCS (26) of [1] and We turn to a qualitative and quantitative discussion of the physical process. We shall comment and analyze the results obtained in [1] in the light of those we have obtained bearing in mind that we can hardly escape rephrasing the physical insights and explanations contained in [1]. Our disagreement is quantitative since we have shown in the first part of this work that the expression (26) of [1] contains errors and a missing term proportional to $\sin(2\phi_0)$. So, our primary task is to assess the importance of this errors and missing term and to what extent they modify the quantitative and qualitative contents of [1].

III.1- The non relativistic-low electric field strength regime

In this regime, we choose as in [1] $\gamma = 1.0053$ for the relativistic parameter and $\varepsilon = 0.05\ a.u$ for the electric field strength. This relativistic parameter corresponds to an incoming electron kinetic energy $T_i = 100\ a.u = 2.7\ keV$. We plot in the upper part (a) of Figure 1 the non relativistic DCS given by Eq.(34) of [1] and in the lower part (b) of the same figure, the generalized Dirac-Volkov DCS given either by Eq.(26) of [1] or Eq.(11) of our work as a function of the final electron energy scaled to the photon energy. The scattering angle is large enough so that an important number of photons can be exchanged in the course of the collision. In this low-intensity regime, the envelope of the non relativistic DCS is qualitatively different from the envelope for the Dirac-Volkov and Klein-Gordon DCSs. The observed cutoffs occur at $s_{min} = -522$ and $s_{max} = 582$ for the non relativistic DCS and $s_{min} = -474$ and $s_{max} = 474$ both for the Dirac-Volkov and Klein-Gordon DCSs since the argument that appears in the ordinary Bessel functions is the same for both DCSs. So the comments made in [1] concerning the interpretation of the envelope obtained do not apply for the Dirac-Volkov and Klein-Gordon cases. While the spectrum of Figure (1.a) of our work (which is identical to that of Figure (1.a) of [1]) exhibits an overall asymmetric envelope with peaks of negative energy transfer higher than peaks of positive energy transfer, this asymmetry is less pronounced in the case of the Dirac-Volkov and spinless particle DCSs. This emphasized asymmetry in the non relativistic case can easily be traced back by a close look at Eq.(34) of [1]. Indeed, the non relativistic DCS depends on $J_s^2(z)$ (z depends only weakly on s) so the asymmetry can only come from the dependence of the modulus of the final momentum q_f on the number of the transferred photons s according to Eq.(25) of [1]. As mentioned in [1], we have an enhancement of negative over positive-energy transfer cross-section. The DCSs fall of abruptly beyond the points where the argument of the Bessel functions equal to the order. For the Dirac-Volkov and Klein-Gordon DCSs, this cutoff occurs (up to machine precision) numerically for $s = \pm 474$ and an argument z of the ordinary Bessel functions almost constant and equal to 380.016. However Figure (1.b) shows a visual cutoff for $s = \pm 392$. For the non relativistic DCS, the numerical cutoff occurs (again up to machine precision) for $s = -606$ and $s = 685$. The visual cutoff occurs for $s = -500$ and $s = 590$. The difference $\Delta s = 20$ between our results and that of [1] is just a matter of convention. We now analyze the angular distributions. We have summed as in [1] ± 100 peaks around the elastic one in order to draw the angular dependence of the DCS. In Figure 6. Of [1], the accumulated DCS is shown for an electric field strength$\varepsilon = 0.05\ a.u$. The computer code we have written calculates the Dirac-Volkov DCS (11) of our work, the Dirac-Volkov DCS (26) of [1], the spinless particle DCS and the non relativistic DCS. At least, in the non relativistic regime, our results and that of [1] agree very well and are both close to the results for a spinless particle. We give in Figure 3 the angular distribution of the various DCSs.

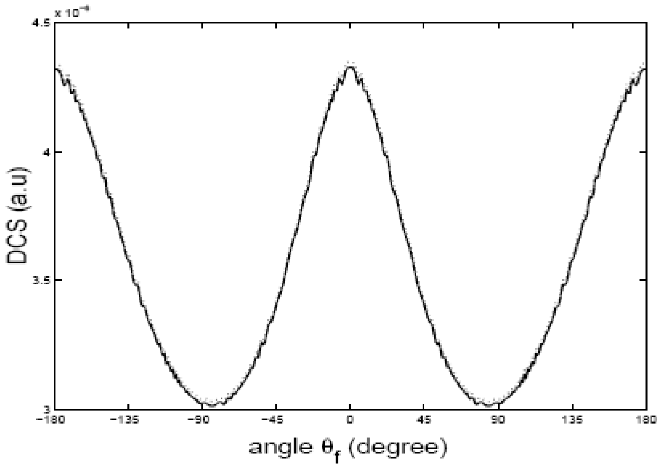


Fig. 2: Summed differential cross sections $\frac{d\sigma}{d\Omega}$ of ±100 peaks around the elastic one as a function of the angle θ_f for a relativistic parameter $\gamma = 1.0053$ and an electric field strength $\varepsilon = 0.05\ a.u$ The solid line denotes the result for Dirac-Volkov electrons, the long dashed one sketches the values for $d\sigma/d\Omega_{DV}$[1] and the short dashed is the result for spinless particles

Apart from minor differences, all three calculations exhibit maxima for $\theta_f = 0^\circ$ and $\pm 180^\circ$ a giggling oscillatory behaviour (as in [1]) and minima slightly shifted from $\pm 90^\circ$ (at $\pm 84^\circ$ Let aside the order of magnitude, we have in our case, three DCSs that are close to each other and not as differentiated as shown in Figure 6. of [1]. So, this adds to the controversy. Even if we use the expression for the Dirac-Volkov DCS given by Eq.(26) of [1], we have a different figure for the non relativistic regime. If we now increase the electric field strength from $\varepsilon = 0.05\ a.u$ to $\varepsilon = 1.00\ a.u$ the agreement remains good

between the three relativistic calculations. There is still a maximum at $\theta_f = 0^c$ $ while the minima are shifted towards $\pm 117^c$ To give an idea the small differences between our result and the result of [1] for the Dirac-Volkov DCS, we have plotted in the upper part (a) of Figure 4, the ratio of the DCS given by Eq.(26) of [1] to the DCS given by Eq.(11) of our work as a function of the angle $\theta_{f,}$ $\varepsilon = 1.00\ a.u$. The ratio R is defined by

$$R = \frac{d\sigma/d\Omega_{DV}{}^{[1]}}{d\sigma/d\Omega_{DV}} \qquad (16)$$

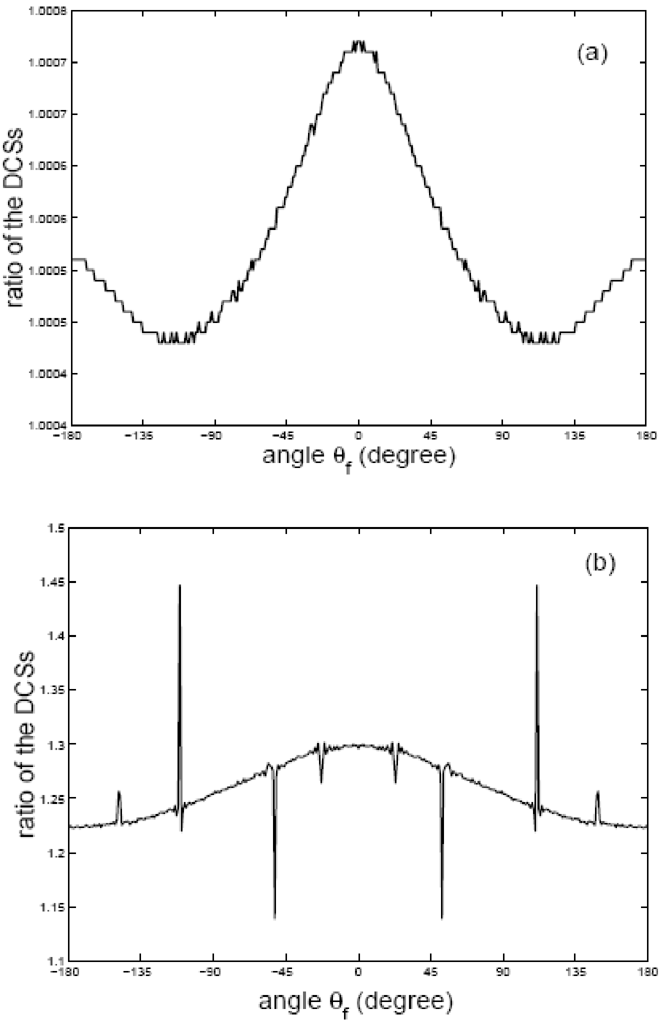


Fig 3 : Ratio R of the two Dirac-Volkov DCS for $\gamma = 1.0053$, $\varepsilon = 1.00\ a.u$ and $s = \pm 100$. (b): Ratio R of the two Dirac-Volkov DCS for $\gamma = 1.0053$, $\varepsilon = 5.00\ a.u$ and $s = \pm 100$.

The deviations from the expected value 1 are shown and have the same shape as the corresponding DCS. However, for increasing electric field strength, the values for this ratio are not close to 1. For a relativistic parameter $\gamma = 1.0053$ and for an electric field strength $\varepsilon = 5.00\ a.u$ and $s = \pm 100$, our results for the Dirac-Volkov DCS and the corresponding results of [1] do not agree at all. In the lower part (b) of Figure (4), there is an over estimation varying from 22.5 % to 30 % with some peaks giving an over estimation of up to 45 % for the DCS (26) of [1] compared to the corresponding DCS (11}) of this work. All these peaks are nearly multiples or submultiples of an angle close to $\pi/4$

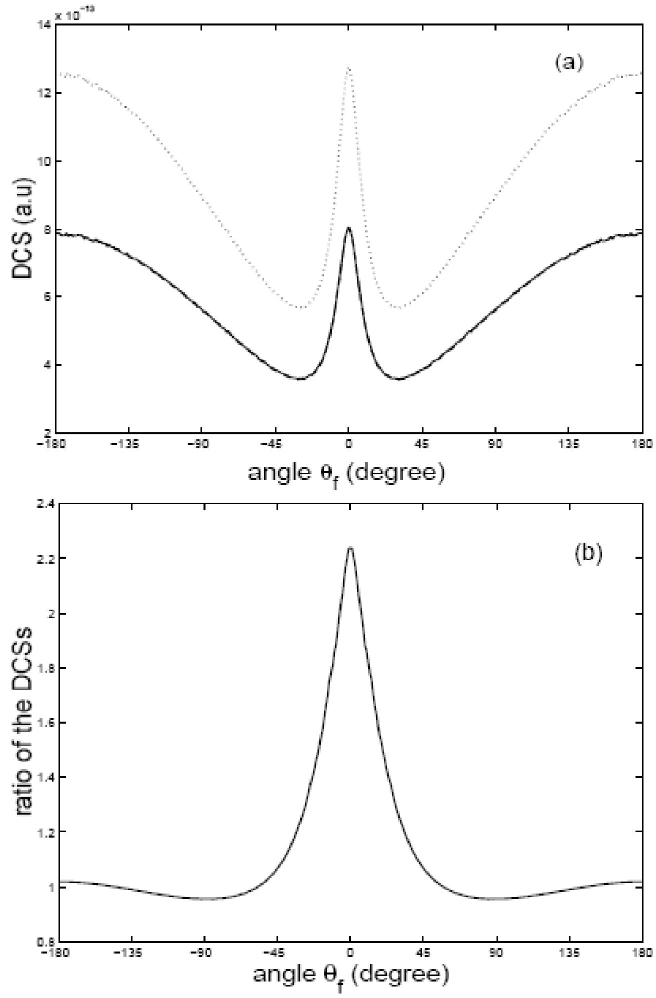
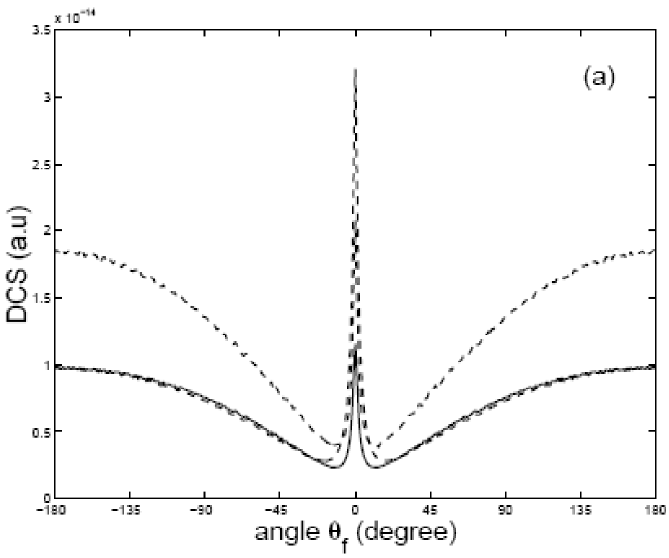


Fig 4 : (a) Summed differential cross sections $\frac{d\sigma}{d\Omega}$ of ± 100 peaks around the elastic one as a function of the angle θ_f for a relativistic parameter $\gamma = 2.0$ and an electric field strength $\varepsilon = 1.00\ a.u$. The solid line denotes the result for the Dirac-Volkov electrons, the long dashed one sketches the values for $d\sigma/d\Omega_{DV}{}^{[1]}$ and the short dashed is the result for spinless particles. (b): Ratio $R$of the two Dirac-Volkov DCSs for a relativistic parameter $\gamma = 2.0$ and an electric field strength $\varepsilon = 5.89\ a.u$

III-2. Relativistic-strong electric field strength regime

For the relativistic regime, we have chosen the parameters of [1] $\gamma = 2$ which corresponds to an incoming electron total energy $E_i = 2c^2$ or a $T_i = 0.5116MeV$. The electric field strength is now $\varepsilon = 1.00\ a.u.$ In this regime, dressing effects are important. The envelope of the energy distribution of the scattered electrons is similar to the one displayed in the lower part (b) of Figure 1 However, there is a more important asymmetry than in the non relativistic regime with was to be expected. The corresponding cutoffs are $(-170000, 194000)$ for $d\sigma/d\Omega_f{}^{(NR)}$ and $(-66000, 64000)$ for $d\sigma/d\Omega_{DV}$, $d\sigma/d\Omega_{DV}{}^{[1]}$ and $d\sigma/d\Omega_f{}^{(KG)}$ The three relativistic calculations lead to angular distributions peaked in the direction of the laser propagation $\theta_f = 0^\circ$. The two Dirac-Volkov DCSs (solid line and long dashed line) are slightly different only in the vicinity of the two minima located at $\theta_f \simeq \pm 33^\circ$ In this regime, the shape of the ratio R is similar to that of the corresponding DCSs. This ratio is equal to 1 for $\theta_f = 180^\circ$ but there is now an overall amplitude of 8.10^3 around the expected value 1. If we increase the electric field strength from $\varepsilon = 1.00\ a.u$ $ to $\varepsilon = 5.89\ a.u$ and keep the same value of the relativistic parameter $\gamma = 2,$ the difference between our Dirac-Volkov results and the corresponding results of [1]$ becomes important.



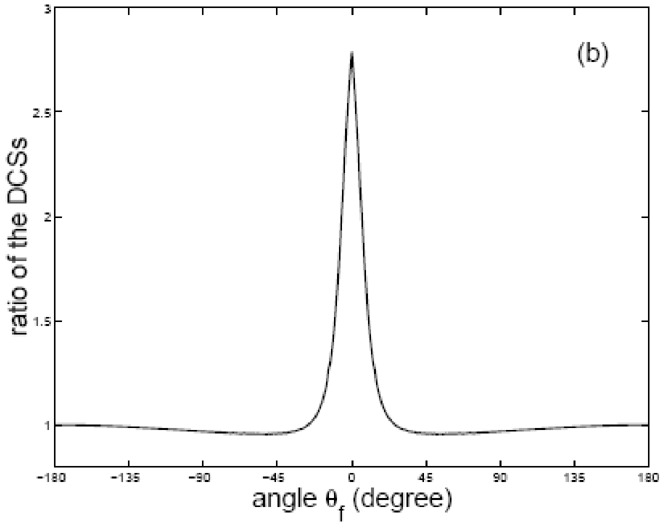


Fig 5 : (a): Summed differential cross section $\frac{d\sigma}{d\Omega}$ of ± 100 peaks around the elastic one as a function of the angle θ_f for a relativistic parameter $\gamma = 5.0$ and an elastic field strength $= 5.89\ a.u$. Th solid line denotes the result for Dirac-Volkov electrons, the long dashed one sketches the values for $d\sigma/d\Omega_{DV}{}^{[1]}$ and the short dashed is the result for spinless particles. (b): Ratio R of the two Dirac-Volkov DCSs for the same values of the relevant parameters, $\gamma = 5.00$ and $\varepsilon = 5.89\ a.u$

In the upper part(a) of Figure 5, we give the two Dirac-Volkov DCSs and in the lower part (b) of the same figure we give the ratio R of the two DCSs for the relativistic parameter $\gamma = 2.00$ and an electric field strength $\varepsilon = 5.89\ a.u$ For the angles $\theta_f = \pm 180^\circ$, the ratio is $R \simeq 1.01$ while for the peak in the direction of the laser propagation, $\theta_f = 0^\circ$, the ratio is $R \simeq 2.34$

III-3. Relativistic and high electric field strength regime

To study this regime, we use the same parameters as in [1], that is a relativistic parameter $\gamma = 5.00$ or an incoming electron kinetic energy $$T_i = 4c^2\ a.u = 2.045\ MeV$ The electric field strength is $\varepsilon = 5.89\ a.u.$ In the upper part (a) of Figure 6, we show the various DCSs. For angles $ $\theta_f = \pm 180^\circ$, the agreement between our results and the results of [1] is good but deteriorates for small values of θ_f. For $\theta_f = 0^\circ$, the result of our work gives a value (scaled in 10^{14} a.u) $d\sigma/d\Omega_f{}^{[DV]} = 1.15$ while de corresponding result found using Eq.(26) of

[1] is $d\sigma/d\Omega_{DV}{}^{[1]} = 3.204$. Our results (solid line) are always smaller than the results for spinless particles while those obtained using Eq.(26) of [1] are greater than $d\sigma/d\Omega_f{}^{[KG]}$ for small angles around the direction of the laser propagation. In the lower part (b) of Figure 5, we show the ratio R defined by Eq.(16). For $\theta_f = 0^\circ$, this ratio is $R = 2.787$

V. Conclusion

In this work, we derived the correct expression of the first Born differential cross section for the scattering of the Dirac-Volkov electron by a Coulomb potential of a nucleus in the presence of a strong laser field. We have given the correct relativistic generalization of the Bunkin and Fedorov treatment [7] that is valid for an arbitrary geometry. We are adamant that the core of the whole controversy stems from the fact that in [1], the vector η^μ of our work has not been properly dealt with while it is the common method to use when a trace contains a γ^0 matrix. Any standard QED textbook introduces this very elementary method. Comparison of our numerical calculations [9] with those of Szymanowski et al. [1] shows qualitative and quantitative differences when the incoming total electron energy and the electric field strength are increased particularly in the direction of the laser propagation. The difference between our results and those of [1] can only be traced back to the mistakes and the omitted term in Eq.(26) of [1]. The corrections that we made allowed us to study other processes that were published in Physical Review A., namely an first article concerning the relativistic electronic dressing in laser assisted electron-hydrogen elastic collisions [10], another concerning the process of Mott scattering in an elliptically polarized laser field [11] as well as a third work dealing with the process of Mott scattering of polarized electrons in a strong laser field [12]. For the difficult process of ionization of atomic hydrogen by electron impact, we published an article concerning the importance of the relativistic electronic dressing in laser-assisted ionization of atomic hydrogen by electron impact [13]. All these works relied heavily on the corrections that we made in this work.

VI. References

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