Modelling and simulation of dusty plasma sheath

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The aim of this paper is to study the behaviour of sheath structure in dusty plasma with collisions, and simulation of the effects collisionality on the plasma sheath using the Runge-Kutta routine. Exact numerical solutions of the model are used to determine the collisional dependence of the sheath width and the ion impact energy at the wall.

Key Words: sheath, dusty plasmas, sheath width, collision parameter, impact energy
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I. INTRODUCTION:

When a plasma is contact with a negatively biased surface, such as electrode or wall, a strong, localized electric field appears between the plasma and that this surface. This ion rich boundary layer, called the sheath, confines electrons in, and expels ions from, the plasma. The energy that ions gain as they fall through the sheath regulates both the physical and chemical processes that occur at surface contacting the plasma. Such plasma-surface interactions are important, for example, in plasma processing. Ions collisions in the sheath can significantly reduce the ion impact energy on the surface, and so it is worthwhile to include them in a sheath model [1].

There is interest recently in low temperature plasmas containing negatively charged micron-size dust particles. Such plasmas are often found near solid objects or walls, such as the planetary rings and comets, artificial satellites, the workpiecein plasma assisted material and chemical processing. It is thus of practical interest to investigate the interaction between a dusty plasma and a solid boundary [2].

It has been mentioned that dusty plasma is an unique multicomponent plasma and is composed of dispersed macroscopic dust charged grains that forms a colloidal type suspension in any given parent plasma background.[3] Dusty plasma represents the most common form of astrophysical, laboratory and industrial plasma. The typically micron-sized dust grains normally acquire negative charges to high order of magnitude with respect to normal electronic charge. The masse of the dust grains can have very high value too, up to $10^6 - 10^8$ times the proton mass. Thus the dust charges to mass ratio are different than the normal ionic species in multicomponent plasmas. Furthermore, the dust charge, mass and size, in general behave as dynamic variables, [4, 5] and produce novel effects on collective degrees of plasma dynamics. Thus, the theoretical modelling of dusty plasma requires the consideration of dust charge fluctuation dynamics. However, under certain limits, a constant dust charge model could be justified for selective dust plasma parameter domain. [6].

In the absence of dust, there are two regions in the near wall plasma; the plasma sheath it self, where main drop of electric field potential occurs, and the preheat, where the potential drop is small. In the presence of dust, there are three distinguished layers: the plasma boundary presheath (containing no dust), the dust cloud (there is main drop of the electric field potential) and the wall plasma layer (containing no dust [2])

In the sec. II we present the mathematical formulation and basic equations that are used to describe the sheath and the numerical calculation of the sheath thickness. In sec. III we present the differential equations describing the sheath structure in dusty plasma. In sec. IV numerical calculation of the sheath thickness and dust energy at wall impact are presented. In sec. V we presented the approximate solutions.

II. SHEATH MODEL OF COLLISIONAL PLASMA

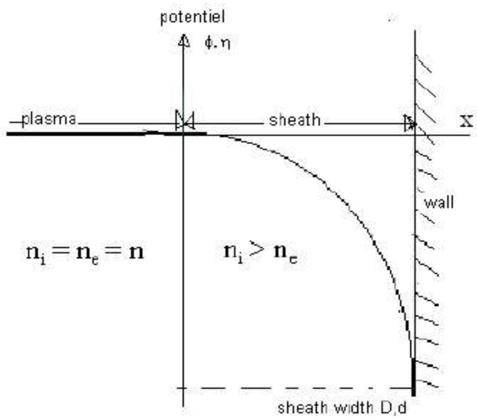
1. Sheath model

We consider unmagnetised, charge neutral plasma in contact with a planar wall.

In the plasma both the density of electrons and the density of ions n_i are equal to the plasma density n_0. The potential in the sheath is ϕ ?, and the wall is held at a negative potential ?ϕ_ω.

Consequently, a sheath forms to separate the plasma from the wall. Ions enter the sheath as a cold beam with a velocity v_0 and strike the wall with a velocity v_ω. Ions experience a collisional drag inside the sheath. The boundary between the plasma and the sheath is at x = 0, and

the sheath thickness is D, That is, the wall is at x = D, The sheath is assumed to be source free. Given these assumptions, our goal is to frame a self -consistent formulation of the collision.



Model system for the sheath

2. The dynamical governing equations in plasma sheath

We consider governing equations based on a two-fluid model. The electrons are thermalized so their density obeys the Boltzmann relation:

$$n_e = n_0 \exp\left(e\phi / k_B T_e\right) \qquad (1)$$

Where: e = elementary charge, k_B = Boltzmann's constant T_e = electron Temperature.

The cold ions obey the source-free, steady state equation of continuity.

$$\nabla.\left(n_i v_i\right) = 0 \qquad (2)$$

And equation of motion

$$m_i \left(v_i \nabla\right) v_i = -e\nabla\phi - F_{ci} \qquad (3)$$

Where: m_i =mass of the ion

The ion fluid travels through the sheath it experiences a drag force:

$$F_{ci} = m_i n_n v_i^2 \sigma \qquad (4)$$

Where n_n : Neutral gas density

σ : The momentum transfers cross section for collisions between ions and neutrals

$$\sigma\left(v_i\right) = \sigma_s \left(\frac{v_i}{C_s}\right)^{\gamma} \qquad (5)$$

Where

$C_s = \sqrt{\dfrac{k_B T_e}{m_i}}$: Ion acoustic speed

? σ_s = cross section

γ = dimensionless parameter ranging from 0 to -1

Combining equations (1)-(5) with Poisson's equation:

$$\varepsilon_0 \nabla^2 \phi = -e\left(n_i - n_e\right) \qquad (6)$$

Where ε_0 = permittivity constant.

We find two differential equations:

$$v_i \frac{d}{dx} v_i = -\frac{e}{m_i}\frac{d}{dx}\phi - n_n \sigma_s \frac{v_i^{2+\gamma}}{C_s^{\gamma}} \qquad (7)$$

$$\frac{d^2}{dx^2}\phi = -\frac{e}{\varepsilon_0} n_0 \left(\frac{v_0}{v_i} - \exp\left(\frac{e\phi}{k_B T_e}\right)\right) \qquad (8)$$

The governing equations can be made dimensionless by an appropriate choice of variables. [7] Like, v_i , x, ? ϕ and D are scaled by :

$$u_i = \frac{v_i}{C_s}, \quad \xi = \frac{x}{\lambda_D}, \quad \eta = -\frac{e\phi}{k_B T_e} \quad \text{and} \quad d = \frac{D}{\lambda_D}$$

λ_D is the Debye length: $\lambda_D = \sqrt{\dfrac{\varepsilon_o k_B T_e}{n_o e^2}}$

The degree of collisionality in the sheath is parameterised by $\alpha = \dfrac{\lambda_D}{\lambda_{mfp}}$

Where $\lambda_{mfp} = \dfrac{1}{n_n \sigma_S}$ is the mean free path

After the dimensionless variables in equations (7) and (8), those equations become:

$$u_i u_i' = \eta' - \alpha u_i^{2+\gamma} \qquad (9)$$

$$\eta'' = \frac{u_{io}}{u_i} - \exp\left(-\eta\right) \qquad (10)$$

Where the prime denotes differentiation with respect to the spatial coordinate ξ .

$$n_e = n_0 \exp\left(e\phi / k_B T_e\right)$$

3. Numerical solutions

The governing equations were solved exactly for the electric potential η_ω and ion velocity $u(\xi)$ by integrating them numerically with a Runge-Kutta routine [8]. We obtain:

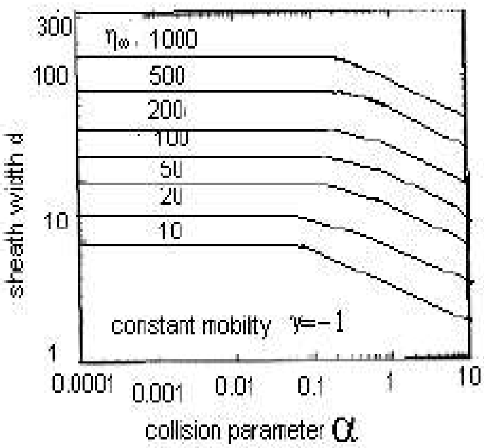


FIG. 1: Graphical representation of the Sheath-width with collisional parameter.

III. THEORY OF SHEATH IN COLLISIONAL PLASMA

The aim of this section is to study the effect of collisionality in dust plasma. We have considered here the near wall region of an unmagnetized dusty plasma witch consists of electrons, ions, micron size dust particles and neutral particles. Since the dust particles are much heavier than both the electrons and ions, the latter can be assumed to in thermal equilibrium, which dusts as a cold fluid. The neutrals are taken as immobile.

Here, since we have used fluid theory for dusts, we cane ignore the variations in shape, size and charge separations among the individual dust particles. This is so because these variations are succulently small and that they cannot be distinguished on the fluid element. Moreover though the dusts are massive with respect to the ions and electrons, due to their inertia and positive ion-dust interactions, the dusts will possess a drift velocity. For convenience, a steady state $\left(i.e.\ \frac{\partial}{\partial t} = 0 \right)$ plasma we assume here as one-dimensional.

The dynamics of the electrons and ions are treated as a neutral background in plasma and are assumed to follow respectively the following Boltzmann relations:

$$n_i = n_{i0} \exp\left(-e\phi / k_B T_i \right) \quad (11)$$

The cold dust fluid obeys the source free, steady state of continuity equation:

$$n_d v_d = n_{d0} v_{d0}$$

And momentum transfer equation

$$m_d \left(v_d \nabla \right) v_d = Z_d e \nabla \phi - F_{cd}$$

Where m_d , n_d , v_d and Z_d are respectively the dust particle mass, density, velocity and charge number and n_{d0}, v_{d0} are dust density and velocity at the sheath edge respectively.

The collisional effects between the dust and the neutrals are introduced. We use the collisional force term F_{cd} , which is given by [9]:

$$F_{cd} = m_d n_n v_d^2 \sigma$$

Where n_n is the neutral gas density and ?σ momentum transferring cross section for the collisions between the dust charged grains and neutrals.

Elastic and charge-exchange collisions contribute to this cross-section ?σ , which depends on the dust's relative velocity v_d . Poisson equation now becomes [10]:

$$\nabla^2 \phi = -4\pi e \left(n_i - n_e - Z_d n_d \right) \quad (12)$$

For strong dust-neutral collisions the movements of dusts are mobility limited.

There for, here we are interested only about the constant dust mobility case

(i.e. γ = -1). Hence, we denote consider the case of constant dust mean free path

(i.e. γ = 0)

Combining eqs (11) to (13) we find two coupled, differentials equation describing the sheath structure as:

$$v_d \frac{d}{dx} v_d = \frac{Z_d e}{m_d} \frac{d}{dx} \phi - n_n \sigma_s \frac{v_d^{2+\gamma}}{C_d^{\gamma}} \quad (13)$$

And

$$\nabla^2 \phi = -4\pi e \left(n_i - n_e - Z_d n_d \right) \quad (14)$$

Along with the parameters:

$$\delta = \frac{n_{io}}{n_{eo}},\ Z_d \frac{n_{do}}{n_{eo}} = \delta - 1,\ \theta = \frac{T_e}{T_i}$$

Now. Normalizing the governing equations by an appropriate choice of variables:

$$u_d = \frac{v_d}{C_d}, \eta = -\frac{e\phi}{k_B T_e}, \xi = \frac{x}{\lambda_D}, \alpha = \lambda_D n_n \sigma_s$$

Based on these non-dimensional parameters the basic equations reduce to:

$$u_d u_d^{'} = -Z_d \eta^{'} - \alpha u_d^{2+\gamma} \quad (15)$$

$$\eta^{''} = \delta \exp(\eta\theta) - \exp(-\eta) + (1-\delta)\frac{u_{do}}{u_d} \quad (16)$$

IV. RESULTS AND COMMENTS

In the following figure, we plot the sheath thickness d and the ion impact energy ε_ω as the function of the collision parameter

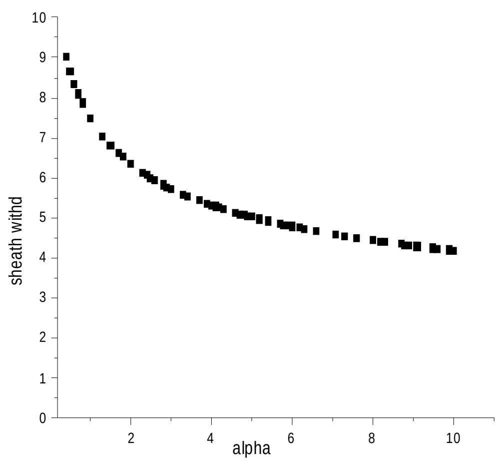


FIG. 2: Graphical representation of the sheath-width with collisional parameter

We find that, sheath thickness d decreases with increasing collisionality α

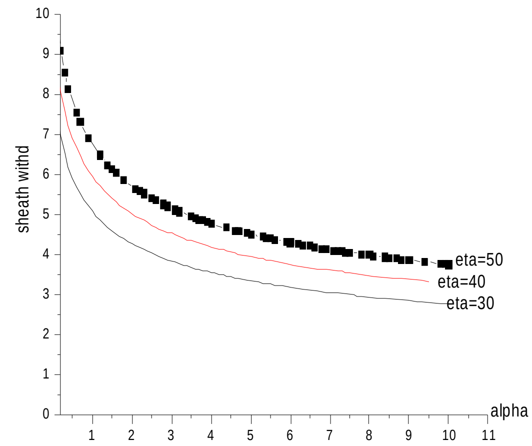


FIG. 3: Graphical representation of the sheath-width with collisional parameter for various wall potentials.

It is observed that, the electric potential η varies not only with ξ and α, but also with the energy dependence of the cross-section, characterized by γ. We find that, sheath thickness d decreases with increasing collisionality α, we find also that, sheath thickness increases when the electric potential increases

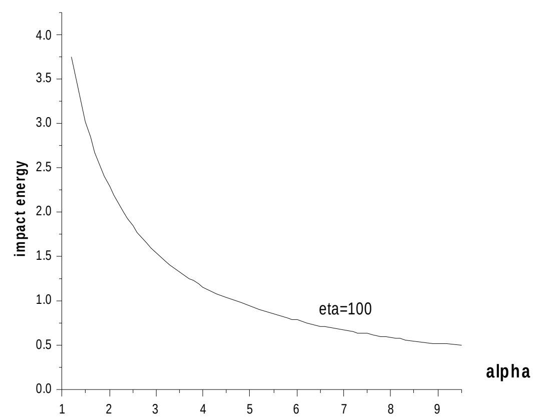


FIG.4: Graphical representation of the impact energy with collisional parameter

It is found that, impact energy decreases asymptotically with increasing collisionality α.

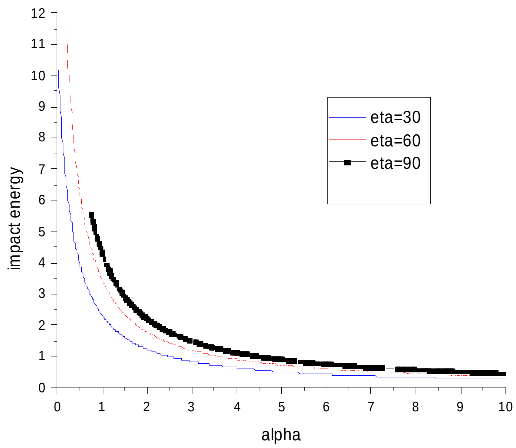


FIG.5: Graphical representation of the impact energy with collisional parameter for various wall potentials.

We find that, impact energy decreases asymptotically with increasing collisionality α , and increases also when the electric potential increases

V. APPROXIMATE SOLUTIONS

In the limit of strong collisions, the collision parameter is large the equations of motion are simplified by neglecting the convective term on the left hand side. The equations thus become:

$$\varepsilon_\omega = \frac{1}{2}\left(\frac{\eta_\omega '}{\alpha}\right)^{\frac{2}{2+\gamma}}$$

For strong dust-neutral collisions the Movements of dusts are mobility limited. Therefore, here we are interested only about the constant dust mobility case (i.e. γ = -1)
The dust impact energy on the wall is

$$\varepsilon_\omega = \frac{1}{2}u_\omega^2 = \frac{1}{2}\left(z_d \frac{\eta_\omega '}{\alpha}\right)^2$$

Where η_ω and $\eta_\omega '$ are given by equations

$$\eta = 3.2^{\frac{3}{2}}\alpha^{\frac{1}{2}}\left(\frac{\delta - 1}{z_d}\right)^{\frac{1}{2}}\frac{3}{2}\xi^{\frac{3}{2}}$$

$$\eta '_\omega = 3.2^{\frac{3}{2}}\alpha^{\frac{1}{2}}\left(\frac{\delta - 1}{z_d}\right)^{\frac{1}{2}}\frac{3}{2}\xi^{\frac{1}{2}}$$

We find

$$\varepsilon_\omega = \frac{1}{2} z_d^{\frac{11}{6}} 3^{\frac{7}{3}} \eta_\omega^{\frac{1}{3}} u_0^{\frac{1}{3}} (\delta - 1)^{\frac{1}{6}} \alpha^{\frac{-5}{3}}$$

From this equation dust impact energy decreases with increasing collisionality.

CONCLUSION

In this work we have presented a fluid model for the collisional dusty plasma sheath. We have considered the case when electrons and ions remain in thermal equilibrium while the dust charged grains are the inertial species. For the case of strong dust-neutral collisions, it is found that the sheath width is reduced by collisions. It is observed also that impact energy decreases asymptotically with increasing collisionality α , and increases also when the electric potential increases. Approximate solutions of this model appropriate for the collisionality-dominated sheath were derived. We have showing that the impact energy decreases with the great values of parameter collision α .

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