Dynamic model of fracture

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We study the failure properties of heterogeneous materials within the framework of the fiber bundle model subject to the global load-sharing rule in which the load failing elements is shared equally among all surviving elements. We develop a simulation technique by using the Langevin equation in order to investigate some characteristics of our model. It is found that the behavior of time to failure t_f decreases with an exponential law and the avalanche size distribution present a power law.

Keywords: fracture, fiber bundle model, heterogeneous material, Langevin equation.

I. INTRODUCTION

Fracture of heterogeneous materials has received a lot of attention in the last years due to its economic and human cost. It covers a wide range of phenomena like material science, rock mechanics and earth sciences. Recently the development reached by statistical physics has had a perform impact on this phenomenon.

Many authors [1-9] are trying to study this fracture phenomenon through different models as through direct experiences.

A very important class of approaches to the fracture problem is the well- known fiber bundle model's (FBM) that were introduced by seminal works of Daniels [10] and Coleman [11] and have been the subject of intense research during the last several years [12-15].

Fiber bundles are being classified into two groups:

Static FBM which simulate the failure of materials by quasistatic loading and dynamic or time dependent FBM which a constant load is maintained on the system and the fibers break by fatigue after a period of time. Once the fibers begin to fail one can choose among two load transfer schemes. Global load sharing (GLS) scheme, in which the load of a broken fiber is equally shared with all intact fibers in the whole system this model is known as democratic fiber bundle, assumes long-range interaction among the fibers which makes it a mean-field approximation that can be solved analytically [16,17]. The other scheme is the local load sharing (LLS), which the terminal load of the failed fiber is given equally to all the intact neighbors. This case assumes short-range interaction among the fibers; models based on LLS rules have not been solved analytically.

In this paper, we develop a Langevin dynamics (LD) simulation in order to introduce the thermal noise and the friction between the fibers.

The rest of the paper is organized as follows. In the next section, we describe the FBM model under the GLS rule. In section 3, we present the result of the Langevin dynamic simulation. Finally, section 4 is devoted to conclusions.

II.THE MODEL

The fiber bundle model (FBM) model consists of N fibers arranged on a square lattice of side length L, i.e. $N=L^2$, where N fibers are connected in parallel to each other and clamped between their two ends. The geometrical structure of the model is illustrated in Figure 1. The bundle supports an uniaxial load $F = \sum_{i=1}^{N} f_i$ and the breaking elongation threshold $(\delta_c)_i$ of the fibers are assumed to be different for different fiber (i). GLS model assumes global load sharing, i.e., the intact fibers share the applied load F equally where f_i is the initial applied stress (load per fiber). Each fiber has a different random elongation $(\delta_c)_i$, taken from an uniform distribution.

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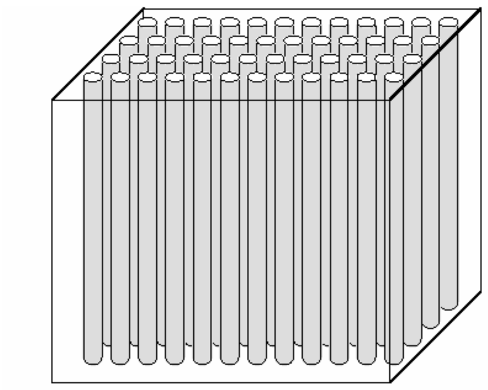

FIG. 1 A schematic illustration of the fiber bundle model.

The failure of a simple fiber under an external load is illustrated in Figure 2. Initially the fiber has a length l_0, and then when we apply a load the fiber dilates and the length becomes: $l=l_0+\delta$, Here δ is the elongation of the fiber.
When δ exceeds the elongation threshold value δth, the fiber is removed and its load is transferred equally to all the intact ones.

II. LANGEVIN DYNAMICS SIMULATION OF THE FAILURE PROCESS

The Langevin equation is a stochastic differential equation in which two force terms have been added to Newton's second law to approximate the effects of neglected degrees of freedom. One term represents a frictional force, the other a random force $\vec{R}$, which describes here the thermal noise.
The dynamics of our system is completely determined by the Langevin equation:

$$m_i \frac{d^2 z_i}{dt^2} = -m_i \gamma_i \dot{z}_i + f_i(z_i) + \vec{R}_i(t)$$

Where $f_i(z_i)$, is the force applied on fiber i. m_i is the fiber masse and γ_i is the friction coefficient.

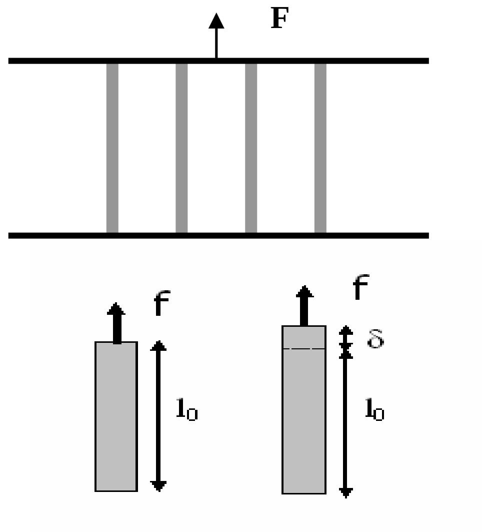

FIG. 2 An illustration of the damage of a fiber.

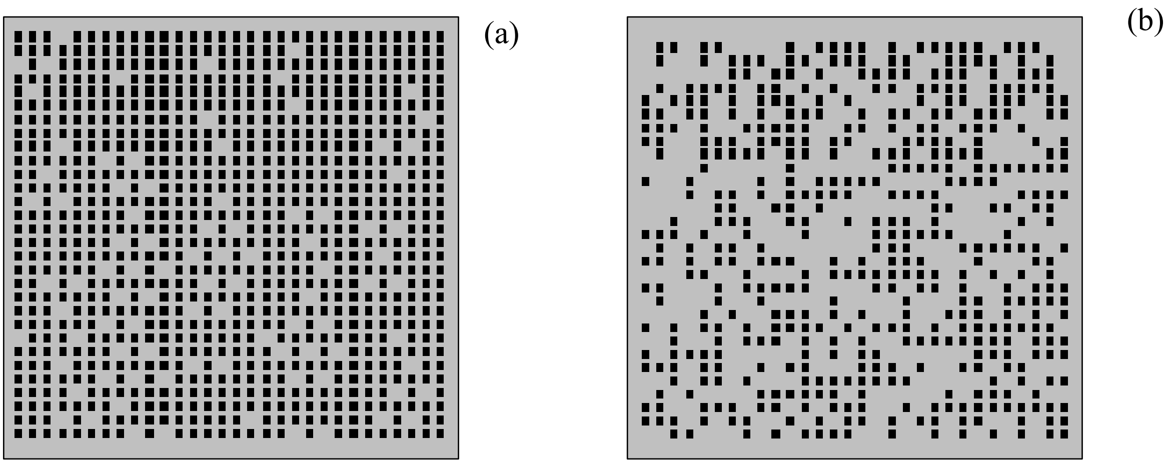

FIG. 3 The damage of the fibers. (a), in the early stage of the simulation. (b), the final crack. Undamaged fibers are represented by black squares and the damaged ones are shown by the absent squares.

The friction coefficient is related to the fluctuations of the random force $\vec{R}_i(t)$ by the fluctuation-dissipation theorem:

$$\left\langle \vec{R}_i(t) \right\rangle = 0$$

$$\left\langle \vec{R}_i(t)\vec{R}_i(t') \right\rangle = 2m_i\gamma_i k_B T\,\delta(t-t')$$

The angular brackets denote here an average, and k_B is Boltzmann's constant.

In numerical simulation, the cycle of complete breakdown of the material is performed many times in order to average out the effect of fluctuation.

Let us first examine the geometry of our material. Figure 3 shows the evolution of crack of the system.

We start the simulation with undamaged material: all the fibers are intact. In the early stage of the process only small rearrangements take place. This is also evident by observing the structure of the system (Fig. 3 a). Increasing load the micro cracks system grow until the final crack of the material (Fig. 3 b).

In figure 4 we plot the size of broken fibers S_b versus time for two different forces and for the same system size $L=32$. (Fig. 4 a) correspond to $f/f_c=0.2$ and (Fig. 4 b) correspond to $f/f_c=0.4$. In the beginning of the simulation, after a load F is applied on the bundle, only weak fibers breaks immediately. After this the total load is redistributed globally among all intact fibers. This redistribution causes a secondary failure which in general causes further failure and so on. We observe that when we grow the force the failure time decreases.

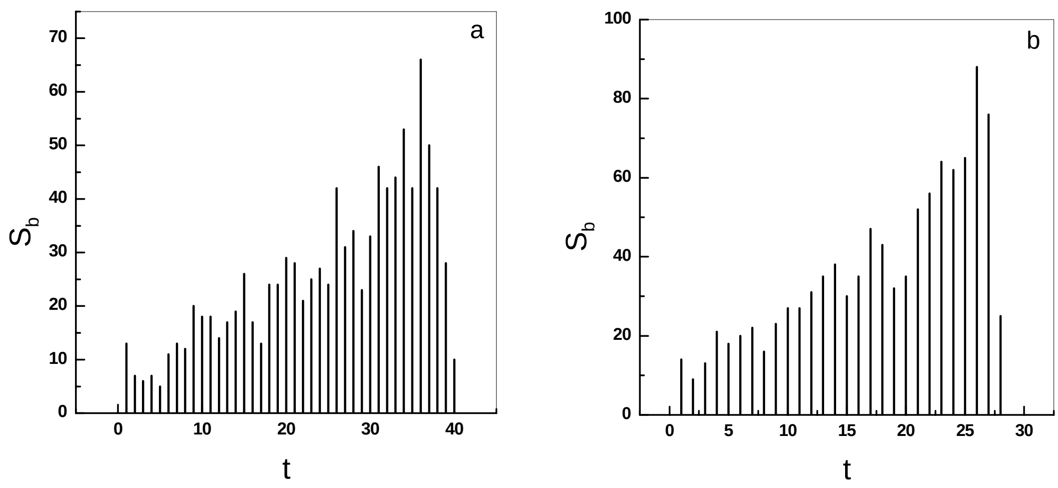

FIG. 4 The size of broken fibers versus time

Now we explore the behavior of time to failure t_f as a function of the load, which is determined when all the fibers are broken. We found that t_f presents a power law $t_f \sim (f_c - f)^{-\beta}$ with an exponent $\beta=1/2$. As we see in Fig. 5a.And also we examine the effects of noise ($T\neq0$) for the same parameter; the rupture is accelerated in the presence of noise, as shown in Fig. 5b.

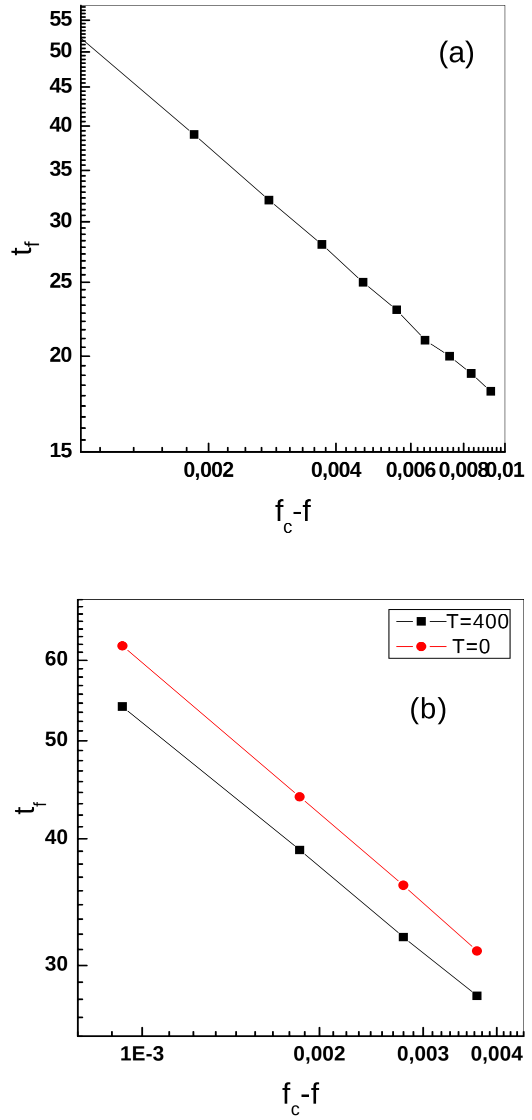

FIG. 5 The behavior of time to failure t_f

The avalanche size distribution characterizes the fracture process by reflecting the precursory activities toward complete failure. This can be related to the acoustic emissions observed in material failure.

We define the avalanche size distribution as the number of fibers that break between two steps of time. It has been shown by analytical means [12, 16, 17] that in the case of GLS the avalanche size distribution n(s) follows a universal power law: $n(s)\sim s^{-\alpha}$, with an exponent $\alpha=5/2$. In our LD simulation we found the similar behaviour as we see in figure 6.

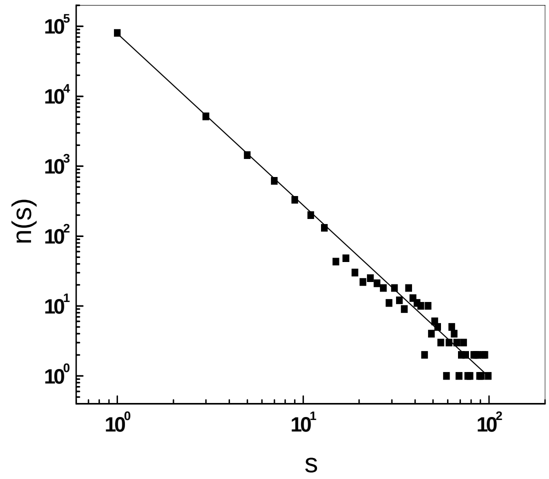

FIG. 6 Avalanche size distributions

Figure 7 shows the behavior of the ratio $\rho=N_i/N$, which defines the fraction of unbroken fibers, with time for different values of the load. We observe that the system fails as the applied load increased. We conclude that the lifetime decreases when the load increases.

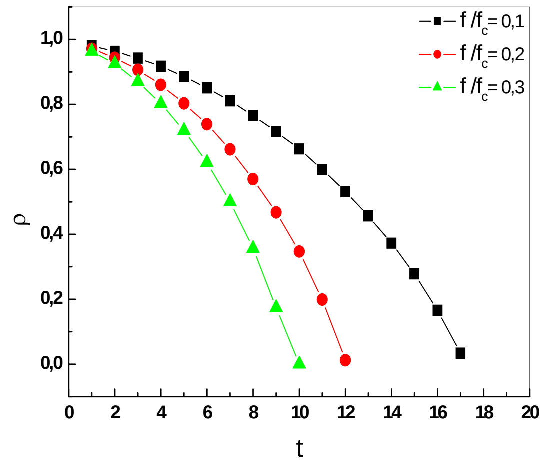

FIG. 7 Behavior of the fraction of unbroken fibers for different forces and for system size L=128.

We present in figure 8 another parameter which is the number of failed fibers.

We can remark that the number of failed fibers increases with time until the final breakdown; also we can see that when we increase the load the system fails rapidly.

Then we examine the effect of thermal noise for this parameter. This thermal noise has the effect to accelerate the material breakdown as we can see in the figure.

The results have been obtained for a system of $L=128$ and a force $f/f_c=0.1$.

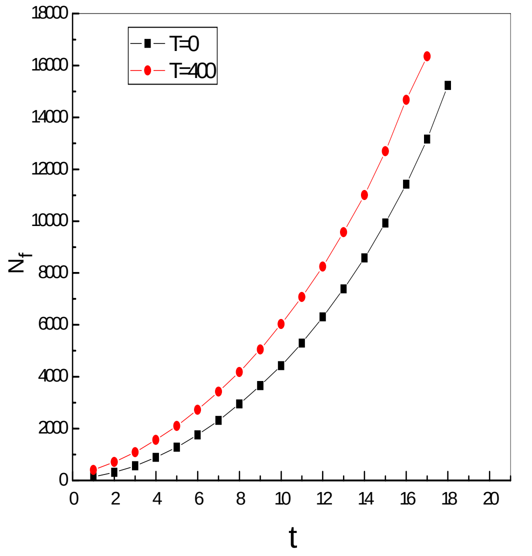

FIG. 8 Number of failed fibers as a function of time with thermal noise ($T \neq 0$) and without thermal noise.

IV. CONCLUSION

In summary, we have studied the fracture of fiber bundle model by means the Langevin dynamics simulation which allows us to introduce the thermal noise and the friction between fibers. We have also investigated the behavior of some properties of this model, which are in agreement with some recent works on fiber bundle model.

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