The Optimal Velocity Traffic Flow Models With Open Boundary

H. Ez-Zahraouy*, Z. Benrihane, A. Benyoussef
Laboratoire de Magnétisme et de Physique des Hautes Energies
Departement de physique, Faculté des Sciences,
B.P.1014, Rabat, Morocco

The effects of the open boundaries on the dynamical behavior of the optimal velocity traffic flow models with a delay time τ allowing the car to reach its optimal velocity is studied using numerical simulations. The particles could enter the chain with a given injecting rate probability α, and could leave the system with a given extracting rate probability β. In the absence of the variation of the delay time $\Delta\tau$, it is found that the transition from unstable to metastable and from metastable to stable state occur under the effect of the probabilities rates α and β. However, for a fixed value of α, there exist a critical value of the extraction rate β_{c1} above which the wave density disappears and the metastable state appears and a critical value β_{c2} above which the metastable state disappears while the stable state appears. β_{c1} and β_{c2} depend on the values of α and the variation of the delay time $\Delta\tau$. Indeed β_{c1} and β_{c2} increase when increasing α and/or decreasing $\Delta\tau$. The flow of vehicles is calculated as a function of α, β and $\Delta\tau$ for a fixed value of τ. Phase diagrams in the (α,β) plane exhibits four different phases namely, unstable, metastable, stable. The transition line between stable phase and the unstable one is curved and it is of first order type. While the transition between stable (unstable) phase and the metastable phase are of second order type. The region of the metastable phase shrinks with increasing the variation of the delay time $\Delta\tau$ and disappears completely above a critical value $\Delta\tau_c$.

PACS number : 05.50.+q , 64.60.Cn , 75.30.Kz , 82.20.wt
Keywords: Traffic flow, Open boundary, Optimal velocity, Delay time, Numerical simulation.
*Corresponding author . E-mail address: ezahamid@fsr.ac.ma

I-Introduction

Recently, traffic problems have attracted considerable attention[1-3], due to the fact that traffic behavior is important in our lives. When car density increases, traffic jams appear. A variety of approaches have been applied to describe the collective properties of traffic flow: car-following models [4-9], cellular automaton models [10-14], gas kinetic models [15-17], and hydrodynamic models [18,19]. The traffic flow models are classified into the deterministic and stochastic models. Nagel and Schreckenberg [10] have introduced a stochastic cellular automaton model. It has been shown that the start-stop waves (traffic jams) appear in the congested traffic region as observed in real freeway traffic. Bando et al. [6] have proposed the deterministic optimal velocity model in which a car accelerates or decelerates according to the dynamic equation of car motion with the optimal velocity function. Traffic flow is a kind of many-body systems of strongly interaction cars. Recent studies reveal physical phenomena such as the non-equilibrium phase transitions and nonlinear waves [7-19] the jamming transitions between the freely

moving traffic and jammed traffic are very similar to the conventional phase transitions and critical phenomena: the freely moving and jammed traffic correspond to the gas and liquid phases, respectively. The traffic behavior has been studied by microscopic and macroscopic models [4,5].

In many works, the jamming transitions and the density waves have been investigated for the system of vehicles with the same characteristic. The car following models with the optimal velocity function are a favorable one of the microscopic traffic models and have been studied in great detail by the use of the numerical and analytical methods [7,8,9-20]. In these models, the optimal velocity function is the same for all the vehicles and the equations of motion of all the vehicles are given by the same differential equations. Generally, a vehicle has the characteristic different from the other. The degree of the acceleration or deceleration of a vehicle is different from each other. It will be excepted that the difference among the characteristics of vehicles has an important effect on the traffic flow. Thus , it will be necessary to include the different characteristics of vehicles into the car following models.

In modern traffic theory, it is well known that traffic jams occur in the high density region and propagate as kink-antikink density waves [6-9]. In the past, traffic jams have been treated as the soliton density wave [4,5] or triangular shock wave [26]. The nonlinear waves depend strongly on the traffic models [26]. Until now, it has been unclear as to whether or not the triangular shock wave occurs in modern traffic models. The car-following models with optimal velocity function are favorable among microscopic traffic models and have been studied in great detail using of the numerical and analytical methods [20-25]. Recently, Nagatani has studied the effect of the velocity fluctuations of the leader vehicle [22], the velocity-dependent sensitivity [18], the next-nearest-neighbor interaction [23]and the effects of the variation of the delay time $\Delta\tau$ to reach the optimal velocity in the periodic boundaries using numerical simulations[24]. In the latter case and depending on the value of $\Delta\tau$, Nagatani showed the existence of two regime namely, kink jam and uniform velocity free traffic.

Our main contribution in this paper is to study the effects of open boundaries (injecting and extracting rates) on the traffic flow behavior, average velocity and phase diagram of such a model [24] using numerical simulations. However, it is found that phase diagrams in the (α,β) plane exhibits three different phases namely, unstable, metastable, stable. The transition line between stable phase and unstable one is curved and it is of first order type. While the transitions between stable (unstable) phase and metastable phase are of second order one. Furthermore the region of the metastable phase, in the (α,β) plane shrinks with the increase of the variation of the delay time and disappears completely above a critical value $\Delta\tau_c$, that's depend on the values of injecting and extracting rates. The paper is organized as follows: the model is defined In section II. numerical simulation, results and discussion are presented in section III. Section IV is reserved to the conclusions.

II- Model

We consider a one-dimensional road of length L with open boundary conditions the particles are injected with a given rate probability α at one end of the road and are extracted with a given rate probability β at the opposite end. We present the car-following models with the optimal velocity function [5-10-12]. Newell [4] and Whitham [5] have analyzed the traffic model described by the following equation of motion of car j:

$$\frac{dx_j(t+\tau)}{dt}=V(\Delta x_j(t)) \qquad (1)$$

Where $x_j(t)$ is the position of the vehicle j at time t, $\Delta x_j(t)=x_{j+1}(t)-x_j(t)$ is the headway of vehicle j at time t, and τ is the delay time (how is allows for the time lag that it takes the car velocity to reach the optimal velocity $V(\Delta x_j(t))$ when the traffic flow is varying. $V(\Delta x_j(t))$ is the optimal velocity of vehicle j and is given by:

$$V(\Delta x_j(t))=\frac{V_{max}}{2}(\tanh(\Delta x_j(t)-h_c)+\tanh(h_c)) \qquad (2)$$

Where h_c is the safety distance and V_{max} is the maximal velocity of vehicle j when other vehicles do not exist. In the original car following models, the optimal velocity is the same for all the vehicles. Thus, the optimal velocity function of each vehicle is different from each other. Generally, it is necessary that the optimal velocity function has an upper bound(maximal velocity). Also, it is important that the optimal velocity function has the turning point. Otherwise, one cannot have the rebus density wave representing the traffic jams.

The idea of the above car following model is that a driver adjusts the vehicle velocity $\frac{dx_j(t+\tau)}{dt}$ according to the observed headway $\Delta x_j(t)$. The delay time τ allows for the time lag that it takes the velocity $\frac{dx_j(t)}{dt}$ of each vehicle to reach the optimal velocity $V(\Delta x_j(t))$ of each vehicle when the traffic flow is varying. By taylor-expanding, Eq.(1), one obtains the differential equation model [6]

$$\frac{dx_j^{2}(t)}{dt^{2}}=a\left(V(\Delta x_j(t))-\frac{dx_j(t)}{dt}\right) \qquad (3)$$

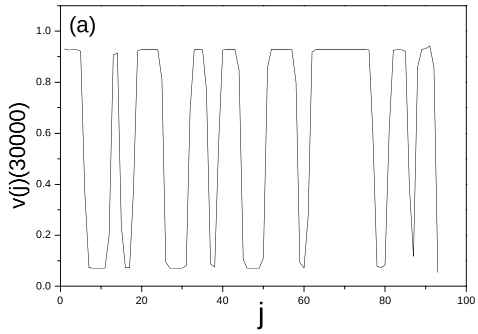
Where $a=1/\tau$ is the sensitivity of a driver Furthermore, by transforming the time derivative to the difference in Eq (1), one can obtain the difference equation model

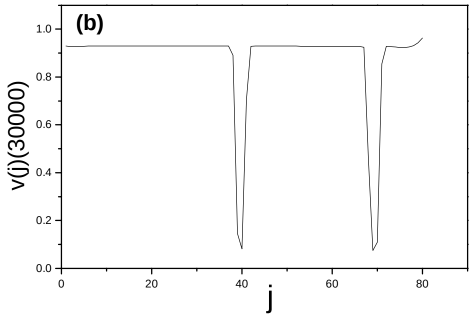
$$x_j(t+2\tau)=x_j(t+\tau)+\tau V(x_j(t)) \qquad (4)$$

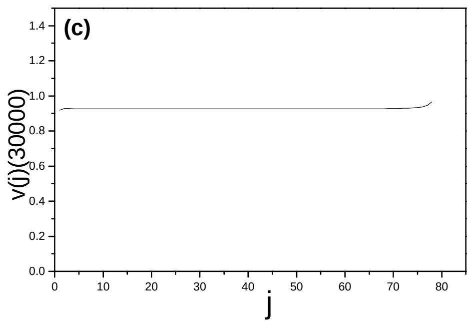
We consider such a case that the dimensionless delay time of vehicle j is uncorrected with other vehicles and is given by:

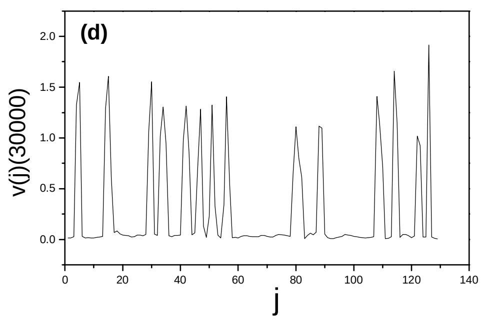
$$\tau_j =<\tau>+\Delta\tau\ [2rnd(j)-1.0] \qquad (5)$$

Where rnd(j) is the random number between zero and unity, $<\tau>$ is the average value of the dimensionless delay time, and $\Delta\tau$ is the strength of the variation of the dimensionless delay time.









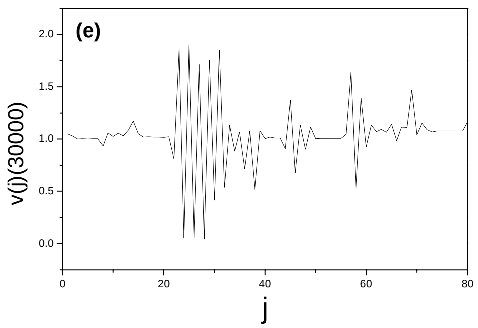


Fig. 1: Velocity profile obtained (a) β=0.2, (b) β=0.4, (c) β=1, and $<\tau>$=0.5, $\Delta\tau$ = 0.0 While (d) β=0.2, (e) β=0.4 and $<\tau>$ = 0.75, $\Delta\tau$ = 0.5 at t = 30000, h_c = 5.0, α=1, and L=500.

III-Simulation and results

In our simulation the rule described above is updated in parallel dynamics, i.e. during one updating procedure the new particle positions do not influence the rest and only previous positions have to be taken into account. The initial conditions are chosen as follows: $\Delta x_j(0) = \Delta x_0$ for all j, $\Delta x_j(1) = \Delta x_0$ for $j \neq 50,51$, $\Delta x_j(1) = \Delta x_0 -0.1$ for j = 50 and $\Delta x_j(1) = \Delta x_0 +0.1$ for j = 51, where the initial number of particles is N=100 and h_c=5. Cars are numbered as 1,2,3,....N, the injecting and extracting rate can be do as follows:

if the position of the first particle in the system is superior to the safety distance h_c and the rate injecting probability is superior to an random number between zero and unity, to this moment a new particle is added to the system and it is going to occupy the first position in the chain. In the same time step, if the position of the leading car in the system is superior to the difference between the size of the chain and the safety distance h_c ($L-h_c$) and the rate extracting probability is superior then an random number between zero and unity, however the leading car removed the system.

In order to study the effects of the open boundaries, in the absence of the variation of the dimensionless delay time $\Delta\tau$, on the traffic flow behavior, Figs.1(a)-1(c) show the velocity profiles obtained at t= 30000, τ=0.5, for fixed values of α and β. However, Fig.1(a) shows the kink density waves representing the traffic jams in the conventional car following model. It is clear that the vehicle velocities oscillate discontinuously between two different values , high and low velocities. Hence, the higher value correspond to the optimal velocity. The traffic flow appears to be irregular in the velocity profile and the density waves propagate upstream. However, there exist a critical value β_{c1} above which the kink jams become weaker, and the number of the peaks decrease and then the traffic flow becomes to be regular (Fig. 1.b), this situation is called metastable phase. Hence, in this case, we have a congested traffic with a single and/or a few density waves which propagates backward. When the value of β becomes larger than a critical value β_{c2} ($\beta_{c2} > \beta_{c1}$), the kink jams disappear and the velocity profile becomes uniform over all vehicles in the road, and then the traffic flow exhibits the uniform-velocity phase (Fig. 1.c). Figs : 1(d)-1(e) shows the combination effects of both open boundaries and finite value of $\Delta\tau$ =0.25 on the velocity profiles at t= 30000, τ=0.75 and for a fixed value of α and β. However, Fig. 1(d) shows the oscillatory congested traffic with the kink-antikink form [22]. In the same way when increasing the value of β, the kink-antikink form disappears and we show the oscillatory free traffic where vehicles oscillate around the optimal velocity [24] (Fig. 1.e). Even in absence of

dimensionless delay time, however, it is found that the effects of open boundaries permits to produce the same features in the traffic flow obtained by Nagatani [24] in the periodic conditions and in presence of dimensionless delay time.

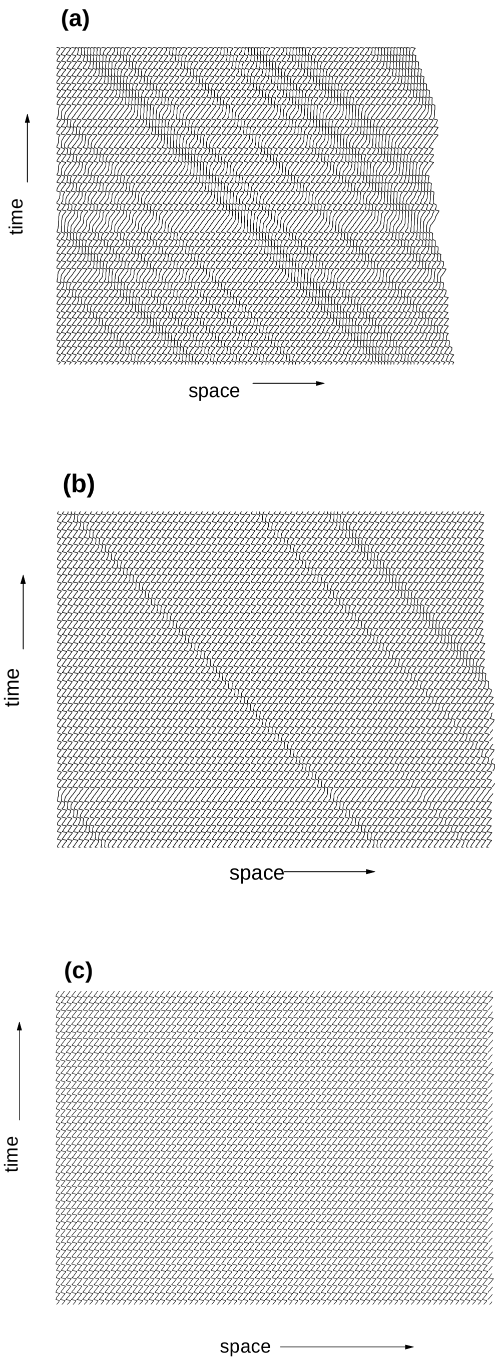


Fig. 2: Space-time plot for $<\tau> = 0.5$, $h_c = 5.0$, $\Delta\tau = 0.0$, $\alpha = 1$, and $L = 500$. (a) $\beta = 0.2$, (b) $\beta = 0.4$, (c) $\beta = 1$.

In order to examine the behavior of the flow density regime in the open boundaries, we study the space-time evolution for various values of extracting rate probability β. As a result, three types of traffic flow behavior are distinguished: (1) a coexisting phase in which the kink-antikink density wave appears (unstable state),(2)weaker density wave (metastable state), and (3) a freely moving phase (stable state). To illustrate these features, Figs 2(a)-2(c) shows the typical traffic patterns that is represented the different behavior (unstable, metastable, stable) for the traffic behavior induced by a fixed value of injecting and extracting rate probability α and β. The pattern in Fig.2(a) exhibits the space-time evolution for the coexisting phase (the alternative high velocity and low velocity) after a sufficiently large time. It is clear that the density wave appears as the traffic jam. Fig.2(b) exhibits the space-time evolution for the uniformly congested phase, where the density wave becomes weaker and propagate backward. The pattern in Fig.2(c) exhibits the space-time evolution for the freely moving phase where all vehicles move with the same velocity. Each velocity profile in the Figs 1(a)-1(c) corresponds to patterns (a)-(c) in (Fig.2).

In the case $h_c=5.0$ and , $\tau=0.5$ and for several values of $\Delta\tau$ phase diagram in the (α,β) plane shows that the system exhibits three phases, unstable metasatble and stable phases (Fig.3)

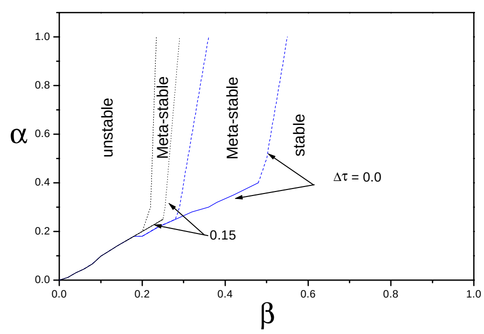


Fig. 3: Phase diagram in the (α,β) plane for $\tau=0.5$. Dashed lines correspond to second order transition, while solid lines

correspond to first order transition. The number accompanying each curve denotes the value of $\Delta\tau$.

It is clear that the transition from unstable to metastable and from metastable to stable state occur under the effect of the probabilities rates α and β. However, for a fixed value of α, there exist a critical value of the extraction rate β_{c1} above which the wave density disappears and the metastable state appears and a critical value β_{c2} ($\beta_{c1} < \beta_{c2}$) above which the metastable state disappears while the stable state appears. β_{c1} and β_{c2} depend on the values of α and the variation of the delay time $\Delta\tau$ Indeed β_{c1} and β_{c2} increase when increasing α and/or decreasing $\Delta\tau$. Furthermore the region of the metastable phase, in the (α,β) plane shrinks with the increase of the variation of the delay time and disappears completely above a critical value $\Delta\tau_c$ that's depend on the values of injecting and extracting rates.

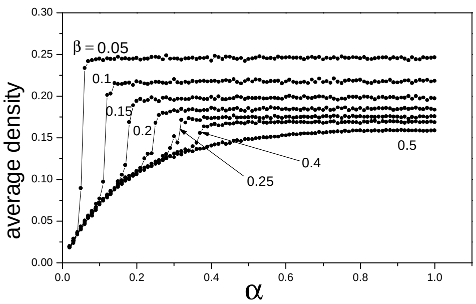


Fig. 4: The variation of the average density versus the injecting rate probability α for $<\tau>$ = 0.5, h_c = 5.0, $\Delta\tau$ = 0.0, and L=500. The number accompanying each curve denotes the value of β.

The behavior of density as a function of the injecting (or extracting) rate for a fixed value of extracting (or injecting) rate (Fig.4) shows that the transitions between unstable state (stable state) (for $\beta>\beta_c$) and the metastable one are of second order type. While the transitions between metastable (unstable) and stable state (for $\beta<\beta_c$) are of first order type. Indeed, at the first order transition, the density undergoes a discontinuity, while the density behaves continuously at the second order transition [11,27].

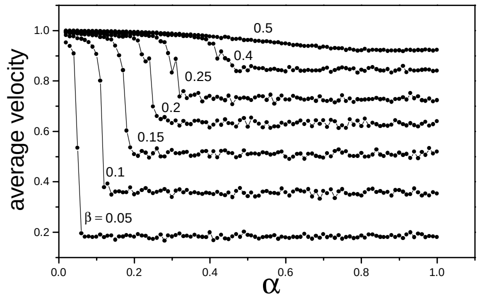


Fig. 5: The variation of the global mean velocity versus the injecting rate probability α for several values of β, with $<\tau>$ = 0.5, h_c = 5.0, $\Delta\tau$ = 0.0, and L=500 .

The critical value β_c depends on the value of the variation of the delay time $\Delta\tau$. Indeed β_c decreases when increasing $\Delta\tau$. Indeed for $\Delta\tau$=0, β_c =0.5; while for $\Delta\tau$=0.15, β_c=0.25. The corresponding velocity behavior as a function of the injecting rates is given in (Fig.5). It is found that the average velocity undergoes a jump from high to low values at the first order transition such jumps is obtained, in our previous work, in the case of two dimensional cellular automaton traffic flow models [27]

IV-Conclusion

Using numerical simulation method, we have studied the effects of open boundaries conditions on the dynamical behavior of the optimal velocity traffic flow models with a delay time τ. We have shown in the absence of the variation of the delay time $\Delta\tau$, that the transition from unstable to metastable and from metastable to stable state occur under the effect of the probabilities rates α and β. However, for a fixed value of α, there exist a critical value of the extracting rate β_{c1} above which the

wave density phase disappears and the metastable state appears and a critical value β_{c2} above which the metastable state disappears while the stable state appears. β_{c1} and β_{c2} depend on the values of α and the variation of the delay time $\Delta\tau$. Indeed β_{c1} and β_{c2} increase when increasing α and/or decreasing $\Delta\tau$. The flow of vehicles is calculated as a function of α, β and $\Delta\tau$ for a fixed value of τ. We have established Phase diagrams in the (α,β) plane that exhibits three different phases namely, unstable, metastable and stable

References

[1] traffic and Granular Flow, edited by D.E. Wolf, M. Schreckenberg, and A. Bachem (World Scientific, singapore, 1996).
[2] D.Helbing, Traffic and related self-driven many-particle systems, Rev. Mod. Phys. 73, 1067-1141 (2001).
[3] Traffic and Granular Flow 97, edited by M. Schreckenberg and D.E. Wolf (springer, singapore, 1998).
[4] G.F. Newell, Oper. Res. 9, 209 (1961).
[5] G.B. Whitham, Proc. R. Soc. London. Ser. A 428, 49 (1990).
[6] M. Bando, K. Hasibe, A. Nakayama, A. Shibata, and Y. Sagiyma, Phys. Rev. E 51, 1035 (1995).
[7] T. Komatsu and S. Sasa. Phys. Rev. E 52, 5574 (1995).
[8] T. Nagatani and K. Nakanishi, Phys. Rev. E 57, 6415 (1998).
[9] T. Nagatani, K. Nakanishi, and H. Emmerich, J. Phys. A 31, 5431 (1998).
[10] K. Nagel, M. Schreckenberg, J. phys. I 2 2221 (1992).
[11] A. Benyoussef, H. Chakib, and H. Ez-Zahraouy, Eur. Phys. J. B 8, 275-280 (1999).
[12] A. Benyoussef, N. Boccara, H. Chakib, and H. Ez-Zahraouy, Chinese journal of physics 39, 428 (2001).
[13] T. Nagatani, Phys. Rev. E 48, 3290 (1993).
[14] D. Helbing and M. Schreckenberg, Phys. Rev. E 59, R2505 (1999).
[15] D. Helbing, Phys. Rev. E 53, 2366 (1996).
[16] M. Treiber, A. Hennecke, and D. Helbing, Phys. Rev. E 59, 239 (1999).
[17] T. Nagatani, Physica A 237 ,67 (1997).
[18] T. Nagatani, Physica A 261 ,599 (1998).
[19] H.Y. Lee, H.W. Lee, and D. Kim, Phys. Rev. E 59, 5101 (1999).
[20] N. Mitarai, H. Nakanishi, J. phys. Soc. Jpn. 68 (1999) 4857.
[21] T. Nagatani, Phys. Rev. E 61, 3564 (2000).
[22] T. Nagatani, Phys. Rev. E 61, 3534 (2000).
[23] T. Nagatani, Phys. Rev. E 60, 6395 (1999).
[24] T. Nagatani, Physica A 284 (2000) 405-420.
[25] T. Nagatani, Physica A 253 (1998) 353-365.
[26] K. Nagel, Phys. Rev. E 53, 4655 (1996).
[27] A. Benyoussef, H. Chakib, and H. Ez-Zahraouy, To appear in Phys. Rev. E (2003).