Wavelet phase evaluation extended to digital speckle pattern interferometry.

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A wavelet algorithm developed to improve metrology based on electronic speckle pattern intérferometry (ESPI) is presented. The wavelet algorithm is based on the calculation of the continuous wavelet transform of a modulated speckle correlation fringes .The Paul wavelet is used , the extraction of the maximum scales of the modulus of the wavelet transform leads simply to the phase gradient distribution. The advantage of the method is to provide phase distribution, with a high accuracy, from a single interferogram without unwrapping step.

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I. .INTRODUCTION

Digital Speckle Pattern Interferometry, DSPI, is a powerful 'non-contact' technique used to accurately measure static and dynamic displacements and deformations of surfaces down to sub micron scale[1]. DSPI has wide ranging areas of application in the 'full-field' studying deformations, vibrations, defects and damages assessment. This technique is based on speckle effect [2] and on coding speckle phase in interferograms. We obtain the displacement fields by digitally correlating the images of two speckle interferograms recorded before and after deformation. General principle of DSPI technique and a review of setups and applications are given in [3-4].

The speckle is first used as an information carrier leading to a fringe pattern, and then becomes a noise making it necessary to perform a filtering stage.

From the filtered correlation fringes, we propose a wavelet technique to extract the phase.

The main advantage of this non contact technique is that it enables a real-time correlation fringes to be displayed leading to a real-time phase extraction.

In this paper we extend the wavelet phase evaluation algorithm [5] to speckle correlation interferometry. The paper first describes the speckle effect and the DSPI technique. Finally, results of numerical simulations are presented.

II. SPECKLE EFFECT

When optically rough surfaces are illuminated with coherent light, the scattered beams have a random spatial variation of intensity known as the speckle effect. This 'speckle pattern' bears no obvious relationship to the illuminated object and appears chaotic and unordered and is best described by methods of probability and statistics. The speckle pattern formed in space is due to the self-interference of scattered waves from a number of scatterers and requires a coherent source for generation (Fig.1). The structure of the speckle pattern depends on the coherence properties of the illumination beam and on the surface characteristics of the diffuse object.

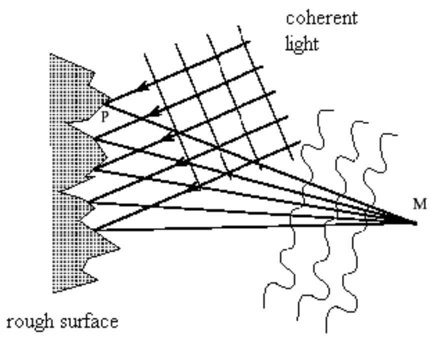


FIG.1. Random wave scattering by a rough surface

The probability distribution of the intensity is equivalent to the problem of the random walk . The electric field of the speckle pattern at a point in space may be regarded as the sum of the contributions from all the illuminated scattering elements of the rough surface .

$$E(x,y,z)=\sum_{k=1}^{N}|a_k|\exp(j\phi_k) \tag{1}$$

where N is the number of the scattering elements and a_k and ϕ_k are the amplitude and phase of the contribution due to the kth scatterer.

The Central Limit Theorem states that a random variable resulting from the sum of a large number of independent random variables results in a Gaussian distribution in the limit when the number approaches infinity. Provided that the number of scatterers is large, the application of the CLT results in the following conclusions :

a) the real and imaginary components of the field are independent, zero mean ,and identically distributed Gaussian variables .

b) the intensity I (x,y,z) has a negative exponential probability distribution.

$$p(I) = \frac{1}{\langle I \rangle} \exp\left(-\frac{I}{\langle I \rangle}\right) \quad (2)$$

The associated phase ϕ is uniformly distributed between ($-\pi$ to π).

$$p(\phi) = \frac{1}{2\pi} \quad (3)$$

In this paper we are concerned with the speckle pattern which is observed in the image of a rough surface produced by a finite-sized thin lens. A schematic diagram of the setup of such a system is shown in Fig. 2. The granularity in the image arises from interferences between closely spaced and randomly phased scatterers within the diffuser. The amplitude of the light at any point in the image plane will be the sum of the components scattered from an area in the object, that area being the area of resolution of the imaging system.

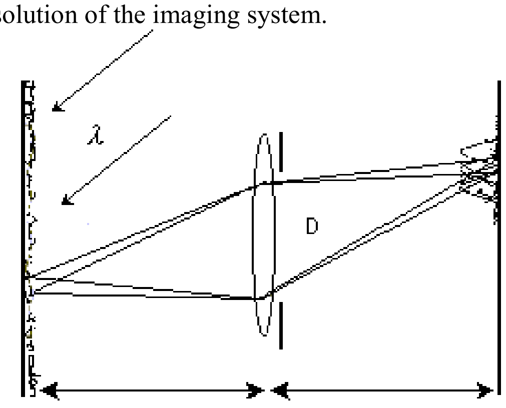


FIG.2. Schematic setup

Within the paraxial approximation, optical propagation through any complex optical system, described by an ABCD ray transfer matrix, can be formulated by Collins formulas [6]. Collins has obtained an analytic form for the resulting complex field amplitude.

Let $U(x_0,y_0)$ be the field on the input plane denoted by (x_0,y_0) located at z=0. The diffraction integral relating the fields across the input and output planes can be expressed as:

$$U(x,y) = \frac{j}{\lambda B} \exp(-jkL) \iint U(x_0,y_0) \exp[-\frac{jk}{2B}(D(x^2+y^2) - 2(xx_0+yy_0) + A(x_0^2+y_0^2)]\, dx_0 dy_0 \quad (4)$$

where (x,y) are transverse coordinates of the output plane, k is the optical wave number, λ is the free-space wavelength, L is the optical distance along the z axis, and A, B, D [7] are the ray matrix elements for the complete optical system between input and output planes. When the input and output planes are in free space, its determinant is unity (i.e. AD-BC=1).

The phase is uniformly distributed between $+\pi$ and $-\pi$ and the speckle sizes are estimated from the auto correlation function [2]. A simple modification of the aperture size permit us to change the speckle size. Figures (3) and (4) show a speckle pattern and its intensity probability distribution.

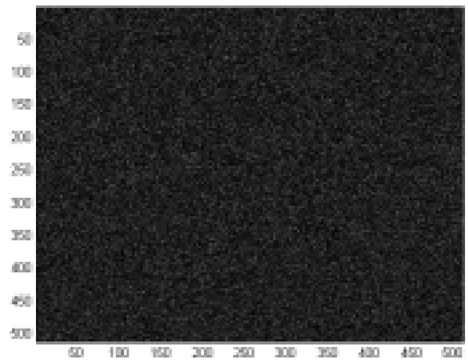


FIG.3.Speckle pattern for an average size of 3 pixels

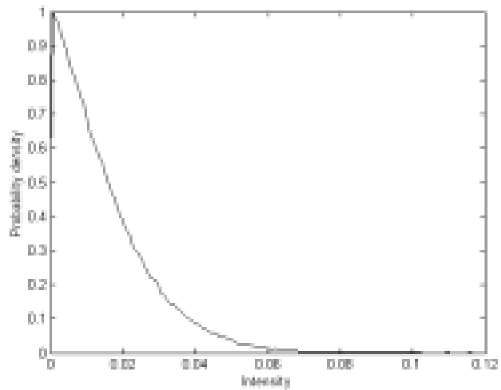


FIG.4. Probability density function of the Intensity.

III. DSPI TECHNIQUE

The basic principle DSPI technique is that the speckle pattern intensity distribution is a function of the relative phases of two interfering plane waves inside each resolution cell of an imaging setup. Displacement of the surface affects the intensity received in each speckle cell on the image.

The intensity distribution of a reference speckled image (before displacement)

$$I_1(x,y) = I_0(x,y)[1 + V(x,y)\cos(\phi_s(x,y))] \quad (5)$$

where I_0 is the bias intensity, V the visibility and ϕ_s is the original phase from the speckle that appears as the high frequency and apparently random pixel-by-pixel intensity variation.
After displacement, the intensity distribution becomes

$$I_2(x,y) = I_0(x,y)[1+V(x,y)\cos(\phi_s(x,y)+\varphi(x,y))] \quad (6)$$

where φ is the phase change in the light resulting from the displacement.We assume that the displacements are sufficiently small, that speckle decorrelation effects can be ignored.

The speckle correlation fringes are obtained by subtraction of a reference speckled image from images of displaced surfaces. The intensity distribution in the speckle correlogram is given by

$$I(x,y) = I_2 - I_1 = 2I_0 V \sin\left(\frac{\varphi}{2}\right)\sin\left(\phi_s + \frac{\varphi}{2}\right) \quad (7)$$

The desired information in the sin(φ/2) fringe term may be enhanced by appropriate filtering processes to remove the high frequency sin (ϕ_s + φ/2) noise.

IV. WAVELET PHASE EVALUATION TECHNIQUE

In this study, the phase mapping of speckle correlation fringes is obtained by our wavelet phase evaluation method [5]. From a modulated fringe pattern, the continuous wavelet transform is used to extract the localized spatial frequencies. The phase gradient is performed simply from the modulus of the wavelet transform by extracting the extremum scales. The phase distribution is obtained by integration.

The one-dimensional wavelet transform of the modulated correlation fringe pattern intensity, in the y direction, is given by

$$W(x,s,\xi) = \frac{1}{\sqrt{s}}\int_{-\infty}^{+\infty} I_0 V (\psi_{s,\xi}(y))^* dy - \frac{1}{\sqrt{s}}\int_{-\infty}^{+\infty} I_0 V \cos(my+\varphi(x,y))(\psi_{s,\xi}(y))^* dy \quad (8)$$

where my is the phase modulated carrier and $\psi^*_{s,\xi}$ (y) is the conjugate analyzing wavelet obtained for the shift ξ and the scale s.

Exploiting the wavelet localization property and assuming a slow variation of the intensity bias and the visibility, the wavelet transform becomes

$$W(x,s,\xi) = \frac{I_0(x,\xi)V(x,\xi)}{\sqrt{s}}\int_{-\infty}^{+\infty} \cos(my + \varphi(x,\xi) + (y-\xi)\frac{\partial\varphi}{\partial y}(x,\xi))(\psi_{s,\xi}(y))^* dy \quad (9)$$

By introducing Paul mother wavelet of order n [8] formulated by

$$\Psi(\mathrm{x}) = \frac{2^{\mathrm{n}}\ \mathrm{n}!\,(1-\mathrm{i\,x})^{-(\mathrm{n}+1)}}{2\pi\ \sqrt{(2\ \mathrm{n})!/2}} \quad (10)$$

and using Parseval identity, the modulus of the wavelet transform is equal to

$$\left|\mathrm{W}(\mathrm{x,s,\xi})\right| = \left(\frac{\mathrm{I_0(x,\xi)V(x,\xi)m_l^n}}{(2\mathrm{n})!}\right)\mathrm{s}^{\mathrm{n}+1/2}\exp(-\mathrm{sm_l}) \quad (11)$$

where m_l is the localized spatial frequencies given by

$$\mathrm{m_l} = \mathrm{m} + \frac{\partial\varphi}{\partial \mathrm{y}}(\mathrm{x},\xi) \quad (12)$$

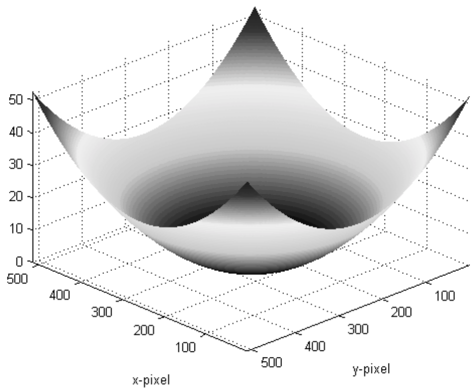
The phase gradient is performed simply from the modulus of the wavelet transform by extracting the extremum scales S

$$\frac{\partial\varphi}{\partial \mathrm{y}}(\mathrm{x},\xi) = \frac{2\ \mathrm{n}+1}{2\ \mathrm{S}(\mathrm{x},\xi)} - \mathrm{m} \quad (13)$$

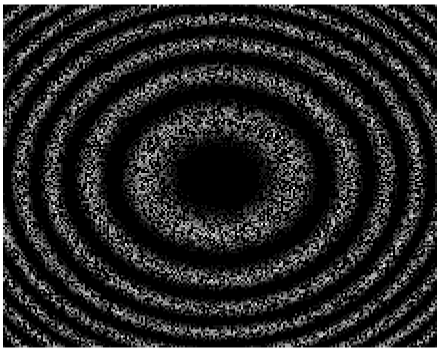
This leads to the phase by integration.
To avoid discontinuities and hence spurious, large values of the CWT at the edges of the data. We extend the fringe pattern at its left-and right-band edges, using zero padding method.

V. NUMERICAL SIMULATION RESULTS

The simulation consists in generating numerically speckle correlation fringes of a given phase change resulting from surface displacement. We illustrate Fig.5 (b) the simulated speckle correlation fringes obtained for the tested phase change presented Fig.5.(a).



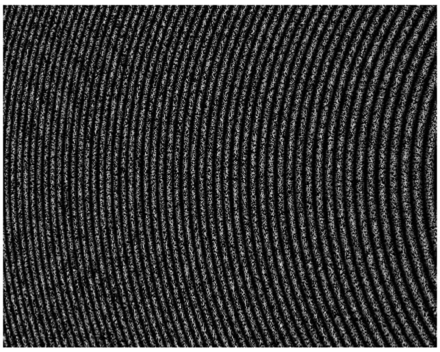
(a)



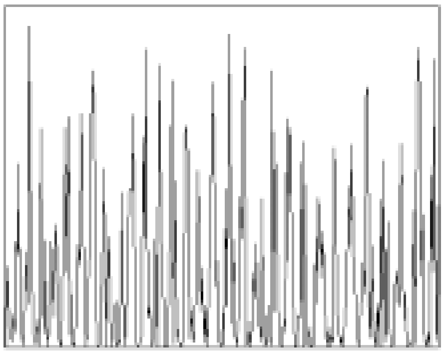
(b)

FIG.5 (a)Tested phase change;
(b) Correlation fringes

When the tested phase modulates a high frequency spatial carrier, we obtain the correlogram shown in Fig.6(a). Fig.6(b) represents a line intensity distribution in this correlogram.



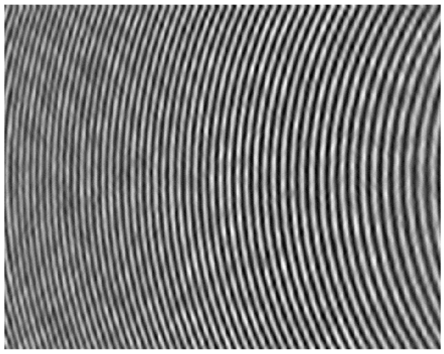
(a)



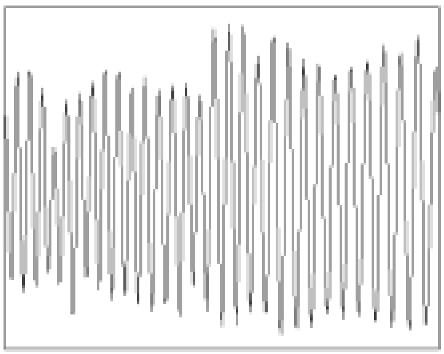
(b)

FIG.6 Noisily modulated correlation fringes

To perform the wavelet phase algorithm, the speckle noise must be removed from modulated correlation fringes by an adequate filtering process. In this study we have used the discrete stationary wavelet transform SWT [9] for reducing speckle noise and maintaining correlogram features effectively .The SWT performs a multilevel 2-D stationary wavelet decomposition. The filtered modulated correlation fringes image is shown in Fig.7.



(a)

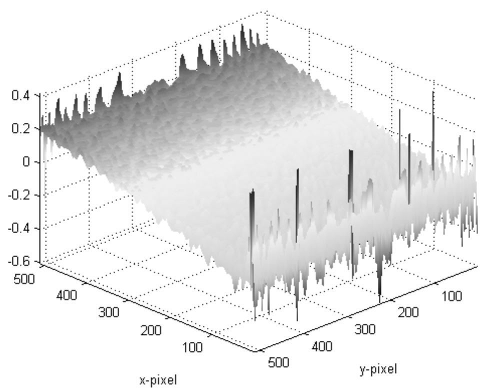


(b)

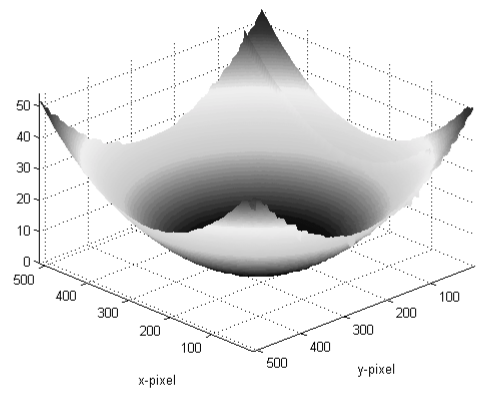
FIG.7 (a) Filtered modulated correlation fringes

(b) Line intensity distribution

Finally, simulation results are illustrated in Fig.8, where the retrieved phase gradient distribution is shown in Fig.8.(a) while the obtained phase distribution is presented Fig.8.(b).



(a)



(b)

FIG.8 (a) Phase gradient (b) phase distribution

A simple comparison of the exact proposed phase and its derivative with the obtained results confirms the ability of this technique to extract, with a high accuracy, the phase gradient and the phase distribution from noisily correlation fringes. We estimated that the accuracy of the wavelet phase evaluation technique was 0.12 rms.
The simulation study and the SWT de-noising are computed by MATLAB

5. CONCLUSION

In this study a simple method in order to simulate the speckle effect was presented and the wavelet phase evaluation method was extended to DSPI. The ability of this technique to extract, with a high accuracy, the phase gradient was demonstrated. The phase distribution is obtained simply by integration from a single correlogram, there is no need to perform phase unwrapping. We have established that the stationary wavelet transform SWT used for de-noising is appropriate to remove the speckle noise in correlation fringes.

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