Disordered Cellular automata traffic flow models

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In this paper, we extend the Nagel-Schreckenberg (NaSch) model by introducing disordered acceleration and deceleration terms. The disorder leads to segregated states where the flow is constant at intermediate densities for high values of breaking probability *p*. Within the model we present a density wave behavior appears below a critical value of *p*. Such result was found in car following models with an optimal velocity. The behavior of the gap distribution shows that the traffic exhibits a self organized criticality for high values of *p* and random deceleration.

PACS numbers: 05.60.+w, 45.70.Vn, 02.50.Ey

Introduction

Recently, traffic problems have attracted considerable attention, due to the fact that traffic behavior is important in our life. In recent years many non-equilibrium systems have been modeled [1,2] as systems of interacting particles driven far from equilibrium. A special class of such models includes, for example, car- following models, cellular automaton models, hydrodynamic models[3] and gaz kinetic models[4] . For a detailed study of all above approaches and theories we refer the reader to the review article ref. 5.

For their simplicities, cellular automaton (CA) models have been used to analyze traffic problems. The NaSch model and its slow to start variant[6] is the simplest one that reproduces the mean features of the real traffic. It is a stochastic CA model based on some pertinent rules. The model was intensively studied using both analytical and numerical methods[7] .

In this work we suggest and study numerically an extension of the NaSch[8,9] model that presents many features of real traffic. In our model, unlike in the NaSch model, the particles (cars) may accelerate or decelerate randomly more than one unit in single update step. We mention that such phenomenon were separately observed in different class of models. In sec II we define our model, while the numerical results are presented in sec III. In sec IV we conclude.

II. Model

We consider the cellular automata model introduced by NaSch to describe single-lane highway traffic. The model consists of a one-dimensional array of L cells with periodic boundary conditions. Every cell can either be empty or occupied by one car with velocity $v = 0,1,2,\ldots,v_{\max}$. Let g_n denotes the number of empty cells in front of a car . The CA rules consist of the following steps that are performed in parallel for all cars:

Step1: Acceleration.

If $v_n < v_{max}$, the velocity of the n-th car is increased by a_n sites, i.e.

$$v_n \rightarrow min(v_n + a_n, v_{max}) \tag{1}$$

where $a_n = [p_n q_n] + 1$.

Step2: Deceleration.

If $g_n \leq v_n$, the velocity of the n-th car is reduced to $g_n - 1$, i.e.

$$v_n \rightarrow min(v_n, g_n - 1) \tag{2}$$

Step3: Randomization.

If $v_n > 0$, the velocity of the n-th car is decreased randomly by d_n sites with probability p, i.e.

$$v_n \rightarrow max(0, v_n - d_n) \tag{3}$$

with the probability p
where $d_n = [q_n g_n] + 1$.

Step4: Vehicle movement.

Each car moves forward according its new velocity v_n obtained from the steps 1-3, i.e.

$$x_n = x_n + v_n \tag{4}$$

The symbol $[A]$ denotes the integer part of A and p_n , q_n are random variables distributed in the interval $[c,1]$ according to the distribution laws:

$$f(p) = \frac{n+1}{(1-c)^{n+1}}(p-c)^n \tag{5}$$

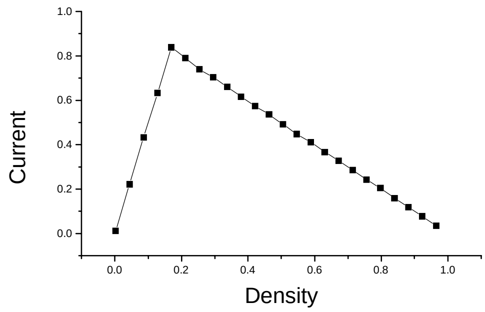
and

$$g(q) = \frac{n+1}{(1-c)^{n+1}}(1-q)^n \tag{6}$$

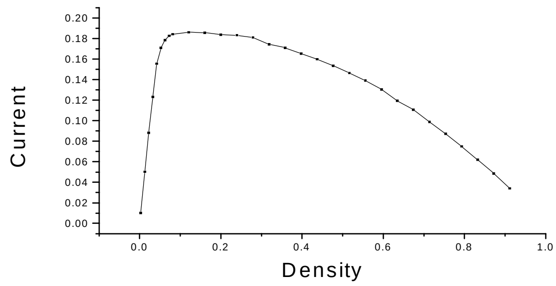
The choice of such distributions gives advantages to fast drivers.

III. Results and discussion.

The model defined above is investigated using numerical simulations for a ring of size L. Different varieties of the fundamental diagram are obtained depending on the model parameters. In this article we will restrict ourselves to the mean results of the model. In Fig.1 we present the fundamental diagram (j . ρ). For low values of p we observe that it has quite similar form to that of the NaSch model while for higher values of p we can distinguish three different phases depending on the density of particles. As for the NaSch model, the fundamental diagram exhibits high and low density phases. But at intermediate densities one observes a segregated phase where the flow is constant. In addition it gets a slower



value than in the NaSch model.



(a)
(b)

Fig1: The fundamental diagram of the NaSh model with random acceleration and deceleration variables. (a): $p = 0$. (b):$p = 0.6$

So the distinction between the models should be presented at microscopic level. In Fig 2 we present a space-time diagram of the system. Each dot corresponds to a particle at a given time step.

One can distinguishes two macroscopic regions, free phase and jamming phase which is caused by the slowest cars. The evolution of the jamming phase strip in time is independent on the global densities in the segregated phase. Consequently, the average flow is constant. Such behavior was observed in the NaSch model with an external perturbation namely the single defect site and the on-and-off ramp models(9,11,12). The microscopic structure of the states in systems with single defect site and ramps is qualitatively similar to the one obtained in our disordered model. Thus, the segregated phase may be generated in traffic flow by the intrinsic dynamics of the system without introducing any external parameter.

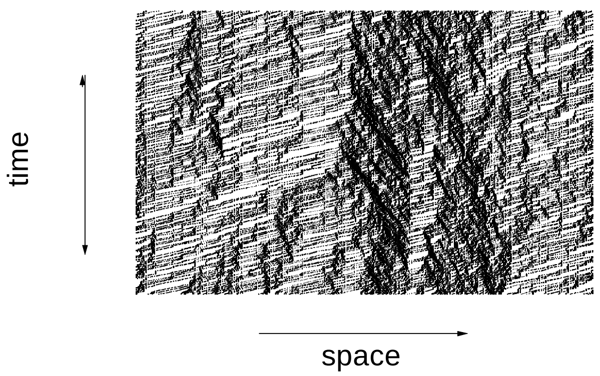


Fig2: space-time diagram for p = 0.6 at density ρ =0.25 showing a segregated phase.

At vanishing values of the randomization parameter p our model exhibits a density wave behavior that was never detected neither in the NaSch model or other CA variant models. Fig3. presents the density wave appearing in the congested phase. When car density is high in highway, traffic jams occur and propagate as density waves. The typical density wave has the kink-antikink form. We establish that the oscillatory traffic is qualitatively similar to that found in the car-following models(10) and we can explain these oscillations using the velocity v_{shock} of the 'domain wall' between the two stationary low and high density regions, of densities respectively ρ^- and ρ^+, obtained from mass conservation law:

$$v_{shock} = \frac{j(\rho_+) - j(\rho_-)}{\rho_+ - \rho_-}$$

Depending on the sign of v_{shock}, the shock wave may propagate either in the same or the opposite direction of the vehicles.

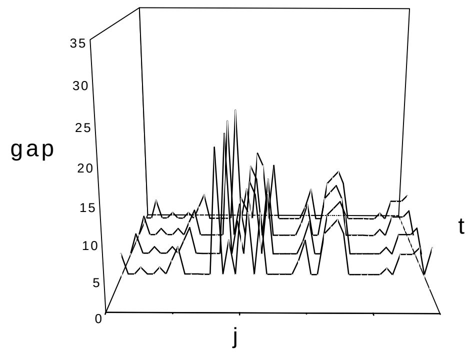


Fig 3: The gap distribution vs time, t, for p = 0 in the case of careless drivers, i.e q_n=0 for all vehicles.

Indeed, If the flux of entering cars is more (less) important than the leaving ones ; i.e. $j^+ > j^-$ ($j^+ > j^-$), $v_{\text{shock}} > 0$ ($v_{\text{shock}} < 0$) and the shock wave propagates in the same (opposite) direction of the vehicles. Thus, the domain wall oscillates in time.

Features of self organization in traffic flow were detected in empirical investigations(13)

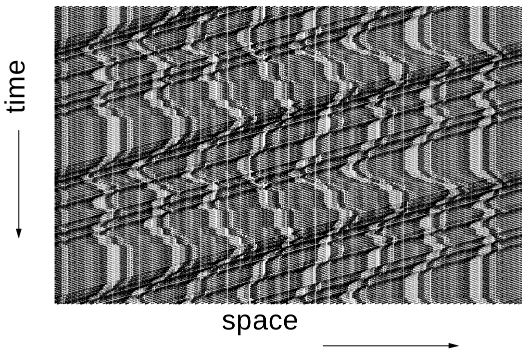


Fig4 : Space-time diagram for p =0 showing a density wave behavior.

There is a pinch region characterized by self-formation of small narrow jams that may merge leading to wide moving jams. This transformation determines a scale in distances between stop-and-go patterns. By drawing the gap distribution in the case of random accelerations, for some values of randomization parameter p and at low densities, we remark that our model exhibits a power law distribution which is a sign of self organized criticality (SOC). The finite size analysis collapses of the form.

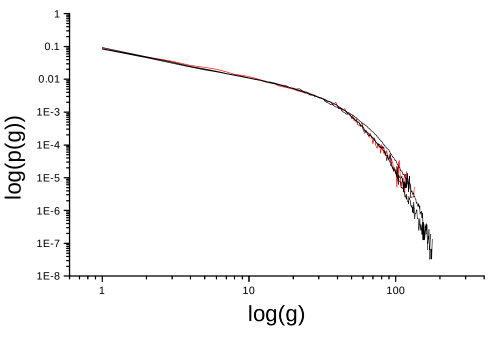


Fig 5: Distribution of gap for four different system sizes L ranging from 430 to 3440, for random distributed deceleration.

$$P(g)= g^{-\tau} f(g/L^{\nu})$$

The critical exponents τ and ν vary continuously with the randomization parameter p and the maximum velocity $v_{\max}$.

Conclusion

The investigation of the effect of random acceleration and deceleration on the traffic flow has shown that this kind of disorder can change the macroscopic proprieties of the basic NaSch traffic flow model.

Our disordered traffic flow model produces many features of real traffic. Namely the appearance of a segregated phase at intermediate densities where the flow gets a constant value within the intrinsic dynamics of the traffic flow and without any external perturbation. We note that such behavior was observed in the standard NaSch model by introducing a defect site [9,11] or an on and off- ramps[12]. The model we suggest shows that the CA traffic flow approach may reproduce some features that were observed in some continuous model as the optimal velocity model. Indeed, our model exhibits a density wave behavior in the case of fast drivers, i.e $p \to 0$. Finally, in the case of random deceleration at critical density and for high values of randomization parameter p the gap distribution of our model exhibits a power law behavior which is a hall mark of the self organized criticality phenomena. We think that the model we suggest may be developed to include the whole features and peculiarities observed in real traffic flow and then leads to a nice theoretical explanations.

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