The effect of mixture lenghts of vehicles on the traffic flow behavior in one-dimensional cellular automaton

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The effect of mixture lengths of vehicles on the asymmetric exclusion model is studied using numerical simulations for both open and periodic boundaries in parallel dynamics. The vehicles are filed from their length, the small cars Type 1 occupy one cell whereas the big ones Type 2 takes two. In the case of open boundaries two varieties of models are presented. The former model corresponds to a chain with two entries where densities are calculated as a function of the injecting rates α_1 and α_2 of vehicles type 1 and type 2 respectively, and the phase diagram (α_1, α_2) is presented for a fixed value of the extracting rate β. In this case the first order transition from low to high density phases occurs at $\alpha_1+\alpha_2=\beta$ and disappears for $\alpha_2>\beta$. The latter model correspond to a chain with one entry, where α is the injecting rate of vehicles independently of their nature. Type1 and type2 are injected with α_1 and α_2 respectively, where $\alpha_2=n\alpha$, n is the concentration of type2 and $\alpha_2\leq\alpha_1\leq\alpha$. Densities are calculated as a function of the injecting rates α, and the phase diagrams (α,β) are established for different values of n. In this case the gap which is a characteristic of the first order transition vanishes with increasing α for $n \neq 0$.However, the first order transition between high and low densities exhibit an end point above which the global density undergoes a continuous passage. The end point coordinate depends strongly on the value of n. In the periodic boundaries case, the presence of vehicles type2 in the chain leads to a modification in the fundamental phase diagram (current, density). Indeed, the maximal current value decreases with increasing the concentration of vehicles type 2, and occurs at higher values of the global density in contrast with what was found by Schadschneider et al. [20].

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I.Introduction

Without doubt, an efficient transportation system is essential for the functioning and success of modern industrialized societies. But the days when freeways were free are over. The increasing problems of roadway traffic raise the following questions: Is it still affordable and publicly acceptable to expand the infrastructure? will drivers still buy cars when streets are effectively turned into parking lots? Automobile companies worried about their future market, have spent considerable amounts of money for research on traffic flow and on how the available infrastructure could be used more efficiently by new technologies.

Indeed, traffic flow is an interesting field of inter-disciplinary research, it attracted the interest of many researchers from different disciplines. Like mathematicians, chemists, engineer, physicists have addressed the problems of traffic flow many years ago and they have been trying to understand the fundamental principals governing the flow of vehicular traffic using theoretical approaches based on concepts and techniques of statistical physics[1,2,3]. The approach of a physicist is usually quite different from that of traffic engineer. A physicist try to develop a model of traffic by incorporating only the most essential ingredients which are absolutely necessary to describe the general features of typical real traffic. There are two different ways for modelling traffic: The macroscopic models which are based on fluid-dynamical description, and the microscopic ones where attention is explicitly focused on individual vehicles which is represented by a particle. The interaction is determined by the way the vehicles influence each others movement. In other words in the microscopic theories traffic flow is considered as a system of interacting particles driven far from equilibrium. Thus, it offers the possibility to study various fundamental aspects of the dynamics of truly non equilibrium systems which are of current interest in statistical physics[4,5]. Within the conceptual framework of microscopic approach, the particle hopping models describe traffic in terms of a stochastic dynamics of individual vehicles which are usually formulated using the language of cellular automata(CA)[6]. In general, CA are idealization of physical systems in which both space and time are assumed to be discrete and each of the interacting units can have only a finite number of discrete states, thus in CA models of

traffic the position, speed, acceleration as well as time are treated as discrete variable and the lane is represented by a one-dimensional lattice, each site represents a cell which can be either empty or occupied by at most one vehicle at given instant of time. The computational efficiency of CA is the main advantage of this approach. Much efforts were concentrated on stochastic CA models of traffic flow first proposed by Nagel and Schreckenberg [7] and subsequently studied by other authors using a variety of techniques[8,9]. The stochastic dynamics of interacting particles have been studied in the mathematical and physical literature[10]. In the physical case, driven lattice gases with hard core repulsion provide models for diffusion of particles through narrow pores and for hopping conductivity[11], and belong to the general class of non-equilibrium models which includes driven diffusing systems[12,13]. They are closely linked to growth process[14,15], and can also be formulated as traffic jam or queuing problems[16]. The fully asymmetric exclusion model AEM which corresponds to the case where particles hop only in one direction, can be divided into four classes according to the dynamics(sequential or parallel) and the choice of boundary conditions(open or periodic). In the sequential dynamics each particle has a probability Δt(time intervale) of jumping to its right-hand neighbour if this neighbouring site is empty. This model has been solved exactly in one dimension with open boundaries conditions[17,18]. Several phase transitions were found, and the exact results are illustrated for continuous time dynamics(i.e. infinitesimal Δt), where only one particle can move during Δt. The parallel update of the asymmetric exclusion model can be introduced to obtain the phase diagram using Monte-Carlo simulations Δt. Recently Zia and al [22] studied the protein synthesis using Totally asymmetric exclusion process (TASEP) with extended objects i.e system with particles of length l>1. Using numerical and analytical tools for random sequential updating Zia and al generalized the well studied l=1 TASEP model to particles with extended lengths l>1. The aim of this paper is to study the effect of mixture lenghts l=1 and l=2 of vehicles on the AEM in the case of parallel dynamics(i.e. Δt =1) in both open and periodic boundaries conditions by introducing two types of vehicles and using Monte-Carlo simulation. Our goal is to extend the AEM in order to address the issue of mixture lengths observed in real traffic. This paper is organized as follow, section II is devoted to explain the three models i.e. open boundaries with two entries, open boundaries with one entry and periodic boundaries. In the third section we present the main results obtained in these models with a critical discussions. The last section is devoted to a conclusion.

II. Definition of the models

As we said in the introduction, the main idea of this work is to study the influence of mixture lengths of vehicles on density, current and phase diagram in the AEM in both open and periodic boundaries for Δt=1 i.e. in the case of parallel dynamics . The choice of the AEM is motivated by the simplicity of this model which makes possible to investigate only the effect of mixture lengths on the traffic flow without incorporating other parameters. The vehicles are filed from their length, so throughout this paper type1 means short cars which occupy one site l=1 and type2 the long vehicles occupy two l=2. Both type1 and type2 moves with the same speed Vmax=1. FIG.1. display an example of configurations obtained after three steps for system size L=8.

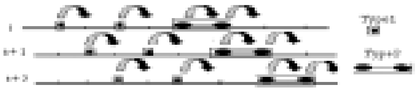

Fig. 1.: Example of configurations obtained after three steps for system size L=8 where both type1 and type2 moves with Vmax=1.

II.1.Open boundaries with two entries

The aim of this choice is to explore the phase diagram(α_1, α_2) for fixed value of the extracting rate β. In this case, type1 are injected in the first site (first entry) with the injecting rate α_1, and type2 with the injecting rate α_2 in the third site (second entry). In order to avoid the on-ramp, both second and third sites must be empty to inject type2, and the distance between the two entries should be at least equal to Vmax=1. Generally one should respect the following rules:

$(I_{E2}-I_{E1})-1 \geq Vmax$.

I_{E2} and $I_{E2}-1$ must be free.

where I_{E2} and I_{E1} are respectively the positions of the first and second entries.

II.2.Open boundaries with one entry

For both practical and theoretical reasons, some times different boundary conditions are used. Imagine a situation where a multilane road containing different kind of vehicles, is reduced to one lane due to road construction. Such situation can be modeled by using our model ie the AEM with two types of vehicles where:

The multilane part of the road acts as a particles reservoir (both types).

The one part lane is represented by a chain with N sites.

α is the injecting rate of cars in the first site.

$\alpha_2= n\alpha$ is the probability that this car is a vehicle type2.
α_1 is the probability that this car is a vehicle type1.
$\alpha_2 \leq \alpha_1 \leq \alpha$.
n the concentration of type2. We assume that type2 could be liken to truck and type1 to cars. Since in the road we have a few type2 in comparison to type1, we take $0 \leq n \leq 0.5$.
The process of the insertion of the vehicles is as follow:
If the first cell is empty a random number R is chosen.
If $R \leq \alpha$ a car is inserted here, but the nature of the vehicle depend on this random number.
If $\alpha_2 \leq R \leq \alpha$ type1 enter the road.
If $R \leq \alpha_2$ type2 enter the road.

II.3.Periodic boundaries.

The implementation of the periodic boundaries means that vehicles which leave the chain re-enter in the opposite side. The periodic boundary conditions for the AEM with one species of particles on a ring has been studied exactly by Schadschneider and Schreckenberg[20]. Here we study the AEM with two types of vehicles for periodic boundaries. To illustrate this situation let us consider a ring of N sites, where:
C_1 is the density of sites occupied by type1 which is the density of type1.
C_2 is the density of sites occupied by type2.
$C= C_1+ C_2$ is the density of sites occupied by both vehicles.
$n= C_2 /C$ is the concentration of sites occupied by type2.
In order to reduce the space parameters (C_1, C_2) we choose $C_1=(1-n)C$ and $C_2=nC$. The main idea of studying this case is to establish the phase diagram (I,C) for different values of n. Thus to measure the local current I_i, N detectors are put at each site. here I is the mean value of the current in the steady state i.e.:

III. Simulations and results.

In our computational studies we have considered a chain with N=1000 sites. Simulation of closed system begin with particles randomly distributed around the ring depending on their densities C_1 and C_2, whereas the open system begin with small number of vehicles evenly distributed in the chain. The systems run for 20 000 MCS to ensure that steady state is reached for periodic case and 40 000 MCS for open systems, at this moment data including the current and density are collected. In order to eliminate the fluctuations 25 initial configurations were randomly chosen.

III.1.Open boundaries with two entries.

Open boundary conditions means that vehicles are injected at one end and are removed at the opposite end. The AEM in the case of parallel dynamics(i.e. $\Delta t=1$)with open boundary conditions exhibits a first order transitions at $\alpha= \beta$ where(α,β) are the injecting and extracting rates respectively[21]. We recall that our aim in this case is to explore the phase diagram (α_1, α_2) for a fixed values of the extracting rate β.
FIG.2. shows the profile of the global density ρ (which is the density of all sites occupied by both types of vehicles) versus α_1 for β =0.3 and various injecting rates α_2.
We distinguish two regions:
The first region is when $(\alpha_1+\alpha_2) < \beta$ in which ρ increase with α_1, but this increasing depend on the value of α_2. Indeed:
$\rho (\alpha_1, \alpha_2=0) < \rho (\alpha_1, \alpha_2=0.1) < \rho (\alpha_1, \alpha_2=0.2)$.
In the other hand this region correspond to the low density phase due to the low values of ρ.
The second region is when $(\alpha_1 +\alpha_2) > \beta$ in which ρ become constant(about 0.77) and does not depend on α_1 and α_2. This is the high density phase.
At $(\alpha_1+\alpha_2)_c=\beta$ the global density ρ is discontinuous and undergoes a jam between the two regions which is a characteristic of the first order transition.
We can see that the system undergoes the usual first order transition at $\alpha_1=\beta$ for $\alpha_2=0.0$ in agreement with[21]. Also we remark that the density takes its higher values when $\alpha_2 > \beta$ even for $\alpha_1=0.0$, this means that there is no low density phase in this case i.e. the system prefer to be in the high density phase. In fact Type1 and Type2 are not correlated in the beginning of the road due to the conditions mentioned in section II.1, this explain why the first order transition occurs at $(\alpha_1+\alpha_2)_c=\beta$. For $\alpha_2 > \beta$ this first order transition disappear due to the great proportion of Type2 in the road, indeed the number of type2 which enter the chain is greater than those which leave it, leading to high density phase independently of α_1.

FIG.2

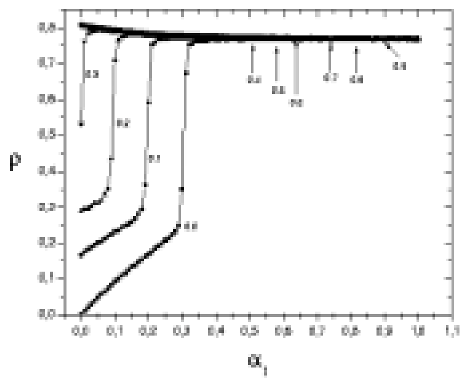

Fig. 2: The variation of the global density ρ versus α_1 for different values of α_2 and β=0.3 in the case of open boundaries with two entries, the number accompanying each curve denote the values of α_2.

Collecting all of these results we obtain the phase diagram (α_1, α_2) for β=0.3 in FIG.3. The same results can be obtained for others values of β.

FIG.3

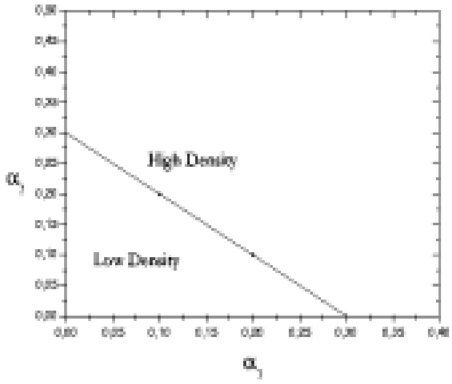

Fig. 3: The phase diagram (α_1, α_2) for β=0.3 in the case of open boundaries with two entries.

III.2.Open boundaries with one entries.

In this case the type1 and type2 particles enter the chain in the same entry. As we said in the above section The AEM with open boundary conditions and parallel dynamics exhibits a first order transitions[21], in which ρ is discontinuous and undergoes a gap between the low and high density phases at α_c=β. In our model the gap decrease when increasing α and vanish for its higher values. FIG.4. shows the profile of the global density ρ versus α for α_2=nα and various extracting rates β, here n=1/4 the concentration of type2.

FIG.4

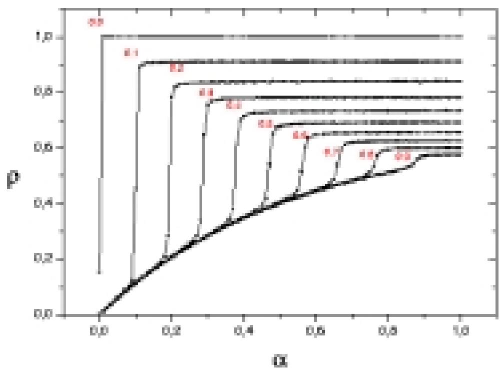

Fig. 4: The variation of the global density ρ as a function of α with α_2=1/4 α and for different values of β in the case of open boundaries with one entry, the number accompanying each curve denote the values of β.

We remark that for β=0.0-0.3 the first order transitions occurs at α_c=β in agreement with [21] and at α_c<β when β=0.4-0.7.

For β=0.8-0.9 no phase transitions is observed.

In fact, in this case about 1/4 of vehicles which enter the road could be type2, and their number become important as long as α increase, we recall that type2 occupy two cells due to their length so it is normal in this situation that the global density in road become larger than the case when the only type of vehicles is type1 as in FIG.5., where we show that the gap(n=1/4) is smaller than the gap(n=0.0) and the transition occurs at α_c<β (n=1/4) instead of α_c =β (n=0.0).

FIG.5

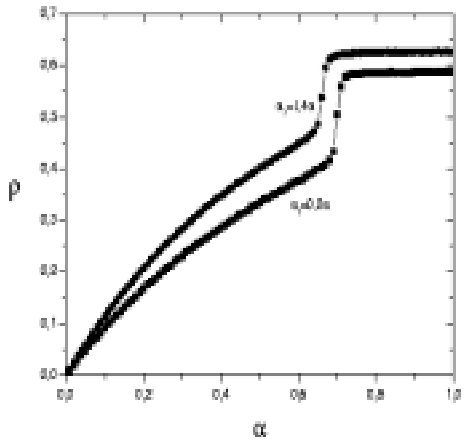

Fig. 5: The variation of the global density ρ as a function of α with α_2 =1/4 α (circles dot) and α_2 =0.0α (square dot) for β =0.7 in the case of open boundaries with one entry.

In the other hand these remarks are not observed when β =0.1-0.3, because the system reach the high density phase for small values of α =0.1-0.3, which means small probability for type2 to enter the road thus the system keep the transition at α_c =β. For β =0.8-0.9 the gap vanish and we have a continuous passage from low to high density phase. FIG.6 display the phase diagram (α,β) for three values of n. we show that the line of first order transitions which separate the low and high density phases in the case of n=0.0 [21] is modified when n=0.25-0.5. For n=0.25 the pseudo line ended at the end point (α =0.7, β =0.74) above which the first order transitions disappear, and for n=0.5 the corresponding end point is (α =0.5, β =0.57).

III.3.Periodic boundaries.

The periodic boundary conditions for the AEM with one species of particles (Type1) on a ring has been studied exactly by Schadschneider and Schreckenberg[20], where the relation between current and density is given as follow:

$$I(C,P)=1/2[1-\sqrt{1-4qC(1-C)}]$$

p is the braking probability and q=1-p is the hopping rate.

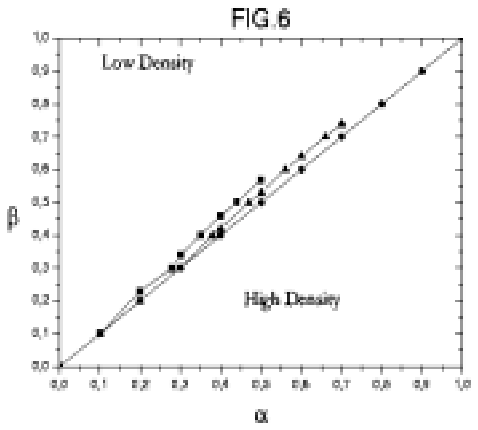

Fig. 6: Phase diagram in the (α , β) plane for several values of n(concentration of type2). Square and up triangle dots correspond respectively to $\alpha_2 = 0.5\alpha$ and $\alpha_2 = 0.25\alpha$ where the pseudo lines of the first order transitions ended at the end points (0.5,0.57) and (0.7,0.74). Circle dots recall the case $\alpha_2 = 0.0\alpha$.

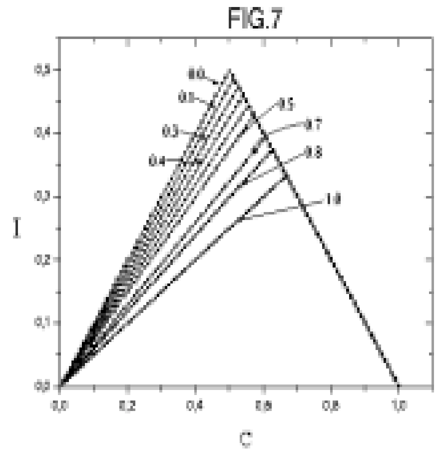

Fig. 7: The fundamental phase diagram (I,C) for several values of n the concentration of type2 the number accompanying each curves denote the values of n.

In the case of p=0 the above expression becomes: $I(C)=1/2[1-\sqrt{1-4C(1-C)}]$ which correspond to our model when n=0 (concentration of type2) i.e only type1 are in the ring. An interesting feature of this expression is that the current is invariant under the operation $C \rightarrow (1-C)$ which interchanges particles and holes. Therefore the fundamental diagram is symmetric about $C_m=1/2$. This symmetry is conserved for all p, but breaks down for fixed value of p and Vmax>1 where the magnitude of C_m decreases and the current increases with increasing Vmax.

In our model, introducing small concentration of type2 leads to a brisure of the symmetry holes particles and the expression of the current I(C) mentioned above doesn't work. **FIG.7.** shows the fundamental diagram (I,C) for different concentration n, we remark that the magnitude of C_m increases and the current decreases continuously when n increases. We can say that so long as C is sufficiently small the vehicles are too far apart to interact mutually, therefore the current increases with C but this increasing is controlled by the concentration n. Indeed Type2 takes lot of time in passing the locals detectors due to their length in contrast to Type1, which reduce the current and shift to the right C_m because the system need a great amount of type2 i.e. high C in order to reach I_m.

In the special case n=1, with random sequential dynamic Zia et al [22] found that the current I isn't symmetric about $C_m=1/2$. Instead, the density $C_m=1/2$ increases from 1/2 to $C_m=0.585$ while the maximal current is lowered from 1/4 to $I_m=0.1715$. With parallel dynamic we find that $C_m=0.669$ and $I_m=0.332$ as in FIG.7.

Conclusion

We have studied the effect of mixture lengths of vehicles on the traffic flow using numerical simulation. For this purpose we have defined two types of vehicles depending on their length, type1 the small cars which occupy one cell l=1, and type2 the long ones takes two l=2. In our investigation we focused on the AEM in parallel dynamic for different kind of boundaries. Thus three varieties of models were presented. The first model correspond to a chain with open boundaries where each types of vehicles enter the road in different positions with the rates α_1, α_2. In this case the global density ρ is discontinuous and undergoes a jam between the low and high density phases which is a characteristic of the first order transition at $(\alpha_1+\alpha_2)_c=\beta$. For $\alpha_2 > \beta$ the system is in the high density phase and no phase transitions is observed. The second model treats the case of open boundaries where both types enter the chain in the same entry. The concentration n of type2 play a major role. Indeed the gap between low and high density decrease when increasing α and vanish for its higher values, and the phase diagram established for three values of n show that the line of the first order transition in the case of n=0.0 [21] is modified when n=0.25-0.5. In these cases the pseudo lines ended at (α=0.7, β=0.74) for n=0.25 and at (α=0.5, β=0.57) for n=0.5 which means that the first order transitions disappear above these end points. The third model correspond to the periodic boundaries where Type2 are present in the chain with a fixed concentration n. The introduction of small concentration of type2 leads to a brisure of the symmetry holes particles and the expression of the current I(c) mentioned in [20] doesn't work, the magnitude of C_m increases and the current decreases continuously when n increases.

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