FANO resonances in solid-fluid one and two dimensional systems

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Abstract: The gaol of this paper is to demonstrate that the propagation of acoustic waves in a single slab made of a homogeneous one dimensional (1D) solid embedded in a fluid at oblique incidence on a slab made of two dimensional (2D) rectangular rods immersed in a fluid, can exhibit transmission zeros near resonances the so-called Fano resonances.

Key words: Acoustic wave; solid-fluid superlattice; Fano resonances; Green function method and FDTD method.

I. Introduction

A great deal of research effort is currently being devoted to the study of acoustic or elastic waves propagation in periodically structured materials, such as the so-called phononic crystals [1-3]. Phononic crystals (PCs) are inhomogeneous elastic media composed of, one, two or three-dimensional periodic arrays of inclusions embedded in a matrix. Several classes of phononic crystals exist and differ mainly by the physical nature of the inclusion and of the matrix. Among them, solid/solid, fluid/fluid and mixed solid/fluid composite systems have received great attention. These structures have potential applications as acoustic filters or wave guides, among others, and present different types of resonant wave phenomena.

Recent studies have investigated elastic waves across a crystal slab made of two [4] or three-dimensional [5] array of structural units which exhibit localized resonance in the transmission spectrum. These resonances are usually associated with the so-called Fano resonance, whose main characteristics can be beautifully explained by a one dimensional mass and spring model [4]. Indeed, the underlying physics of the Fano resonance finds its origin in wave interference which occurs in the system characterized by one or several localized modes that interact with the continuum spectrum, which are analogous in photonic crystals [6] and in a semi-conductor [7].

In a recent paper [8], we have investigated the possibility of the existence of Fano resonance induced by a single slab made of a homogeneous one dimensional (1D) fluid layer between two solids. The Fano resonance is just an internal resonance induced by the discreet modes of the fluid layer surrounded by two solid when these modes fall at the vicinity of the transmission.

The existence of such resonances in 2D and 3D phononic crystals, the so-called locally resonant band-gap materials, has been shown recently [4,5]. Some analytical models have been proposed to explain the origin and the behavior of these resonances [4].

The motivation behind the work presented in this paper is to investigate the existence of Fano resonances in a simple system which consists on a solid slab embedded in a fluid. In particular, we propose two systems that may exhibit Fano resonances depending on the dimension of the structure and the incidence angle. The first structure consists on acoustic waves launched normally ($\theta = 0°$) on a slab constituted of a 2D rectangular rods (see Fig. 1). The second structure consists on a 1D slab but with oblique incidence (see Fig. 3). We show that in the first structure, the 2D system is responsible for the existence of Fano resonances, whereas in the second structure the oblique incidence is responsible for the appearance of Fano resonances. The speed velocities of sound and mass densities of the materials used in this paper are given in reference [8].

II. Case of 2D slab made of rectangular rods

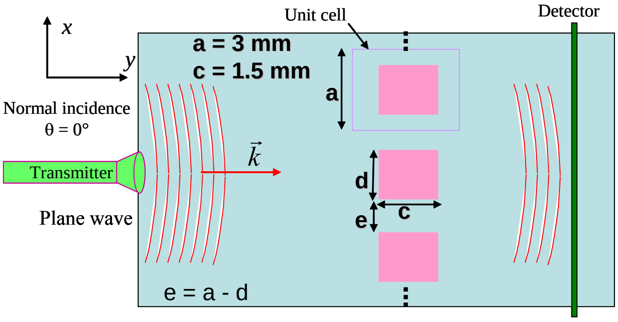

Figure 1: Cross-section of the phononic crystal composed of Plexiglas rods in a water matrix.

Figure 1 displays the cross-section of the phononic crystal considered in this first part. This phononic crystal is composed of Plexiglas rods in a water matrix. a = 3mm describes the mesh of a basic cell, d the length of the rod, $e = d\text{-}a$ the distance separating two successive rods and c the width between the presumedly fixed rods. Note that the study of this system is limited to normal incidence ($\theta = 0°$).

The numerical method used is based on FDTD program developed to calculate the transmission coefficient through perfect phononic crystal with or without a defect. The detail of the FDTD method, applied to phononic crystals, is given in Ref. [9].

The FDTD transmission spectra were obtained numerically by discretizing time and space replacing derivatives by finite differences in the elastic equation. The FDTD grids are composed of three adjacent regions, i.e. a centred region containing the finite phononic crystal sandwich between two homogeneous water regions. A travelling wave packet is launched in the first homogeneous region and propagates along the y direction. Periodic boundary conditions are applied along the x direction and absorbing Mur's boundary conditions are imposed at the free ends of the homogeneous regions along the y direction. The spectrum of the incoming signal is a Gaussian function. The transmission spectrum is obtained by averaging the displacement field at the end of the sample region along the x direction.

Figure 2(a) gives the transmission spectrum along the $\Gamma - y$ direction for e = 0 (i.e. a thin layer along the x axis direction). We can notice that the system does not exhibit transmission zeros. This is due for a normal incidence of the wave front, the elastic wave can be decoupled in two kinds of modes: modes corresponding to pure transverse motion and longitudinal motion, which the longitudinal waves in the guided system (water) are coupled with those of the Plexiglas slab.

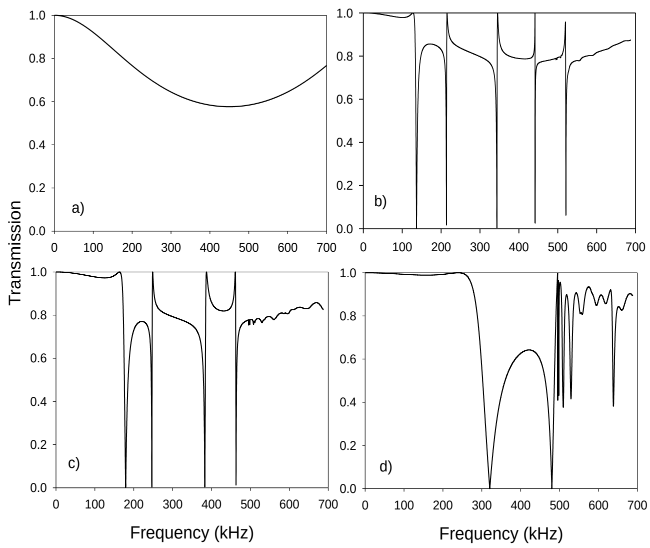

Figure 2: (a) Transmission spectrum corresponding to the phononic crystal described in Fig. 1 for e = 0. (b), (c) and (d) same as (a) but for e = a/20, e = a/10 and e = a/2 respectively.

However, for $e \neq 0$, in figures 2(b), (c) and (d) corresponding, respectively, to $e = \dfrac{a}{20}$, $e = \dfrac{a}{10}$ and $e = \dfrac{a}{2}$, the transmission spectrum exhibit transmission zeros near strong resonances. These resonances are of Fano type in analogy with atomic physics [10] in the optical transmission spectrum. These unusual low frequency characteristics result from a Fano mechanism: an elastic wave traveling inside the structure can interact with localized modes, while part of the wave can use a nonresonant way to travel across the structure. As a consequence, interference between both traveling wave components may occur, which results in resonant peaks in the transmission spectra, as shown in Fig. 2(b-d). Especially Fano resonances are caused by interference of two alternative paths [11], a resonant and a non resonant one.

III. Case of a 1D slab at oblique incidence

The second system consists in a one dimensional structure. The studied geometrie in this case is schematically depicted in Fig. 3, where we consider a finite lamellar

structure sandwiched between two substrates. Note that, all media are assumed to be isotropic elastic media characterized by their mass densities, their transverse velocity, and longitudinal velocity of sound. In this second part, the study is performed by using a Green's function formalism based on the method of interface response theory [12]. In this theory, we calculate the Green's function of a composite system containing a large number of interface that separate different homogeneous media. The dispersion curves as well the transmittance through finite structure are obtained from the knowledge of the corresponding Green's function. The details of the calculations of the Green's function, transmission as well as phase time in the finite-size superlattice are given Ref. [8].

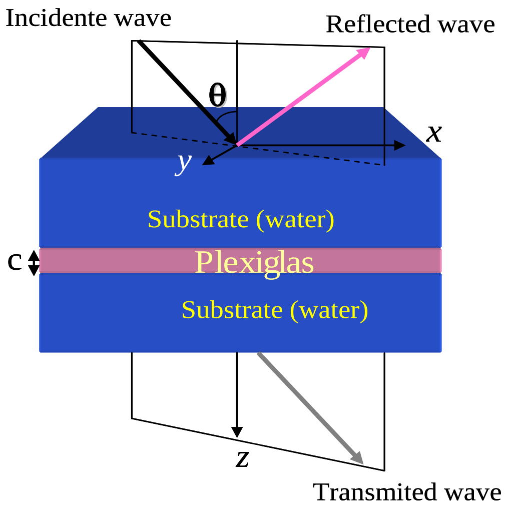

Figure 3: Finite lamellar structure (Plexiglas) sandwiched between two substrates (water).

Figure 4 gives the transmission spectrum of the structure described previously for different incidence angles.

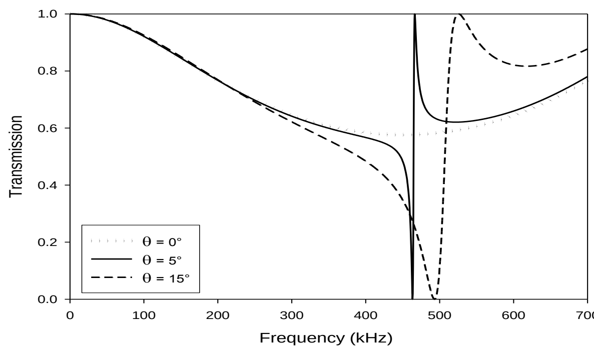

Figure 4: Transmission spectrum corresponding to the structure described in Fig. 3 for different angles: θ = 0° (dotted line), θ = 5° (solid line) and θ = 15° (dashed line).

At normal incidence $\theta = 0°$ (Fig.4, dotted line), the transmission spectrum exhibit no transmission zeros exist. In this case the structure studies is equivalent to the structure described in figure 1 for *e = 0*.

Now, at oblique incident $(\theta \neq 0°)$ figure 4 shows, for a weak angle $(\theta = 5°)$ (solid line) and θ = *15°*, an important dip (Transmission zero) as well as resonances due to destructive interference between transverse and longitudinal waves in the Plexiglas layer. The resonance shows the characteristics of Fano type as in Figs. 2(b)-(d).

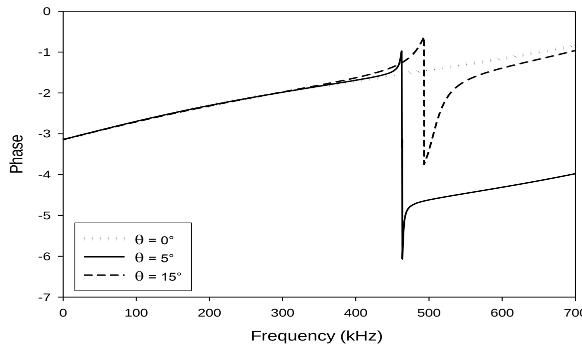

Figure 5: Same as in figure 4 but for the variation of the phase.

Figures 5 show the variation of the phase versus the frequency. One can notice that, at $\theta = 0°$ (dotted line) the phase increases monotonically, however for $\theta \neq 0°$ the phase exhibits a jump of π at the transmission zeros (solid line and short dashed line for $\theta = 5°$ and 15° respectively). Now the derivative of the phase versus the frequency, called the phase time, shows negative delta peaks at the transmission zeros [Fig. 6] which are absent at normal incidence [dotted line]. It is worth to notice that the phase time is different from (equivalent to) the density of states when the system exhibit (do not exhibit) transmission zeros [13].

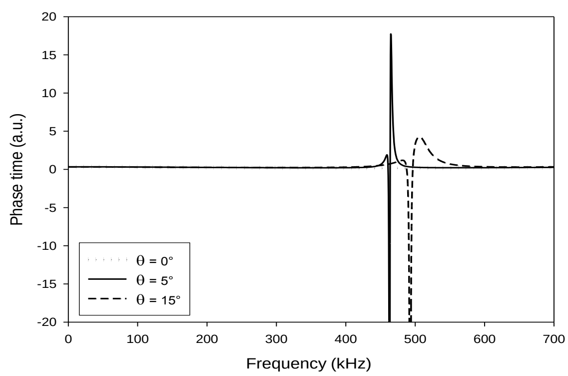

Figure 6: Same as in figure 4 but for the variation of the phase time.

IV. Conclusion

In this paper, we have shown that a single slab made of a homogeneous solid embedded in a fluid can not exhibit transmission zeros and therefore Fano resonances at normal incidence. However, there exist two possibilities that may show these resonances: the first possibility consists on launching obliquely the incident wave on the 1D slab and the second solution consists on keeping the incidence normal but the slab is now constituted of 2D rods instead of being homogeneous.

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