The growth dynamics of the wedding-cake Interfaces.

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In the limit where the ratio of the surfaces diffusion coefficient to the deposition rate $D/F \longrightarrow \infty$, the surface consists of wedding-cake structures. In order to understand the growth dynamics and the scaling properties of theses interfaces, we have calculated the time evolution of its width $\omega(L,t)$ for both one and two dimensional lattice. By the use of the dynamic scaling approach, we find that $\omega(L,t)$ scales with time t and length L as $\omega(L,t) \approx L^{\alpha} f(t/L^{\alpha/\beta})$ where f is a scaling function and α and β are respectively the roughening and the growth exponents. The values of theses exponents are in good agreement with the theoritical ones predicted by the Edwards-Wilkinson equation.

PACS: 05,50.+q-05.60.cd-68.35.Fx

1- INTRODUCTION

Recently there has been a vivid interest in understanding the dynamics of rough interfaces wich can be present in various physical phenomena (Crystal growth, magnetic domain, thin films corrosion,..) [1-9]. When the interfaces grow and roughen due to thermal fluctuations, the origin of the randomness come from the random nature of the deposition and diffusion process. In the limit where the ratio of the surface diffusion coefficient to the deposition rate $D/F \longrightarrow \infty$, the interface consists of wedding-cake structures [10-11]. Microscopically, it originates from an energy barrier rate step edges, which prevents atoms from descending from the atomic layer on which they have been deposited [10]. As a consequence the concentration of adsorbed atoms on top of two-dimensional islands is increased. Such that second-layer nucleation occurs well before the first-layer has been completed. This process repeats itself in subsequent layers giving rise to a wedding-cake like structure of islands on top of islands. One of the effective tool used to study various roughening processes is the dynamic scaling approach. The most important quantities used to characterize the scaling of the interface is the global width parameter $\omega(L,t)$ defined by :

$$\omega(L,t) = \sqrt{\frac{1}{L}\sum_{i=1}^{L}\left[h(i,t) - \bar{h}(t)\right]^2} \qquad (1)$$

where $h(i,t)$ represents the height of interface at site i and time t, $\bar{h}(t)$ is the mean height of interface at time t. L is the system size. It has been shown [5] that the interface width $\omega(L,t)$ scales with time t and system size L as :

$$\omega(L,t) \sim L^{\alpha} f(t/L^{\alpha/\beta}) \qquad (2)$$

where f is a scaling function hence defined by:

$$f(x) \sim \begin{cases} x^{\beta} & if \;\; x >> 1 \\ cst & if \;\; x << 1 \end{cases} \qquad (3)$$

The exponents α and β are respectively called the roughness and the growth exponents.

The Edwards-Wilkinson equation [12], based on Langevin type equation, was the first continuum equation used to study the growth of interface by particles deposition. It has the following form:

$$\frac{\partial h(t)}{\partial t} = \upsilon \nabla^2 h + \eta(x,t) \qquad (4)$$

where $h(t)$ represents the interface height at time t, υ is called surface tension and $\eta(x,t)$ is the noise term. The resolution of this equation gives the exact values of the scaling exponents:

$$\alpha = \frac{3-d}{2}\,;\; \beta = \frac{3-d}{2}\;;\; z = \frac{\alpha}{\beta} = 2 \qquad (5)$$

where d is the dimension of the system and z represents the dynamical exponent.

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In this paper, we have studied the dynamics growth of wedding-cake structures by calculating the roughness and the growth exponents for both one and two dimensional lattice. The obtained results are compared to the ones predicted by the Edwards-Wilkinson equation. We have also verified the universality of the exponents α and β by introducing several geometries for two dimensional lattice.

2- THE MODEL

We consider a integer lattice of sites i with an integer height variable $h\ (i,t)$ defining the position of the surface above i. At time zero, the atoms are randomly deposited with a constant flux F. Atoms arrive on the surface, hop to neighboring sites, provided they remain within the same layer (the deposition and the diffusion processes characterized by the diffusion constant D, take place simultaneously). However, the atoms with more than one neighbour bond (for one dimensional) and more than three neighbour bonds (for two dimensional) are immobile.
The system coverage is given by the following relation:

$$\theta = F.t \qquad (6)$$

The only physical parameter is the ratio D/F. Here we consider the case $D/F >> 1$ where the long-ranged lateral correlation can be observed [10] and the interface consists of wedding-cake-like structures (see figure *1*). Initially, the deposited atom diffuse until it meets another one, they form then an immobile cluster. Similary, if an atom meets an island, it sticks to its edge, and becomes immobile. If an atom arrives on the top of an existing island, it will continue diffusing, eventually falling down if it reaches the edge of the island. Scaling arguments [13-14] and simulations [10, 13, 15, 16] show that the spacing l between first-layer islands for one dimension is of the order

$$l \sim (D/F)^{1/4} \qquad (7)$$

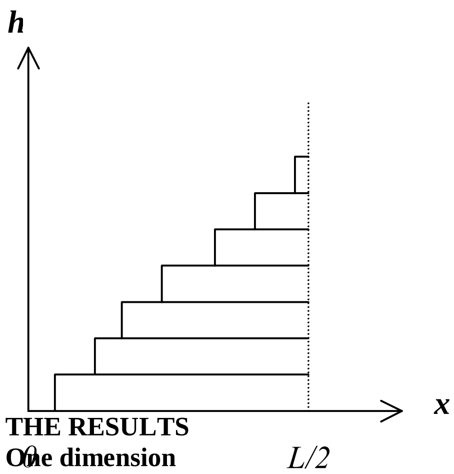

Figure *1* : the geometry of half wedding cake

3- THE RESULTS

a- One dimension

In this case, we have fixed the ratio D/F at 10^8, and we have calculated the interface width $\omega(L,t)$ for several system size L. The results are reported in figure *2*. We observe that the interface width $\omega(L,t)$ presents a two different regimes separated by a cross over time t_x. In the first regime (growth regime $t<<t_x$) the interface width $\omega(L,t)$ increases with the exponent $\beta = 0.50 \pm 0.02$ ($\omega(L,t) \sim t^{\beta}$) (see figure *3*). In the second regime (saturation regime $t>>t_x$), the saturated width $\omega_{sat}(L)$ is sketched as a function of the system size in figure *4*. The slope of the log-log plot leads to an exponent $\alpha = 0.96 \pm 0.01$ ($\omega_{sat}(L) \sim L^{\alpha}$). Theses exponents α and β are very close to the ones predicted by the Edwards-Wilkinson equation [12].

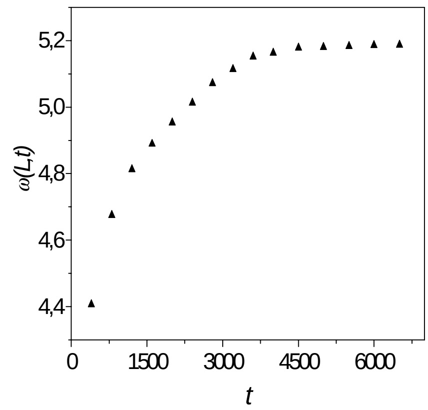

Figure *2*: the temporal variation of the wedding cake interface width for system size $L=80$

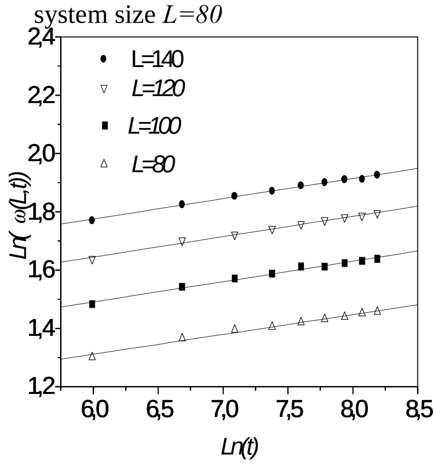

Figure *3*: The wedding cake interface width versus time t, on log-log plot for various system sizes

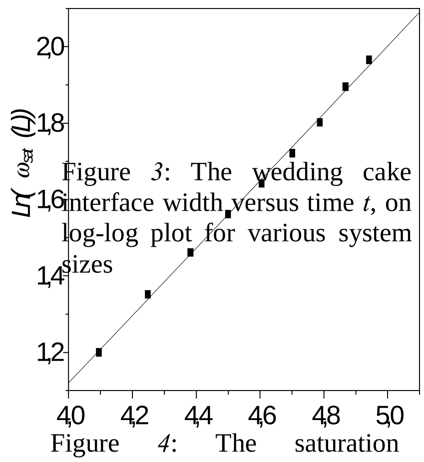

Figure *4*: The saturation wedding cake interface width versus system size on log-log

The wedding-cakes interface generated in this case presents a fractal geometry. In order to calculate its fractal dimension d_f, we have considered the mass-radios relation ($M \sim R^{df}$). We obtain the fractal dimension $d_f = 0.76 \pm 0.02$

geometry	α	β
Square	0.51 ± 0.02	0.25 ± 0.05
Triangular	0.49 ± 0.01	0.24 ± 0.01
Hexagonal	0.52 ± 0.02	0.24 ± 0.01

Table*1*: the values of the exponents α and β for a two dimensional system with three different geometry. We observe that the exponents α and β are respectively near *1/2* and *1/4* as predicted by the EW equation.

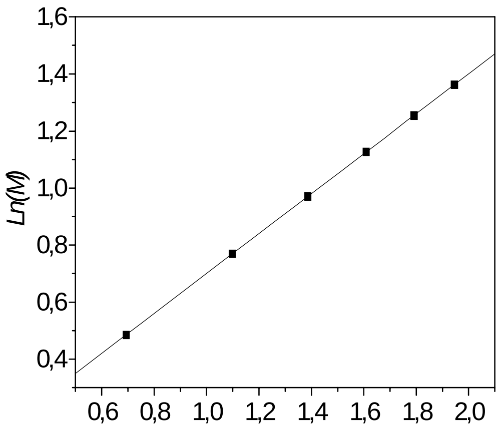

Figure *5*: The mass *M* versus radius *R* on log-log plot.

b- Two dimensions

The extension of the minimal growth model to two dimensions is straight forward in principle. However, in order to check the effect of the system dimension, we have choosen a square lattice and we have calculated the time evolution of the interface width. The simulation results are reported in figures *6* and *7*. This leads to a growth exponent value $\beta = 0.25 \pm 0.05$ and a the one of roughness exponent $\alpha = 0.51 \pm 0.02$, consistent with the critical exponents predicted by the Edwards-Wilkinson equation [12]. The calculation of the fractal dimension of the wedding-cake interface is also done in this case, and we have obtained $d_f = 1.78 \pm 0.02$. In order to test the universality of this exponents α and β, we have studied the system of two dimensions and we have considered various geometries (square, triangular, hexagonal) the results of this exponents α and β are grouped in table *1*. We observe that theses exponents are not affected by the lattice geometry. Thus confirming their universality.

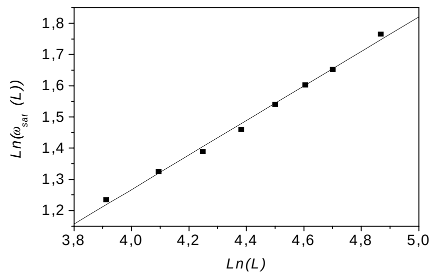

Figure*7*: The saturation wedding cake interface width versus system size on log-log plot.

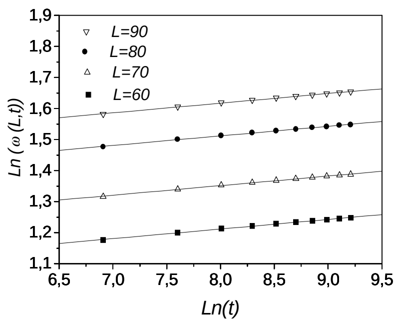

Figure 6: The wedding cake interface width versus time t, on log-log plot for various system sizes.

4- CONCLUSION

In summary we have displayed that the dynamic scaling of the wedding-cakes interface follows the Familly-Viscek law with an exponents α and β very close to the ones predicted by the Edwards-Wilkinson equation. We have also confirmed the universality of theses exponents by introducing a several lattice geometries. The wedding cake interface exhibite also a fractal aspect.

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