Synthesis analysis of the free form curves and surfaces parametrical models

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Parametrical models have become an important mathematical tool for free form curves and surface description. they allow the use of state-of-the-art computers to do the various processing and analysis with respect to shape (calculation of the volume and surface area, vibration analysis, NC programs preparation, etc.). Without these models, the current product design and manufacturing would be difficult.

The paper presents an analysis study for the most important parametrical models of free form curves and surfaces description (Ferguson, Coons, Bezier, B-Spline and rational models). Firstly, it emphasises external and mathematical properties of each model. Secondly, it gives the most interesting interfaces between models. Thus the process of complex surface design and manufacturing would be more efficient in view this analysis. Finally for validation, the study is ended by an application to the design of a car bonnet.

KEY WORDS: Complex surfaces; patch; connection; control shape.

I. INTRODUCTION

At the start of the 1960s J.C Ferguson of the Boeing Aircraft Company in the U.S.A announced a method of describing curve segments, as vectors, using parameters.

A Ferguson curve segment is a cubic vector function with respect to a parameter, obtained by specifying the position vectors and tangent vectors of the starting and ending points of the curve segment. In addition, Ferguson using these curve segments, proposed a method of creating a portion of a surface[1] (called a surface patch), that satisfies the conditions imposed by specifying position vectors, and tangent vectors at 4 points.

In 1964, S.A. Coons[2-3] announced a surface description method and generalised it in 1967. Coons surface patch is defined not only by the position vectors and higher order differential vectors with respect to the 4 corner points of the patch. The position vectors related to the 4 boundary curves and the higher order differential vectors with respect to the directions across boundary curves also defines it. Ferguson curve segments, surface patches and Coons surface patches share a number of problems in control and connection of segments and patches.

A method for solving the connection problem is the Spline curve that is described by different cubic degree curves in different segments between weight and weight. At the positions of weights, that is, at the connecting points between curve segments, connections are determined simultaneously at all connection points.

The Spline method can be thought of as automatically solving the connection problem that exists with Ferguson and Coons curves and surfaces. However the problem of controlling the shape isn't resolved.

P.Bezier of Renault Company in France announced a curve representation that is defined by giving one polygon and smoothing its corners[4]. This method does not require analytical data that are hard to understand intuitively such as tangent vectors and twist vectors. Bezier curves and surfaces use Bernstein Basis functions as blending functions.

Gordon & Riesenfeld proposed curves and surfaces which use Basis Splines as blending functions[5]. These are called B-Spline curves and surfaces. In contrast to Bezier model, which is a convex combination of all of the vertex position vectors, the B-Spline model differs in that it's a convex combination of a number of vertex position vectors in their immediate vicinity. It follows from this that the problem of controlling is improved.

II. ANALYSIS OF MODELS PROPERTIES

This analysis study treats Hermite interpolation, Bezier, Bspline, and rational models. It emphasises external and mathematics properties such as connectivity and controllability. The detailed development of the mathematical definitions of models is not discussed here, but just bi-cubic surfaces definitions of declared models are given because we need them for determining interfaces.

II.1. Hermite interpolation model

Curves and surfaces based on Hermite interpolation (Ferguson and Coons models) can be considered among the first models developed for representing curves and complex surfaces. They are determined by specifying position vectors of points, through which they pass and higher order derivative vectors.

The bi-cubic Coons surface patch can be expressed by the following equation[6]:

$$\boldsymbol{P}_c(u,w) = \boldsymbol{U} M_C B_C M_C^T \boldsymbol{W}^T . \qquad \textbf{(1)}$$

M_C, B_C, $\boldsymbol{U}$ and $\boldsymbol{W}$ are given by:

$$M_C = \begin{bmatrix} 2 & -2 & 1 & 1 \\ -3 & 3 & -2 & -1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

$$B_C = \begin{bmatrix} \boldsymbol{Q}(0,0) & \boldsymbol{Q}(0,1) & \boldsymbol{Q}_w(0,0) & \boldsymbol{Q}_w(0,1) \\ \boldsymbol{Q}(1,0) & \boldsymbol{Q}(1,1) & \boldsymbol{Q}_w(1,0) & \boldsymbol{Q}_w(1,1) \\ \boldsymbol{Q}_u(0,0) & \boldsymbol{Q}_u(0,1) & \boldsymbol{Q}_{uw}(0,0) & \boldsymbol{Q}_{uw}(0,1) \\ \boldsymbol{Q}_u(1,0) & \boldsymbol{Q}_u(1,1) & \boldsymbol{Q}_{uw}(1,0) & \boldsymbol{Q}_{uw}(1,1) \end{bmatrix}$$

$$\boldsymbol{U} = [u^3 \quad u^2 \quad u \quad 1] \qquad \boldsymbol{W} = [w^3 \quad w^2 \quad w \quad 1]$$

Where

u, w : are the surface parameters

$\boldsymbol{Q}$: Position vectors

$\boldsymbol{Q}_u$, $\boldsymbol{Q}_w$: Tangent vectors respectively in the u and w directions

$\boldsymbol{Q}_{u,w}$: Twist vectors

II.1.1. Model defining parameters

The great number of diversified inputs, makes difficult the shape prediction from them, and causes some problems in control shape.

The boundary curves, in the case of surfaces, can be given arbitrarily. Then this model is largely used when we have at one's disposal just the boundary curves, as after an intersection operation between patches.

II.1.2. Connection property

By assuming equality between position and derivative vectors at both ends of boundary curves, we can reach superior orders of continuity without specifying all similar derivative vectors the long of these curves[7]. However, this method of connection is heavy to process, due to the fact that it is indirect.

II.1.3. Control shape property

The presence, in the defining parameters, of vectors other than position vectors makes control delicate. So to control a shape, it's necessary to adjust all types of vectors, specifically position vectors, tangent vectors, and twists vectors. The effects of tangent and twist vectors are difficult to interpret at a glance and differ from point to point. To show this we take nine connected surface patches, lying on a plane and we adjust the different factors. (See figures 1.1 to 1.7).

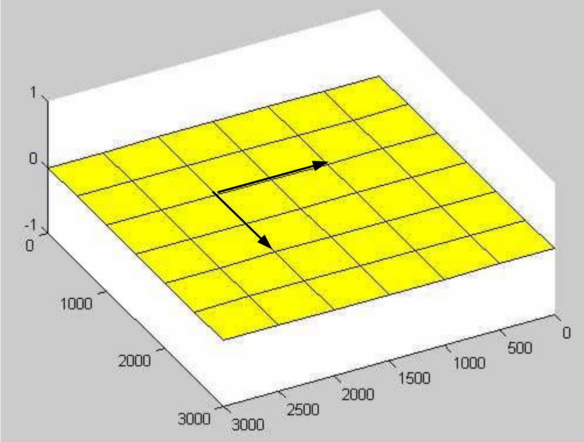

FIG. 1.1. Nine connected surface patches.
Vectors of all 9 surface patches: tangent vector in u-direction (1000,0,0), tangent vector in w-direction (0,1000,0), twist vector (0,0,0).

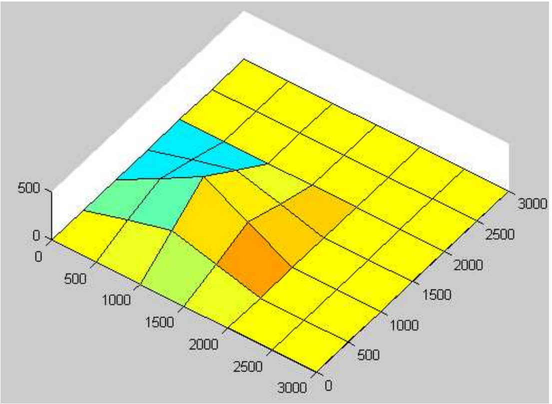

FIG. 1.2. Movement of only one surface patch corner 500 units in the z-direction

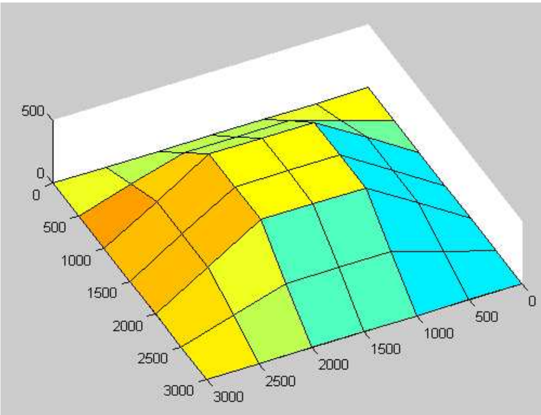

FIG. 1.3. Movement of all four corners of the center surface patch 500 units in the z-direction

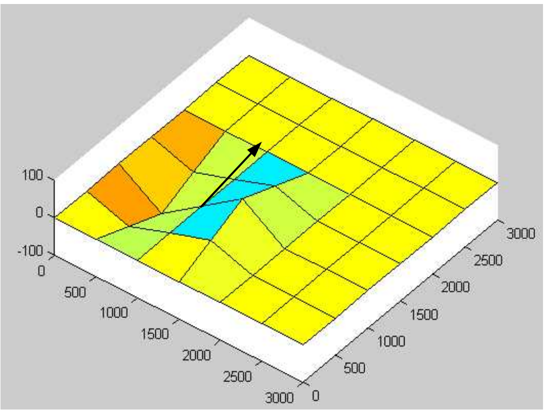

FIG. 1.4. Rotation of the tangent vector in w-direction 30° upward

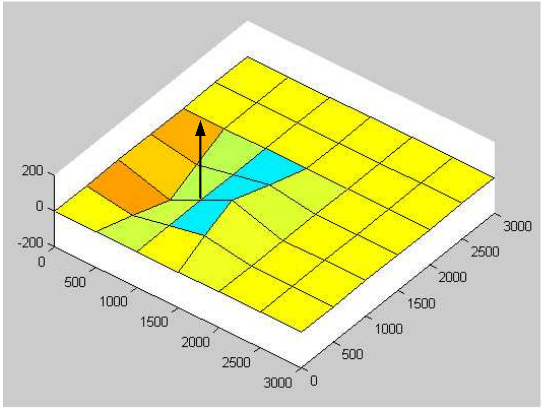

FIG. 1.5. Rotation of the tangent vector in w-direction 90° upward

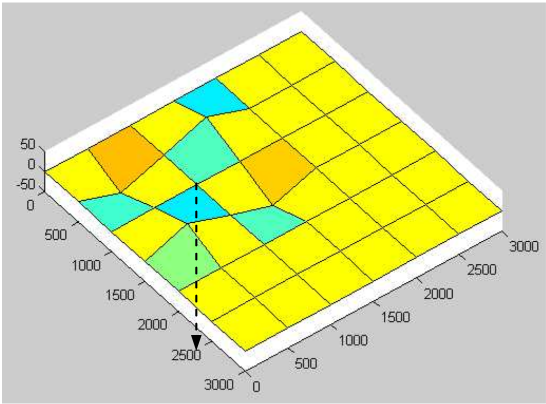

FIG 1.6. Twist vector is taken to be (0,0,-1500)

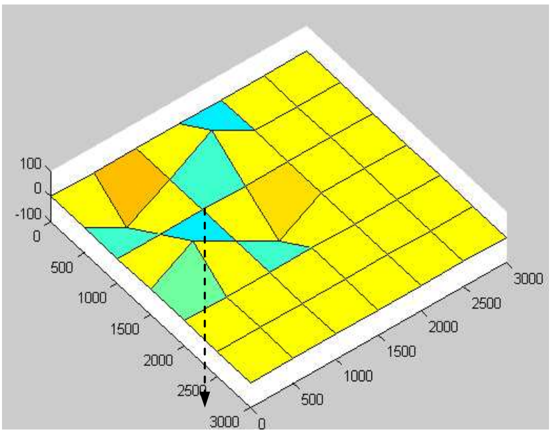

FIG. 1.7. Twist vector is taken to be (0,0,-4000)

II.2. Bezier model

The Bezier model allows solving some problems encountered on Hermite interpolation models, essentially in shape control and connectivity between patches (see 2.1.2 and 2.1.3). The Bezier model has some interesting mathematics properties that generally incite the CAD software producer to use it in their systems.

The Bezier bi-cubic surface can be expressed as follows[6]:

$$\boldsymbol{P}_B(u,w) = \boldsymbol{U}M_B B_B M_B^T \boldsymbol{W}^T . \qquad \textbf{(2)}$$

M_B and B_B are given by:

$$M_B = \begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

$$B_B = \begin{bmatrix} \boldsymbol{Q}_{00} & \boldsymbol{Q}_{01} & \boldsymbol{Q}_{20} & \boldsymbol{Q}_{03} \\ \boldsymbol{Q}_{10} & \boldsymbol{Q}_{11} & \boldsymbol{Q}_{12} & \boldsymbol{Q}_{13} \\ \boldsymbol{Q}_{20} & \boldsymbol{Q}_{21} & \boldsymbol{Q}_{22} & \boldsymbol{Q}_{23} \\ \boldsymbol{Q}_{30} & \boldsymbol{Q}_{31} & \boldsymbol{Q}_{32} & \boldsymbol{Q}_{33} \end{bmatrix}$$

II.2.1. Model defining parameters

Bezier curves and surfaces are defined only by position vectors. They don't require analytical data which are difficult to understand intuitively, such as tangent vectors and twist vectors, as do Hermite interpolation models. They use geometrical data that are simple to manipulate and easy to understand intuitively, such as polygons and polyhedrons.

II.2.2. Recursivite

The recursivite relation among Bernstein basis functions applied to Bezier model allows the reduction of the computation lead-time and the memory consumption.

II.2.3. Convex hull property

To each Bezier curve, we can associate a convex polygon[8] *PCm* built from defining polygon *PGm* of *P(u)* (See figure 2).

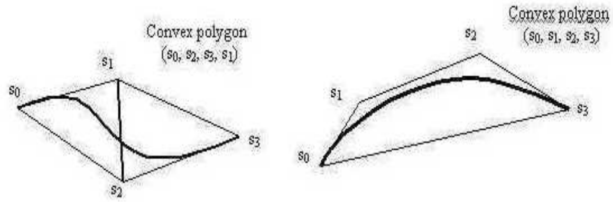

FIG. 2. Examples of convex polygons associated to Bezier curves

The interest of this property appears in the case of research of intersection between curves. The principle consists in comparing convex polygons of each entity to detect a possible intersection (See figure 3). This property allows a great algorithmic performance improvement. Other consequences of this property, if $\boldsymbol{Q}_0 = \boldsymbol{Q}_1 = \boldsymbol{Q}_2 = \cdots = \boldsymbol{Q}_n$, the curve degenerates to the simple point Q_0, in addition if all of the curve defining vectors is placed to be collinear, it's the line that joins Q_0 to Q_n.

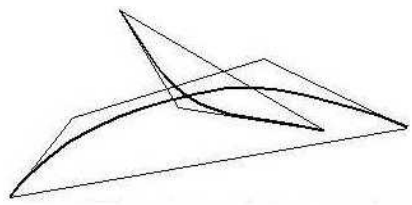

FIG. 3. Intersection research using convex polygons

II.2.4. Variation diminishing property [9]

The number of intersections of an arbitrary line and a Bezier curve is never more than the number of intersections of that straight line with the curve defining polygon. A more accurate interpretation of this property consists in telling that the number of inflection point of a Bezier curve is always inferior or equal to the number of undulation of its characteristic polygon. However, the number of undulation of the characteristic polygon does not predict rigorously the number of curve inflection points. It is not therefore possible to conclude as for the curve geometrical quality from this property in most of situations (See figure 4), except when a characteristic polygon presents no or an only one undulation, we can conclude with certainty that the associated curve will have no or one inflection point respectively.

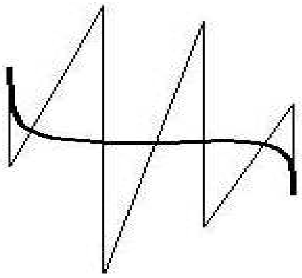

FIG. 4.a. Important number of characteristic polygon undulation $\Rightarrow$ Impossible to detect visually curve inflection points.

FIG. 4.b. No undulation
$\Rightarrow$ No inflection point

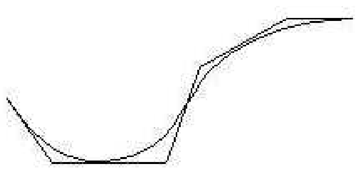

FIG. 4.c. One undulation
$\Rightarrow$ One inflection point

II.2.5. Connection property

Bezier patches and segments can be connected easily to the slope[10)], but when we seek a superior order continuity (as up to curvature), a program that will preserve all continuity conditions with flexibility and generality becomes complicated.

II.2.6. Control shape property

The shape control in Bezier model is made by varying the polygon or the characteristic matrix points, called control knots. These knots have a global but not a local effect on the shape, what makes control difficult when we desire particularly to refine a part of the surface or of the curve. (local ray change of curvature in manufacturing application).

This problem can be overcome by the means of the subdivision of the surface or the curve in patches and segments.

II.3. Bspline model

The Bpsline model is an improvement of the Bezier model. It has benefited from Splines properties to give rise to a strong model in connection and in control.

Let $\mathbf{Q}_{00}, \mathbf{Q}_{01}, \ldots, \mathbf{Q}_{0n}, \ldots, \mathbf{Q}_{m0}, \ldots, \mathbf{Q}_{mn}$ be a lattice of position vectors and $N_{i,K}(u), N_{j,L}(w)$ be the B-Spline functions type3 of order K and L, respectively. A Cartesian product B-Spline surface is given by: [6)]

$$\mathbf{P}(u,w) = \sum_{i=0}^{m}\sum_{j=0}^{n} N_{i,K}(u) N_{j,L}(w) \mathbf{Q}_{ij} \qquad \mathbf{(3)}$$

In matrix form we have:

$$\mathbf{P}(u,w) = \lfloor N_{0,K}(u) \quad N_{1,K}(u) \quad \ldots \quad N_{m,K}(u) \rfloor \times \begin{bmatrix} \mathbf{Q}_{00} & \mathbf{Q}_{01} & \cdots & \mathbf{Q}_{0n} \\ \mathbf{Q}_{10} & \mathbf{Q}_{11} & \cdots & \mathbf{Q}_{1n} \\ & & \vdots & \\ \mathbf{Q}_{m0} & \cdots & \cdots & \mathbf{Q}_{mn} \end{bmatrix} \times \begin{bmatrix} N_{0,L}(w) \\ N_{1,L}(w) \\ \vdots \\ N_{n,L}(w) \end{bmatrix} \qquad \mathbf{(4)}$$

II.3.1. Model Defining parameters

Like in Bezier model, the Bspline model inputs are simple to manipulate. They are all of the same type (position vectors) and in addition allow having an idea on the curves and surfaces shape.

II.3.2. Recursivite property

As Bernstein basis functions, the Bspline basis functions can be calculated easily with recursive formula.

II.3.3. Connection property

In general, C^{M-2} class continuity is automatically maintained between connected segments or patches.

II.3.4. Control shape __ Locality

Bspline model has superior control property, so a curve segment is expressed as a convex combination of local sequence of position vectors $\boldsymbol{Q}_{i-1}, \boldsymbol{Q}_i, \cdots, \boldsymbol{Q}_{i+M-2}$. Consequently, if a certain polygon vertex is varied,

only the part of the curve in the immediate neighbourhood of that vertex is affected. (See figure 5)

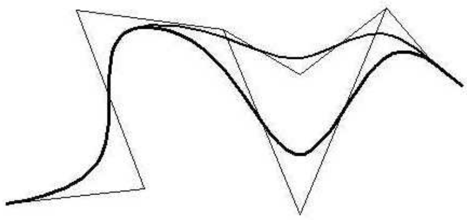

FIG. 5. Local control of a Bspline curve

II.3.5. Convex hull property and variation diminishing property

Bspline model, like Bezier model has convex hull and variation diminishing properties. As the convex hull in Bspline model is made by the nearest M vectors $\boldsymbol{Q}_j$, Bspline model is more faithful to variation in the polygon shape than Bezier model in which the convex hull is a global region made by all position vectors $\boldsymbol{Q}_j$ $(j = 0, 1, \cdots, n)$ (See figure 6)

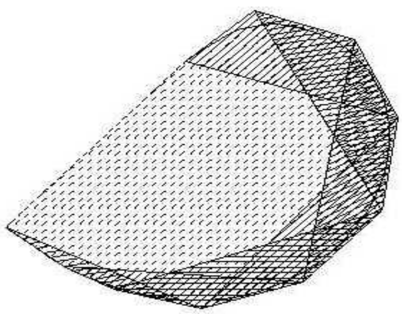

FIG. 6. Relation between convex hulls of Bézier curve segment and Bspline curve.
Slanted line area: Bspline curve convex hull;
Dotted lines: Bezier curve segment convex hull.

All consequences of these properties mentioned in the case of the Bezier model are also maintained

II.4. Rational model

The rational Bezier and Bspline keep respectively the same properties of Bezier and Bspline models. But in addition, they give a solution for the exact conics representation by the introduction of the homogenous coordinate.

III. INTERFACES BETWEEN MODELS

To benefit from strong properties of each model, the determination of interfaces between them is the most evident solution. This part gives three important interfaces: the interface between the bi-cubic Bezier surface and the bi-cubic Coons surface, the interface between the bi-cubic Bspline surface and the bi-cubic Coons surface and finally the interface between the bi-cubic Bezier surface and the bi-cubic Bspline surface.

III.1. Interface between Bezier bi-cubic surface and Coons bi-cubic surface

The relation between the surface defining vectors of a bi-cubic Coons surface and the surface defining vectors of a bi-cubic Bezier surface patch can be derived as follows:

The bi-cubic Coons surface patch can be expressed by:

$$\boldsymbol{P}_c(u, w) = \boldsymbol{U} M_C B_C M_C^T \boldsymbol{W}^T. \qquad \textbf{(5)}$$

The Bezier bi-cubic surface can be expressed as follows:

$$\boldsymbol{P}_B(u, w) = \boldsymbol{U} M_B B_B M_B^T \boldsymbol{W}^T. \qquad \textbf{(6)}$$

Setting

$$\begin{aligned} \boldsymbol{P}_C(u, w) &= \boldsymbol{P}_B(u, w) \\ M_C B_C M_C^T &= M_{\boldsymbol{B}} B_B M_B^T \end{aligned} \qquad \textbf{(7)}$$

This implies:

$$\begin{aligned} B_B &= \left(M_B^{-1} M_C\right) B_C M_C^T \left(M_B^T\right)^{-1} \\ &= \left(M_B^{-1} M_C\right) B_C M_C^T \left(M_B^{-1}\right)^T \end{aligned} \qquad \textbf{(8)}$$

We also have:

$$M_B^{-1} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & \frac{1}{3} & 1 \\ 0 & \frac{1}{3} & \frac{2}{3} & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$$M_B^{-1} M_C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & \frac{1}{3} & 0 \\ 0 & 1 & 0 & -\frac{1}{3} \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

So that:

$$B_B = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & \frac{1}{3} & 0 \\ 0 & 1 & 0 & -\frac{1}{3} \\ 0 & 1 & 0 & 0 \end{bmatrix} \times$$
$$\begin{bmatrix} \mathbf{Q}(0,0) & \mathbf{Q}(0,1) & \mathbf{Q}_w(0,0) & \mathbf{Q}_w(0,1) \\ \mathbf{Q}(1,0) & \mathbf{Q}(1,1) & \mathbf{Q}_w(1,0) & \mathbf{Q}_w(1,1) \\ \mathbf{Q}_u(0,0) & \mathbf{Q}_u(0,1) & \mathbf{Q}_{uw}(0,0) & \mathbf{Q}_{uw}(0,1) \\ \mathbf{Q}_u(1,0) & \mathbf{Q}_u(1,1) & \mathbf{Q}_{uw}(1,0) & \mathbf{Q}_{uw}(1,1) \end{bmatrix} \times$$
$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & \frac{1}{3} & 0 & 0 \\ 0 & 0 & -\frac{1}{3} & 0 \end{bmatrix}$$
$$\equiv \begin{bmatrix} \mathbf{Q}_{00} & \mathbf{Q}_{01} & \mathbf{Q}_{20} & \mathbf{Q}_{03} \\ \mathbf{Q}_{10} & \mathbf{Q}_{11} & \mathbf{Q}_{12} & \mathbf{Q}_{13} \\ \mathbf{Q}_{20} & \mathbf{Q}_{21} & \mathbf{Q}_{22} & \mathbf{Q}_{23} \\ \mathbf{Q}_{30} & \mathbf{Q}_{31} & \mathbf{Q}_{32} & \mathbf{Q}_{33} \end{bmatrix} \quad \textbf{(9)}$$

Therefore, all $\boldsymbol{Q}_{ij}$ are determined (see annex A.1).

By the same procedure, we can determine the surface defining vectors of a bi-cubic Coons surface patch in terms of the defining vectors of Bezier surface patch. (See annex A.2)

III.2. Interface between Bspline bi-cubic surface and Coons bi-cubic surface

Let us look at the relation between a Bspline bi-cubic surface and a Coons bi-cubic surface. The Bspline bi-cubic surface can be expressed by the following equation.

$$\boldsymbol{P}_R(u,w) = \boldsymbol{U}M_R B_R M_R^T \boldsymbol{W}^T . \quad \textbf{(10)}$$

Setting

$$\begin{aligned} \boldsymbol{P}_R(u,w) &= \boldsymbol{P}_C(u,w) \\ M_R B_R M_R^T &= M_C B_C M_C^T \end{aligned} \quad \textbf{(11)}$$

Solving this equation for B_C gives:

$$B_C = \left(M_C^{-1}M_R\right)B_R\left(M_C^{-1}M_R\right)^T \quad \textbf{(12)}$$

Therefore all $\boldsymbol{Q}(i,j)$, $\boldsymbol{Q}_u(i,j)$, $\boldsymbol{Q}_w(i,j)$ and $\boldsymbol{Q}_{uw}(i,j)$ are determined (See annex A.3)

III.3. Interface between Bspline bi-cubic surface and Bezier bi-cubic surface

The relation between a Bspline bi-cubic surface and a Bezier bi-cubic surface can be derived as follows.

Setting

$$\begin{aligned} \boldsymbol{P}_R(u,w) &= \boldsymbol{P}_B(u,w) \\ M_R B_R M_R^T &= M_B B_B M_B^T \end{aligned} \quad \textbf{(11)}$$

Solving this equation for B_B gives:

$$B_B = \left(M_B^{-1}M_R\right)B_R\left(M_B^{-1}M_R\right)^T \quad \textbf{(12)}$$

Therefore, all $\boldsymbol{Q}_{ij}$ are determined (see annex A.4)

IV. ILLUSTRATION

In this illustration, we proceed to the design of a car bonnet, in view to validate the synthesis study and to propose a method allowing the reduction of the design lead time by using more than one model.

Step 1

Given that, we don't have the net of point of the bonnet surface, and that it is very difficult to have it intuitively, we propose at the beginning to use the Coons model which requires only the boundary curves as inputs.

To approach well the bonnet, we will constitute it by seven surface patches, connected just in position.

The seven surface patches are presented by the next figure.

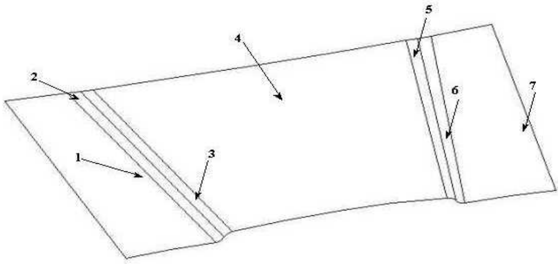

FIG. 7. The boundary curves can be made by the Bspline model, the Bezier model, or by simple geometric elements (line, arc...).

Step 2

In this second stage we connect the seven patches, just in position. Then we will extract the co-ordinates of the all points of the global surface generated.

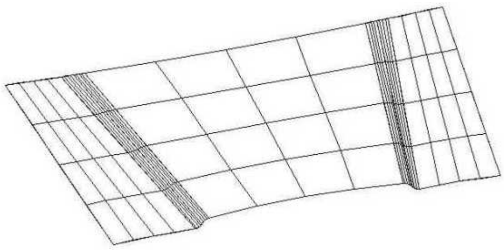

FIG. 8. The seven Coons connected patches for knots extraction

The Co-ordinates of extracted knots are given in annex B.

Step 3

As has been mentioned previously, the Bspline model has the superior qualities of modelling and guarantees automatically continuity up to curvature. For this, we will use it to generate the bonnet surface in only one unit patches.

The set of points extracted previously will be considered as the net surface of the bonnet.

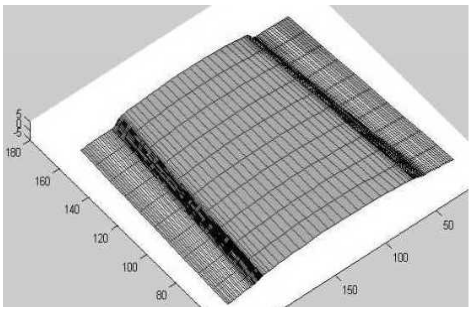

FIG. 9. The bonnet surface genereted by the Bspline model

Now the bonnet generated by the Bspline model is constituted of a set of patches connected automatically up to curvature, and has the local control quality that is very important in the of modelling operation .

To show this superior quality of modelling, we will try to modify just the front edge of the bonnet to render it a little bit highest.

The new modified co-ordinates:

X= 2.94	Y= 44.44	Z= 0.00
X= 55.84	Y= 50.69	Z= 0.00
X= 108.75	Y= 56.94	Z= 0.00
X= 161.65	Y= 63.19	Z= 0.00
X= 214.56	Y= 69.44	Z= 0.00
X= 4.48	Y= 84.43	Z= 0.00
X= 57.15	Y= 87.91	Z= 1.27
X= 109.52	Y= 90.42	Z= 4.06
X= 161.90	Y= 92.93	Z= 6.85
X= 214.56	Y= 96.41	Z= 8.12
X= 5.00	Y= 124.44	Z= 0.00
X= 57.58	Y= 124.44	Z= 1.70
X= 109.78	Y= 124.44	Z= 5.43
X= 161.98	Y= 124.44	Z= 9.16
X= 214.56	Y= 124.44	Z= 10.86
X= 4.48	Y= 164.45	Z= 0.00
X= 57.15	Y= 160.97	Z= 1.27
X= 109.52	Y= 158.46	Z= 4.06
X= 161.90	Y= 155.95	Z= 6.85
X= 214.56	Y= 152.47	Z= 8.12
X= 2.94	Y= 204.44	Z= 0.00
X= 55.84	Y= 198.19	Z= 0.00
X= 108.75	Y= 191.94	Z= 0.00
X= 161.65	Y= 185.69	Z= 0.00
X= 214.56	Y= 179.44	Z= 0.00

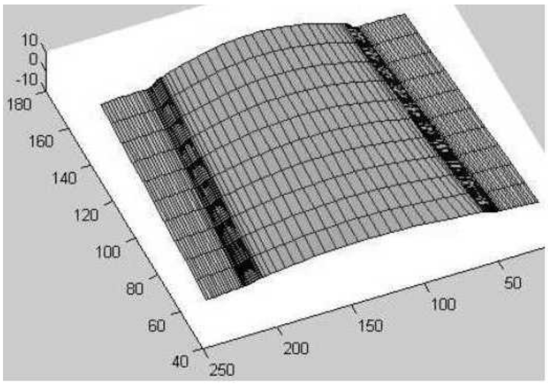

FIG. 10. The modified bonnet surface

The obtained result shows well the local control property of the Bspline model, just the middle patch is modified while the rest of the surface resides invariably.

V. CONCLUSION

Bspline and rational Bspline models seem to be the most interesting models as they offer several advantages, on the plan of representativeness, control and connection. However, it is necessary to tell that the other models have also their reasons to be, either for rapidity generation reason or for processing reason. In order that a software including interfaces between models, would be qualified, powerful and flexible.

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Annex A

A.1. Interface between Bezier bi-cubic surface and Coons bi-cubic surface

$$\mathbf{Q}_{00} = \mathbf{Q}(0,0)$$
$$\mathbf{Q}_{01} = \mathbf{Q}(0,0) + \frac{1}{3}\mathbf{Q}_w(0,0)$$
$$\mathbf{Q}_{02} = \mathbf{Q}(0,1) - \frac{1}{3}\mathbf{Q}_w(0,1)$$
$$\mathbf{Q}_{03} = \mathbf{Q}(0,1)$$
$$\mathbf{Q}_{10} = \mathbf{Q}(0,0) + \frac{1}{3}\mathbf{Q}_u(0,0)$$
$$\mathbf{Q}_{11} = \mathbf{Q}(0,0) + \frac{1}{3}\mathbf{Q}_u(0,0) + \frac{1}{3}\mathbf{Q}_w(0,0) + \frac{1}{9}\mathbf{Q}_{uw}(0,0)$$
$$\mathbf{Q}_{12} = \mathbf{Q}(0,1) + \frac{1}{3}\mathbf{Q}_u(0,1) - \frac{1}{3}\mathbf{Q}_w(0,1) + \frac{1}{9}\mathbf{Q}_{uw}(0,1)$$
$$\mathbf{Q}_{13} = \mathbf{Q}(0,1) + \frac{1}{3}\mathbf{Q}_u(0,1)$$
$$\mathbf{Q}_{20} = \mathbf{Q}(1,0) - \frac{1}{3}\mathbf{Q}_u(1,0)$$
$$\mathbf{Q}_{21} = \mathbf{Q}(1,0) - \frac{1}{3}\mathbf{Q}_u(1,0) + \frac{1}{3}\mathbf{Q}_w(1,0) - \frac{1}{9}\mathbf{Q}_{uw}(1,0)$$
$$\mathbf{Q}_{22} = \mathbf{Q}(1,1) - \frac{1}{3}\mathbf{Q}_u(1,1) - \frac{1}{3}\mathbf{Q}_w(1,1) + \frac{1}{9}\mathbf{Q}_{uw}(1,1)$$
$$\mathbf{Q}_{23} = \mathbf{Q}(1,1) - \frac{1}{3}\mathbf{Q}_u(1,1)$$
$$\mathbf{Q}_{30} = \mathbf{Q}(1,0)$$
$$\mathbf{Q}_{31} = \mathbf{Q}(1,0) + \frac{1}{3}\mathbf{Q}_w(1,0)$$
$$\mathbf{Q}_{32} = \mathbf{Q}(1,1) - \frac{1}{3}\mathbf{Q}_w(1,1)$$
$$\mathbf{Q}_{33} = \mathbf{Q}(1,1)$$

A.2. Interface between a Coons bi-cubic surface and Bezier bi-cubic surface

$$\boldsymbol{Q}(0,0) = \boldsymbol{Q}_{00}$$
$$\boldsymbol{Q}(1,0) = \boldsymbol{Q}_{30}$$
$$\boldsymbol{Q}_u(0,0) = 3(\boldsymbol{Q}_{10} - \boldsymbol{Q}_{00})$$
$$\boldsymbol{Q}_u(1,0) = 3(\boldsymbol{Q}_{30} - \boldsymbol{Q}_{20})$$

$$\boldsymbol{Q}_w(0,0) = 3(\boldsymbol{Q}_{01} - \boldsymbol{Q}_{00})$$
$$\boldsymbol{Q}_w(1,0) = 3(\boldsymbol{Q}_{31} - \boldsymbol{Q}_{30})$$
$$\boldsymbol{Q}_{uw}(0,0) = 9(\boldsymbol{Q}_{00} - \boldsymbol{Q}_{01} - \boldsymbol{Q}_{10} + \boldsymbol{Q}_{11})$$
$$\boldsymbol{Q}_{uw}(1,0) = 9(\boldsymbol{Q}_{20} - \boldsymbol{Q}_{21} - \boldsymbol{Q}_{30} + \boldsymbol{Q}_{31})$$

$$\mathbf{Q}(0,1) = \mathbf{Q}_{03}$$
$$\mathbf{Q}(1,1) = \mathbf{Q}_{33}$$
$$\mathbf{Q}_u(0,1) = 3(\mathbf{Q}_{13} - \mathbf{Q}_{03})$$
$$\mathbf{Q}_u(1,1) = 3(\mathbf{Q}_{33} - \mathbf{Q}_{23})$$

$$\boldsymbol{Q}_w(0,1) = 3(\boldsymbol{Q}_{03} - \boldsymbol{Q}_{02})$$
$$\boldsymbol{Q}_w(1,1) = 3(\boldsymbol{Q}_{33} - \boldsymbol{Q}_{32})$$
$$\boldsymbol{Q}_{uw}(0,1) = 9(\boldsymbol{Q}_{02} - \boldsymbol{Q}_{03} - \boldsymbol{Q}_{12} + \boldsymbol{Q}_{13})$$
$$\boldsymbol{Q}_{uw}(1,1) = 9(\boldsymbol{Q}_{22} - \boldsymbol{Q}_{23} - \boldsymbol{Q}_{32} + \boldsymbol{Q}_{33})$$

A.3. Interface between Bspline bi-cubic surface and Coons bi-cubic surface

$$\begin{aligned}\mathbf{Q}(0,0) = {} & \frac{1}{6}\left(\frac{1}{6}\mathbf{Q}_{i-1,j-1} + \frac{2}{3}\mathbf{Q}_{i,j-1} + \frac{1}{6}\mathbf{Q}_{i+1,j-1}\right) \\ & + \frac{2}{3}\left(\frac{1}{6}\mathbf{Q}_{i-1,j} + \frac{2}{3}\mathbf{Q}_{i,j} + \frac{1}{6}\mathbf{Q}_{i+1,j}\right) \\ & + \frac{1}{6}\left(\frac{1}{6}\mathbf{Q}_{i-1,j+1} + \frac{2}{3}\mathbf{Q}_{i,j+1} + \frac{1}{6}\mathbf{Q}_{i+1,j+1}\right)\end{aligned}$$

$$\begin{aligned}\mathbf{Q}(0,1) = {} & \frac{1}{6}\left(\frac{1}{6}\mathbf{Q}_{i-1,j} + \frac{2}{3}\mathbf{Q}_{i,j} + \frac{1}{6}\mathbf{Q}_{i+1,j}\right) \\ & + \frac{2}{3}\left(\frac{1}{6}\mathbf{Q}_{i-1,j+1} + \frac{2}{3}\mathbf{Q}_{i,j+1} + \frac{1}{6}\mathbf{Q}_{i+1,j+1}\right) \\ & + \frac{1}{6}\left(\frac{1}{6}\mathbf{Q}_{i-1,j+2} + \frac{2}{3}\mathbf{Q}_{i,j+2} + \frac{1}{6}\mathbf{Q}_{i+1,j+2}\right)\end{aligned}$$

$$\begin{aligned}\mathbf{Q}(1,0) = {} & \frac{1}{6}\left(\frac{1}{6}\mathbf{Q}_{i,j-1} + \frac{2}{3}\mathbf{Q}_{i+1,j-1} + \frac{1}{6}\mathbf{Q}_{i+2,j-1}\right) \\ & + \frac{2}{3}\left(\frac{1}{6}\mathbf{Q}_{i,j} + \frac{2}{3}\mathbf{Q}_{i+1,j} + \frac{1}{6}\mathbf{Q}_{i+2,j}\right) \\ & + \frac{1}{6}\left(\frac{1}{6}\mathbf{Q}_{i,j+1} + \frac{2}{3}\mathbf{Q}_{i+1,j+1} + \frac{1}{6}\mathbf{Q}_{i+2,j+1}\right)\end{aligned}$$

$$\begin{aligned}\mathbf{Q}(1,1) = {} & \frac{1}{6}\left(\frac{1}{6}\mathbf{Q}_{i,j} + \frac{2}{3}\mathbf{Q}_{i+1,j} + \frac{1}{6}\mathbf{Q}_{i+2,j}\right) \\ & + \frac{2}{3}\left(\frac{1}{6}\mathbf{Q}_{i,j+1} + \frac{2}{3}\mathbf{Q}_{i+1,j+1} + \frac{1}{6}\mathbf{Q}_{i+2,j+1}\right) \\ & + \frac{1}{6}\left(\frac{1}{6}\mathbf{Q}_{i,j+2} + \frac{2}{3}\mathbf{Q}_{i+1,j+2} + \frac{1}{6}\mathbf{Q}_{i+2,j+2}\right)\end{aligned}$$

$$\begin{aligned}\mathbf{Q}_u(0,0) = {} & \frac{1}{6}\left(\frac{1}{2}\mathbf{Q}_{i+1,j-1} - \frac{1}{2}\mathbf{Q}_{i-1,j-1}\right) \\ & + \frac{2}{3}\left(\frac{1}{2}\mathbf{Q}_{i+1,j} - \frac{1}{2}\mathbf{Q}_{i-1,j}\right) \\ & + \frac{1}{6}\left(\frac{1}{2}\mathbf{Q}_{i+1,j+1} - \frac{1}{2}\mathbf{Q}_{i-1,j+1}\right)\end{aligned}$$

$$\mathbf{Q}_u(0,1)=\frac{1}{6}\left(\frac{1}{2}\mathbf{Q}_{i+1,j}-\frac{1}{2}\mathbf{Q}_{i-1,j}\right)+\frac{2}{3}\left(\frac{1}{2}\mathbf{Q}_{i+1,j+1}-\frac{1}{2}\mathbf{Q}_{i-1,j+1}\right)+\frac{1}{6}\left(\frac{1}{2}\mathbf{Q}_{i+1,j+2}-\frac{1}{2}\mathbf{Q}_{i-1,j+2}\right)$$

$$\mathbf{Q}_u(1,0)=\frac{1}{6}\left(\frac{1}{2}\mathbf{Q}_{i+2,j-1}-\frac{1}{2}\mathbf{Q}_{i,j-1}\right)+\frac{2}{3}\left(\frac{1}{2}\mathbf{Q}_{i+2,j}-\frac{1}{2}\mathbf{Q}_{i,j}\right)+\frac{1}{6}\left(\frac{1}{2}\mathbf{Q}_{i+2,j+1}-\frac{1}{2}\mathbf{Q}_{i,j+1}\right)$$

$$\mathbf{Q}_u(1,1)=\frac{1}{6}\left(\frac{1}{2}\mathbf{Q}_{i+2,j}-\frac{1}{2}\mathbf{Q}_{i,j}\right)+\frac{2}{3}\left(\frac{1}{2}\mathbf{Q}_{i+2,j+1}-\frac{1}{2}\mathbf{Q}_{i,j+1}\right)+\frac{1}{6}\left(\frac{1}{2}\mathbf{Q}_{i+2,j+2}-\frac{1}{2}\mathbf{Q}_{i,j+2}\right)$$

$$\mathbf{Q}_w(0,0)=\frac{1}{2}\left(\frac{1}{6}\mathbf{Q}_{i-1,j+1}+\frac{2}{3}\mathbf{Q}_{i,j+1}+\frac{1}{6}\mathbf{Q}_{i+1,j+1}\right)-\frac{1}{2}\left(\frac{1}{6}\mathbf{Q}_{i-1,j-1}+\frac{2}{3}\mathbf{Q}_{i,j-1}+\frac{1}{6}\mathbf{Q}_{i+1,j-1}\right)$$

$$\mathbf{Q}_w(0,1)=\frac{1}{2}\left(\frac{1}{6}\mathbf{Q}_{i-1,j+2}+\frac{2}{3}\mathbf{Q}_{i,j+2}+\frac{1}{6}\mathbf{Q}_{i+1,j+2}\right)-\frac{1}{2}\left(\frac{1}{6}\mathbf{Q}_{i-1,j}+\frac{2}{3}\mathbf{Q}_{i,j}+\frac{1}{6}\mathbf{Q}_{i+1,j}\right)$$

$$\mathbf{Q}_w(1,0)=\frac{1}{2}\left(\frac{1}{6}\mathbf{Q}_{i,j+1}+\frac{2}{3}\mathbf{Q}_{i-1,j+1}+\frac{1}{6}\mathbf{Q}_{i+2,j+1}\right)-\frac{1}{2}\left(\frac{1}{6}\mathbf{Q}_{i,j-1}+\frac{2}{3}\mathbf{Q}_{i+1,j-1}+\frac{1}{6}\mathbf{Q}_{i+2,j-1}\right)$$

$$\mathbf{Q}_w(1,1)=\frac{1}{2}\left(\frac{1}{6}\mathbf{Q}_{i,j+2}+\frac{2}{3}\mathbf{Q}_{i-1,j+2}+\frac{1}{6}\mathbf{Q}_{i+2,j+2}\right)-\frac{1}{2}\left(\frac{1}{6}\mathbf{Q}_{i,j}+\frac{2}{3}\mathbf{Q}_{i-1,j}+\frac{1}{6}\mathbf{Q}_{i+2,j}\right)$$

$$\mathbf{Q}_{uw}(0,0)=\frac{1}{2}\left(\frac{1}{2}\mathbf{Q}_{i+1,j+1}-\frac{1}{2}\mathbf{Q}_{i-1,j+1}\right)-\frac{1}{2}\left(\frac{1}{2}\mathbf{Q}_{i+1,j-1}-\frac{1}{2}\mathbf{Q}_{i-1,j-1}\right)$$

$$\mathbf{Q}_{uw}(0,1)=\frac{1}{2}\left(\frac{1}{2}\mathbf{Q}_{i+1,j+2}-\frac{1}{2}\mathbf{Q}_{i-1,j+2}\right)-\frac{1}{2}\left(\frac{1}{2}\mathbf{Q}_{i+1,j}-\frac{1}{2}\mathbf{Q}_{i-1,j}\right)$$

$$\mathbf{Q}_{uw}(1,0)=\frac{1}{2}\left(\frac{1}{2}\mathbf{Q}_{i+2,j+1}-\frac{1}{2}\mathbf{Q}_{i,j+1}\right)-\frac{1}{2}\left(\frac{1}{2}\mathbf{Q}_{i+2,j-1}-\frac{1}{2}\mathbf{Q}_{i,j-1}\right)$$

$$\mathbf{Q}_{uw}(1,1)=\frac{1}{2}\left(\frac{1}{2}\mathbf{Q}_{i+2,j+2}-\frac{1}{2}\mathbf{Q}_{i,j+2}\right)-\frac{1}{2}\left(\frac{1}{2}\mathbf{Q}_{i+2,j}-\frac{1}{2}\mathbf{Q}_{i,j}\right)$$

A.4. Interface between Bspline bi-cubic surface and Bezier bi-cubic surface

$$\mathbf{Q}_{00}=\frac{1}{6}\left(\frac{1}{6}\mathbf{Q}_{i-1,j-1}+\frac{2}{3}\mathbf{Q}_{i,j-1}+\frac{1}{6}\mathbf{Q}_{i+1,j-1}\right)+\frac{2}{3}\left(\frac{1}{6}\mathbf{Q}_{i-1,j}+\frac{2}{3}\mathbf{Q}_{i,j}+\frac{1}{6}\mathbf{Q}_{i+1,j}\right)+\frac{1}{6}\left(\frac{1}{6}\mathbf{Q}_{i-1,j+1}+\frac{2}{3}\mathbf{Q}_{i,j+1}+\frac{1}{6}\mathbf{Q}_{i+1,j+1}\right)$$

$$\mathbf{Q}_{01}=\frac{2}{3}\left(\frac{1}{6}\mathbf{Q}_{i-1,j}+\frac{2}{3}\mathbf{Q}_{i,j}+\frac{1}{6}\mathbf{Q}_{i+1,j}\right)+\frac{1}{3}\left(\frac{1}{6}\mathbf{Q}_{i-1,j+1}+\frac{2}{3}\mathbf{Q}_{i,j+1}+\frac{1}{6}\mathbf{Q}_{i+1,j+1}\right)$$

$$\mathbf{Q}_{02}=\frac{1}{3}\left(\frac{1}{6}\mathbf{Q}_{i-1,j}+\frac{2}{3}\mathbf{Q}_{i,j}+\frac{1}{6}\mathbf{Q}_{i+1,j}\right)+\frac{2}{3}\left(\frac{1}{6}\mathbf{Q}_{i-1,j+1}+\frac{2}{3}\mathbf{Q}_{i,j+1}+\frac{1}{6}\mathbf{Q}_{i+1,j+1}\right)$$

$$\mathbf{Q}_{03}=\frac{1}{6}\left(\frac{1}{6}\mathbf{Q}_{i-1,j}+\frac{2}{3}\mathbf{Q}_{i,j}+\frac{1}{6}\mathbf{Q}_{i+1,j}\right)+\frac{2}{3}\left(\frac{1}{6}\mathbf{Q}_{i-1,j+1}+\frac{2}{3}\mathbf{Q}_{i,j+1}+\frac{1}{6}\mathbf{Q}_{i+1,j+1}\right)+\frac{1}{6}\left(\frac{1}{6}\mathbf{Q}_{i-1,j+2}+\frac{2}{3}\mathbf{Q}_{i,j+2}+\frac{1}{6}\mathbf{Q}_{i+1,j+2}\right)$$

$$\mathbf{Q}_{10}=\frac{1}{6}\left(\frac{2}{3}\mathbf{Q}_{i,j-1}+\frac{1}{3}\mathbf{Q}_{i+1,j-1}\right)+\frac{2}{3}\left(\frac{2}{3}\mathbf{Q}_{i,j}+\frac{1}{3}\mathbf{Q}_{i+1,j}\right)+\frac{1}{6}\left(\frac{2}{3}\mathbf{Q}_{i,j+1}+\frac{1}{3}\mathbf{Q}_{i+1,j+1}\right)$$

$$\boldsymbol{Q}_{11}=\frac{2}{3}\left(\frac{2}{3}\boldsymbol{Q}_{i,j}+\frac{1}{3}\boldsymbol{Q}_{i+1,j}\right)+\frac{1}{3}\left(\frac{2}{3}\boldsymbol{Q}_{i,j+1}+\frac{1}{3}\boldsymbol{Q}_{i+1,j+1}\right)$$

$$\boldsymbol{Q}_{12}=\frac{1}{3}\left(\frac{2}{3}\boldsymbol{Q}_{i,j}+\frac{1}{3}\boldsymbol{Q}_{i+1,j}\right)+\frac{2}{3}\left(\frac{2}{3}\boldsymbol{Q}_{i,j+1}+\frac{1}{3}\boldsymbol{Q}_{i+1,j+1}\right)$$

$$\mathbf{Q}_{13}=\frac{1}{6}\left(\frac{2}{3}\mathbf{Q}_{i,j}+\frac{1}{3}\mathbf{Q}_{i+1,j}\right)$$
$$+\frac{2}{3}\left(\frac{2}{3}\mathbf{Q}_{i,j+1}+\frac{1}{3}\mathbf{Q}_{i+1,j+1}\right)+\frac{1}{6}\left(\frac{2}{3}\mathbf{Q}_{i,j+2}+\frac{1}{3}\mathbf{Q}_{i+1,j+2}\right)$$

$$\mathbf{Q}_{20}=\frac{1}{6}\left(\frac{1}{3}\mathbf{Q}_{i,j-1}+\frac{2}{3}\mathbf{Q}_{i+1,j-1}\right)$$
$$+\frac{2}{3}\left(\frac{1}{3}\mathbf{Q}_{i,j}+\frac{2}{3}\mathbf{Q}_{i+1,j}\right)+\frac{1}{6}\left(\frac{1}{3}\mathbf{Q}_{i,j+1}+\frac{2}{3}\mathbf{Q}_{i+1,j+1}\right)$$

$$\boldsymbol{Q}_{21}=\frac{2}{3}\left(\frac{1}{3}\boldsymbol{Q}_{i,j}+\frac{2}{3}\boldsymbol{Q}_{i+1,j}\right)+\frac{1}{3}\left(\frac{1}{3}\boldsymbol{Q}_{i,j+1}+\frac{2}{3}\boldsymbol{Q}_{i+1,j+1}\right)$$

$$\boldsymbol{Q}_{22}=\frac{1}{3}\left(\frac{1}{3}\boldsymbol{Q}_{i,j}+\frac{2}{3}\boldsymbol{Q}_{i+1,j}\right)+\frac{2}{3}\left(\frac{1}{3}\boldsymbol{Q}_{i,j+1}+\frac{2}{3}\boldsymbol{Q}_{i+1,j+1}\right)$$

$$\mathbf{Q}_{23}=\frac{1}{6}\left(\frac{1}{3}\mathbf{Q}_{i,j}+\frac{2}{3}\mathbf{Q}_{i+1,j}\right)$$
$$+\frac{2}{3}\left(\frac{1}{3}\mathbf{Q}_{i,j+1}+\frac{2}{3}\mathbf{Q}_{i+1,j+1}\right)+\frac{1}{6}\left(\frac{1}{3}\mathbf{Q}_{i,j+2}+\frac{2}{3}\mathbf{Q}_{i+1,j+2}\right)$$

$$\mathbf{Q}_{30}=\frac{1}{6}\left(\frac{1}{6}\mathbf{Q}_{i,j-1}+\frac{2}{3}\mathbf{Q}_{i+1,j-1}+\frac{1}{6}\mathbf{Q}_{i+2,j-1}\right)$$
$$+\frac{2}{3}\left(\frac{1}{6}\mathbf{Q}_{i,j}+\frac{2}{3}\mathbf{Q}_{i+1,j}+\frac{1}{6}\mathbf{Q}_{i+2,j}\right)$$
$$+\frac{1}{6}\left(\frac{1}{6}\mathbf{Q}_{i,j+1}+\frac{2}{3}\mathbf{Q}_{i+1,j+1}+\frac{1}{6}\mathbf{Q}_{i+2,j+1}\right)$$

$$\mathbf{Q}_{31}=\frac{2}{3}\left(\frac{1}{6}\mathbf{Q}_{i,j}+\frac{2}{3}\mathbf{Q}_{i+1,j}+\frac{1}{6}\mathbf{Q}_{i+2,j}\right)$$
$$+\frac{1}{3}\left(\frac{1}{6}\mathbf{Q}_{i,j+1}+\frac{2}{3}\mathbf{Q}_{i+1,j+1}+\frac{1}{6}\mathbf{Q}_{i+2,j+1}\right)$$

$$\mathbf{Q}_{32}=\frac{1}{3}\left(\frac{1}{6}\mathbf{Q}_{i,j}+\frac{2}{3}\mathbf{Q}_{i+1,j}+\frac{1}{6}\mathbf{Q}_{i+2,j}\right)$$
$$+\frac{2}{3}\left(\frac{1}{6}\mathbf{Q}_{i,j+1}+\frac{2}{3}\mathbf{Q}_{i+1,j+1}+\frac{1}{6}\mathbf{Q}_{i+2,j+1}\right)$$

$$\mathbf{Q}_{33}=\frac{1}{6}\left(\frac{1}{6}\mathbf{Q}_{i,j}+\frac{2}{3}\mathbf{Q}_{i+1,j}+\frac{1}{6}\mathbf{Q}_{i+2,j}\right)$$
$$+\frac{2}{3}\left(\frac{1}{6}\mathbf{Q}_{i,j+1}+\frac{2}{3}\mathbf{Q}_{i+1,j+1}+\frac{1}{6}\mathbf{Q}_{i+2,j+1}\right)$$
$$+\frac{1}{6}\left(\frac{1}{6}\mathbf{Q}_{i,j+2}+\frac{2}{3}\mathbf{Q}_{i+1,j+1}+\frac{1}{6}\mathbf{Q}_{i+2,j+2}\right)$$

Annex B

B.1 Co-ordinates of extracted knots from coons patches

Patch 1

X=	0.00	Y=	0.00	Z=	0.00
X=	53.64	Y=	3.61	Z=	-1.45
X=	107.28	Y=	7.22	Z=	-2.90
X=	160.92	Y=	10.83	Z=	-4.35
X=	214.56	Y=	14.44	Z=	-5.80
X=	0.61	Y=	7.86	Z=	0.00
X=	54.12	Y=	12.27	Z=	-1.34
X=	107.59	Y=	16.93	Z=	-2.58
X=	161.05	Y=	21.60	Z=	-3.83
X=	214.56	Y=	26.01	Z=	-5.16
X=	1.18	Y=	15.71	Z=	0.00
X=	54.54	Y=	21.19	Z=	-1.26
X=	107.87	Y=	26.66	Z=	-2.41
X=	161.21	Y=	32.13	Z=	-3.56
X=	214.56	Y=	37.60	Z=	-4.82
X=	1.72	Y=	23.58	Z=	0.00
x=	54.91	Y=	30.11	Z=	-1.22
X=	108.14	Y=	36.38	Z=	-2.38
X=	161.36	Y=	42.66	Z=	-3.54
X=	214.56	Y=	49.19	Z=	-4.76
X=	2.21	Y=	31.44	Z=	0.00
X=	55.30	Y=	38.78	Z=	-1.25
X=	108.38	Y=	46.11	Z=	-2.50
x=	161.47	Y=	53.45	Z=	-3.75
X=	214.56	Y=	60.78	Z=	-5.00

Patch 2

X=	2.21	Y=	31.44	Z=	0.00
X=	55.30	Y=	38.78	Z=	-1.25
X=	108.38	Y=	46.11	Z=	-2.50
X=	161.47	Y=	53.45	Z=	-3.75
X=	214.56	Y=	60.78	Z=	-5.00
X=	2.31	Y=	33.12	Z=	0.00
X=	55.37	Y=	40.36	Z=	-1.19
X=	108.43	Y=	47.60	Z=	-2.41
X=	161.49	Y=	54.83	Z=	-3.64
X=	214.56	Y=	62.07	Z=	-4.83
X=	2.40	Y=	34.80	Z=	0.00
X=	55.44	Y=	41.89	Z=	-1.03
X=	108.48	Y=	49.04	Z=	-2.17
X=	161.52	Y=	56.19	Z=	-3.30
X=	214.56	Y=	63.28	Z=	-4.33
X=	2.50	Y=	36.48	Z=	0.00
X=	55.51	Y=	43.39	Z=	-0.82
X=	108.53	Y=	50.40	Z=	-1.77
X=	161.55	Y=	57.40	Z=	-2.71
X=	214.56	Y=	64.32	Z=	-3.54
X=	2.60	Y=	38.16	Z=	0.00
X=	55.59	Y=	44.90	Z=	-0.62
X=	108.58	Y=	51.63	Z=	-1.25
X=	161.57	Y=	58.37	Z=	-1.88
X=	214.56	Y=	65.11	Z=	-2.50

Patch 3

X=	2.60	Y=	38.16	Z=	0.00
X=	55.59	Y=	44.90	Z=	-0.62
X=	108.58	Y=	51.63	Z=	-1.25
X=	161.57	Y=	58.37	Z=	-1.87
X=	214.56	Y=	65.11	Z=	-2.50
X=	2.68	Y=	39.73	Z=	0.00
X=	55.65	Y=	46.32	Z=	-0.43
X=	108.62	Y=	52.82	Z=	-0.73
X=	161.59	Y=	59.32	Z=	-1.04
X=	214.56	Y=	65.90	Z=	-1.46
X=	2.77	Y=	41.30	Z=	0.00
X=	55.72	Y=	47.74	Z=	-0.22
X=	108.66	Y=	54.12	Z=	-0.33
X=	161.61	Y=	60.50	Z=	-0.45
X=	214.56	Y=	66.94	Z=	-0.67
X=	2.85	Y=	42.87	Z=	0.00
X=	55.78	Y=	49.19	Z=	-0.06
X=	108.71	Y=	55.51	Z=	-0.09
X=	161.64	Y=	61.82	Z=	-0.11
X=	214.56	Y=	68.15	Z=	-0.17
X=	2.94	Y=	44.44	Z=	0.00
X=	55.84	Y=	50.69	Z=	0.00
X=	108.75	Y=	56.94	Z=	0.00
X=	161.65	Y=	63.19	Z=	0.00
X=	214.56	Y=	69.44	Z=	0.00

Patch 4

X=	2.94	Y=	44.44	Z=	0.00
X=	55.84	Y=	50.69	Z=	0.00
X=	108.75	Y=	56.94	Z=	0.00
X=	161.65	Y=	63.19	Z=	0.00
X=	214.56	Y=	69.44	Z=	0.00
X=	4.48	Y=	84.43	Z=	0.00
X=	57.15	Y=	87.99	Z=	0.32
X=	109.52	Y=	90.67	Z=	1.01
X=	161.90	Y=	93.35	Z=	1.71
X=	214.56	Y=	96.91	Z=	2.02
X=	5.00	Y=	124.44	Z=	0.00
X=	57.58	Y=	124.44	Z=	0.42
X=	109.78	Y=	124.44	Z=	1.35
X=	161.98	Y=	124.44	Z=	2.27
X=	214.56	Y=	124.44	Z=	2.70
X=	4.48	Y=	164.45	Z=	0.00
X=	57.15	Y=	160.89	Z=	0.32
X=	109.52	Y=	158.21	Z=	1.01
X=	161.90	Y=	155.53	Z=	1.71
X=	214.56	Y=	151.97	Z=	2.02
X=	2.94	Y=	204.44	Z=	0.00
X=	55.84	Y=	198.19	Z=	0.00
X=	108.75	Y=	191.94	Z=	0.00
X=	161.65	Y=	185.69	Z=	0.00
X=	214.56	Y=	179.44	Z=	0.00

Patch 5

X=	2.94	Y=	204.44	Z=	0.00
X=	55.84	Y=	198.19	Z=	0.00
X=	108.75	Y=	191.94	Z=	0.00
X=	161.65	Y=	185.69	Z=	0.00
X=	214.56	Y=	179.44	Z=	0.00
X=	2.85	Y=	206.01	Z=	0.00
X=	55.78	Y=	199.69	Z=	-0.06
X=	108.71	Y=	193.37	Z=	-0.09
X=	161.64	Y=	187.06	Z=	-0.11
X=	214.56	Y=	180.73	Z=	-0.17
X=	2.77	Y=	207.58	Z=	0.00
X=	55.72	Y=	201.14	Z=	-0.22
X=	108.66	Y=	194.76	Z=	-0.33
X=	161.61	Y=	188.38	Z=	-0.45
X=	214.56	Y=	181.94	Z=	-0.67
X=	2.68	Y=	209.15	Z=	0.00
X=	55.65	Y=	202.56	Z=	-0.43
X=	108.62	Y=	196.06	Z=	-0.73
X=	161.59	Y=	189.56	Z=	-1.04
X=	214.56	Y=	182.98	Z=	-1.46
X=	2.60	Y=	210.72	Z=	0.00
X=	55.59	Y=	203.98	Z=	-0.63
X=	108.58	Y=	197.25	Z=	-1.25
X=	161.57	Y=	190.51	Z=	-1.88
X=	214.56	Y=	183.77	Z=	-2.50

Patch 6

X=	2.60	Y=	210.72	Z=	0.00
X=	55.59	Y=	203.98	Z=	-0.63
X=	108.58	Y=	197.25	Z=	-1.25
X=	161.57	Y=	190.51	Z=	-1.88
X=	214.56	Y=	183.77	Z=	-2.50
X=	2.50	Y=	212.40	Z=	0.00
X=	55.51	Y=	205.49	Z=	-0.82
X=	108.53	Y=	198.48	Z=	-1.77
X=	161.55	Y=	191.48	Z=	-2.71
X=	214.56	Y=	184.57	Z=	-3.54
X=	2.40	Y=	214.08	Z=	0.00
X=	55.44	Y=	206.99	Z=	-1.03
X=	108.48	Y=	199.84	Z=	-2.17
X=	161.52	Y=	192.69	Z=	-3.30
X=	214.56	Y=	185.60	Z=	-4.33
X=	2.31	Y=	215.76	Z=	0.00
X=	55.37	Y=	208.52	Z=	-1.19
X=	108.43	Y=	201.28	Z=	-2.41
X=	161.49	Y=	194.05	Z=	-3.64
X=	214.56	Y=	186.81	Z=	-4.83
X=	2.21	Y=	217.44	Z=	0.00
X=	55.30	Y=	210.11	Z=	-1.25
X=	108.38	Y=	202.77	Z=	-2.50
X=	161.47	Y=	195.44	Z=	-3.75
X=	214.56	Y=	188.10	Z=	-5.00

Patch 7

X=	2.21	Y=	217.44	Z=	0.00
X=	55.30	Y=	210.11	Z=	-1.25
X=	108.38	Y=	202.77	Z=	-2.50
X=	161.47	Y=	195.44	Z=	-3.75
X=	214.56	Y=	188.10	Z=	-5.00
X=	1.72	Y=	225.30	Z=	0.00
X=	54.91	Y=	218.77	Z=	-1.22
X=	108.14	Y=	212.50	Z=	-2.38
X=	161.36	Y=	206.22	Z=	-3.54
X=	214.56	Y=	199.69	Z=	-4.76
X=	1.18	Y=	233.17	Z=	0.00
X=	54.54	Y=	227.69	Z=	-1.26
X=	107.87	Y=	222.22	Z=	-2.41
X=	161.21	Y=	216.75	Z=	-3.56
X=	214.56	Y=	211.28	Z=	-4.82
X=	0.61	Y=	241.02	Z=	0.00
X=	54.12	Y=	236.62	Z=	-1.34
X=	107.59	Y=	231.95	Z=	-2.58
X=	161.05	Y=	227.28	Z=	-3.83
X=	214.56	Y=	222.87	Z=	-5.16
X=	0.00	Y=	248.88	Z=	0.00
X=	53.64	Y=	245.27	Z=	-1.45
X=	107.28	Y=	241.66	Z=	-2.90
X=	160.92	Y=	238.05	Z=	-4.35
X=	214.56	Y=	234.44	Z=	-5.80