# Surface anisotropy and dipolar interaction effects on a super-lattice magnetic properties.

**A. Fahmi and A. Qachaou**

*Laboratoire de Physique de la matière condensée, Faculté des sciences, KENITRA*

This work is a contribution to the study of the elementary excitations and magnetic properties of a super-lattice such as $[Co(t_m)/Cu(t_{nm})]_N$, represented by the N atomic planes Heisenberg ferromagnetic system where the nearest neighbour and next nearest neighbour exchange, and dipolar interactions are considered in addition to the surface anisotropy. This study is based on the linear spin waves theory. In the presence of the exchange alone, the excitation spectrum E(k) and the magnetization $< S^z >$ analytical expressions are obtained for N=3, 5 and 10 atomic planes using the Green's function formalism. We also highlighted the existence of N excitation modes in conformity with former studies. A numerical study of the surface anisotropy and dipolar interaction contributions is also reported. A good agreement between the calculated and measured magnetization evolution with an applied magnetic field at 300K is obtained.

Key words: super-lattice, spin waves theory, anisotropy, excitation spectra, magnetization, dipolar interaction.

## 1- INTRODUCTION

The magnetic thin films and the multi-layer materials present a behavior different from that observed in the bulk systems, due mainly to the dimensionality reduction allowing thus a wide applications in particular the tape recording field. The treatment of these systems as quasi-two-dimensional generally gives results coherent with the experiment[1,2]. Several theoretical approaches were proposed to study their magnetic behaviour, in particular the spin waves theory is largely used. The first measurements having led to a good results, were carried out on cobalt monolayer deposited on copper by BLS technique (Brillouin light scattering)[3].

The purpose of this work is to study, using the spin wave theory, the properties of a super-lattice having bcc structure, such as $[Co(t_m)/Cu(t_{nm})]_N$ represented by a ferromagnetic Heisenberg of localised spin with nearest neighbour (NN) and next nearest neighbour (NNN) exchange interaction ($J_1$ and $J_2$) which contributes to the long range order observed at room temperature in a monolayer of $[Co(t_m)/Cu(t_{nm})]_N$ [4]. We admitted that each ultrathin ferromagnetic monolayer constitutes a quasi-two-dimensional system extending on a thickness $t_m$ of some atomic distances, although all its atoms are not necessarily located in the same plane. Thus, the super-lattice $[Co(t_m)/Cu(t_{nm})]_N$ would consist of a succession of N ferromagnetic planes alternate with a nonmagnetic copper layers of $t_{nm}$ thickness. We also took account of the surface anisotropy which plays an important role in the magnetic stability of this type of system as proven by former experimental studies [5,6,7]. In addition, for the films, the effect of the dipolar interactions is as significant as that of the anisotropy, the magnetic properties result from a competition between the two effects. The anisotropy (□) tends to align the moments perpendicular to the film plane whereas the dipolar interaction (D) favours their alignment in the plane. In paragraph IV, we reported a numerical processing of their influence on magnetization by spin. In absence of these two effects (D=□=0), we obtained an exact resolution for N=3,5 and 10 which we present in paragraph III.

## 2- MATHEMATICAL FORMULATION : SPIN HAMILTONIEN

We consider a ferromagnetic system with spin localized on a bcc structure with a lattice parameter a. We suppose that the ferromagnetic film plane is confused with the (xz) plane; the y axis constitutes the normal to the film. The corresponding Hamiltonian can be represented by:

$$H = -J_1 \sum_{<ij>} (S_i^x S_j^x + S_i^y S_j^y + \delta S_i^z S_j^z) - J_2 \sum_{<ij'>} (S_i S_{j'}) - J_\perp \sum_{<ii'>} (S_i S_{i'}) - \alpha \sum_i (S_i^y)^2$$
$$- g\mu_B H \sum_i S_i^z + \frac{g^2 \mu_B^2}{2} \sum_{<ij>} \frac{1}{r_{ij}^3} \left\{ S_i S_j - 3\frac{(S_i r_{ij})(S_j r_{ij})}{r_{ij}^2} \right\} \quad (1)$$

 

$J_1$ and $J_2 > 0$ are respectively the NN and NNN exchange integrals in the same atomic plane. $J_{□}$ indicates the NN exchange integral belonging to two successive planes. □>0 and □>0 represent the surface magnetocrystalline anisotropy, where □ is supposed acting only on the surface spins (plane 1 and N) and whose effect on the system properties is now well established. H is the external magnetic field applied along z axis. The last term corresponds to the dipolar interaction between NN belonging to the same plane ($r_{ij} = |r_i - r_j|$).

The treatment of (1) within the framework of the linear spin wave theory consists in transforming the spin operators into boson operators according to the usual Holtstein-Primakoff transformation [8]:

$$S^+ = \sqrt{2S}a; \quad S^- = \sqrt{2S}a^+ \quad \text{and} \quad S = S - a^+ a \tag{2}$$

The Fourier transform of the boson operators is introduced then to take account of the symmetry translation in the (xz) plane and of the break of this symmetry along the y axis. It is expressed for a site i belonging to the $l^{th}$ plane by [9]:

$$a_i^+ = \frac{1}{\sqrt{L}} \sum_k a^+_{k_{//},l} e^{ik_{//}i} \quad and \quad a_i = \frac{1}{\sqrt{L}} \sum_k a_{k_{//},l} e^{-ik_{//}i} \tag{3}$$

where L is the atoms number per film layer and $k_{//}$ is the wave vector whose direction is parallel to the film surface. Then, the transformed hamiltonian is:

$$H = \sum_{l,m}^{N} \sum_{k_{//}} \left\{ A_{lm}(k) a^+_{k_{//},l} a_{k_{//},m} + \frac{1}{2} B_{lm} \left( a^+_{k_{//},l} a^+_{-k_{//},m} + a_{k_{//},l} a_{-k_{//},m} \right) \right\} \tag{4}$$

where the terms $A_{lm}(k)$ and $B_{lm}(k)$ are defined as:

$$A_{lm}(k) = \left[ 8SJ_1 \left( \delta - \cos\frac{k_x a}{2}\cos\frac{k_z a}{2} \right) + 4SJ_2 \left( 2 - \cos k_x a \cos k_z a \right) + 2SJ_\perp - h + DS\left( 2 + \cos\frac{k_x a}{2}\cos\frac{k_z a}{2} \right) \right] \delta_{lm} \tag{5-a}$$

$$+ \left( 4\alpha S \cos\frac{k_x a}{2}\cos\frac{k_z a}{2} \right)\left( \delta_{l,1} + \delta_{l,N} \right) - 2SJ_\perp \left( \delta_{l,m+1} + \delta_{l,m-1} \right) + 2SJ_\perp \left( 1 - \delta_{l,1} + \delta_{l,N} \right)$$

$$B_{lm}(k) = \left[ -3SD \cos\frac{k_x a}{2}\cos\frac{k_z a}{2} \right] \delta_{l,m} - \left[ 2S\alpha \cos\frac{k_x a}{2}\cos\frac{k_z a}{2} \right] \left( \delta_{l,1} + \delta_{l,N} \right) \tag{5-b}$$

with $D = \dfrac{g^2 \mu_B^2}{(a/\sqrt{2})^3}$ and h = g□$_B$H

In (4), we kept only the quadratic terms corresponding to the elementary excitations. Its diagonalization and consequently the determination of the excitation spectrum as well as the magnetization were carried out by using the Green's function method [10]. We considered the following retarded Green's functions :

$$G_{l,m} = \langle\langle a_{k_{//},l}, a^+_{k_{//},m} \rangle\rangle \quad \text{and} \quad G'_{l,m} = \langle\langle a^+_{k_{//},l}, a^+_{k_{//},m} \rangle\rangle \tag{6}$$

The motion equations of these Green's functions enabled us to obtain a 2N equations system represented in the following matrix form:

$$\begin{pmatrix} \underline{A} & \underline{B} \\ -\underline{B} & -\underline{A} \end{pmatrix} \begin{pmatrix} \underline{G} & 0 \\ \underline{G'} & 0 \end{pmatrix} = \frac{-1}{2\pi} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \tag{7}$$

where $\underline{A}$ and $\underline{B}$ are N order matrices, their elements $A_{lm}(k)$ and $B_{lm}(k)$ are given by the expressions (5-a) and (5-b).

The elementary excitations energies are obtained from the poles of the matrix $\begin{pmatrix} G & 0 \\ G' & 0 \end{pmatrix}$ as the (2N×2N) matrix: $\underline{M} = \begin{pmatrix} A & B \\ -B & -A \end{pmatrix}$ eigenvalues.

In the dipolar interactions and surface anisotropy presence, the analytical resolution of this system becomes more complex. Thus, at first we have carried out an analytical resolution in their absence, next a numerical resolution is done taking account of the various interactions.

### III- ANALYTICAL RESOLUTION :EXCHANGE EFFECT ONLY

For systems engaging transition metals, the exchange interaction forces $J_i$ are more important than □ and D. Thus, an exact resolution is possible by omitting their contribution (D=□=0). The magnetic field (h=g□$_B$H) has for simple effect to

move the mode frequencies. The boson Hamiltonian is reduced thus to:

$$H = \sum_{lm} \sum_{k_{//}} A_{lm}(k_{//}) a^{+}_{k_{//},l} a_{k_{//},m} \tag{8}$$

where $A_{lm}(k)$ are given by:

$$A_{lm}(k) = \left[ 8SJ_1 \left( \delta - \cos\frac{k_x a}{2} \cos\frac{k_z a}{2} \right) + 4SJ_2 \left(2 - \cos k_x a \cos k_z a\right) + 2SJ_\perp - h \right] \delta_{lm}$$
$$-2SJ_\perp \left(\delta_{l,m+1} + \delta_{l,m-1}\right) + 2SJ_\perp \left(1 - (\delta_{l,1} + \delta_{l,N})\right)$$

**3.1-Excitation spectra :**

The spin wave energies E(k) are calculated by solving the equations system according to:

$$Det[\underline{A}-E\underline{I}]=0 \tag{9}$$

Where:

$$(\underline{A}-E\underline{I}) = \begin{pmatrix} A-E & -W & 0 & \cdots & 0 \\ -W & A+W-E & -W & & \vdots \\ 0 & \ddots & \ddots & \ddots & 0 \\ \vdots & & -W & A+W-E & -W \\ 0 & \cdots & 0 & -W & A-E \end{pmatrix} \tag{10}$$

is tridiagonal, with: $A=A_{11}$ and $W=-A_{12}$. In this form, the matrix ($\underline{A}$-E$\underline{I}$) is decomposable into a product of two triangular matrices :*($\underline{A}$-E$\underline{I}$)=* $\underline{b}.\underline{c}$ such as: det($\underline{A}$-E$\underline{I}$)=det($\underline{c}$) with:

$$\underline{b} = \begin{pmatrix} 1 & 0 & \cdots & & 0 \\ b_{12} & 1 & \ddots & & \vdots \\ \vdots & b_{23} & \ddots & 0 & \\ \vdots & & \ddots & 1 & 0 \\ 0 & \cdots\ 0 & \cdots & b_{N-1,N} & 1 \end{pmatrix} \text{ and } \underline{c} = \begin{pmatrix} c_{11} & c_{21} & 0 & & \cdots & 0 \\ 0 & c_{22} & c_{32} & \ddots & & \\ \vdots & & & \ddots & c_{N-2,N-1} & \vdots \\ \vdots & & & \ddots & c_{N-1,N-1} & c_{N,N-1} \\ 0 & & \cdots & 0\ \cdots & 0 & c_{NN} \end{pmatrix}$$

A recurrence relation between the $\underline{c}$ diagonal elements is obtained :

$c_{ii} = (A-E)_{ii} - \dfrac{(A-E)_{i,i-1}(A-E)_{i-1,i}}{c_{i-1,i-1}}$ where $(A-E)_{i,i-1}(A-E)_{i-1,i} = W^2$, and a secular equation for a super-lattice with N planes is given by:

$$\det(A-E) = (A_{11}-E)\prod_{i=2}^{N} \left\{ (A-E)_{ii} - \frac{W^2}{c_{i-1,i-1}} \right\} = 0 \tag{11}$$

Thus, for the three examples (N=3, 5 and 10), we have:

$$\det_3 = (A-E)\left[(A-E)(A+W-E) - W^2\right] - W^2(A-E) = 0 \tag{11-a}$$

$$\det_5 = (A-E)^2(A+W-E)^3 - 2\left[(A-E)^2(A+W-E) + (A-E)(A+W-E)^2\right] W^2$$
$$+\left[2(A-E)+(A+W-E)\right]W^4 = 0 \tag{11-b}$$

$$\det_{10} = (A-E)^2(A+W-E)^8 - \left[7(A-E)^2(A+W-E)^6 + 2(A-E)(A+W-E)^7\right]W^2 + \left[15(A-E)^2(A+W-E)^4 + 12(A-E)(A+W-E)^5 + (A+W-E)^6\right]W^4 - \left[10(A-E)^2(A+W-E)^2 + 20(A-E)(A+W-E)^3 + 5(A+W-E)^4\right]W^6 + \left[(A-E)^2 + 8(A-E)(A+W-E) + 6(A+W-E)^2\right]W_4^8 - W^{10} = 0 \quad \text{(11-c)}$$

leading to the excitation spectrum E(k).

a-For a super-lattice with N=3 planes, the expressions of the three energy values are:

$E_i = A(k) + \varepsilon_i^3 W$ with

$$\varepsilon_i^3 = \{-1 \; ; \; 0 \text{ and } 2\} \quad (12)$$

*b*- For N=5, we have obtained 5 distinct solutions:

$E_i = A(k) + \varepsilon_i^5 W$ with

$$\varepsilon_i^5 = \left\{(1 \pm \sqrt{5})/2 \; ; \; -1 \text{ and } (3 \pm \sqrt{5})/2\right\} \quad (13)$$

*c*- For N=10, there is 10 distinct solutions.

$E_i = A(k) + \varepsilon_i^{10} W$ with

$$\varepsilon_i^{10} = \left\{(2 \pm \sqrt{10 \pm 2\sqrt{5}})/2 \; ; \; \pm 1 \text{ and } (2 \pm \sqrt{6 \pm 2\sqrt{5}})/2\right\} \quad (14)$$

**3.2- Magnetization by spin :**

The super-lattice magnetization by spin expression is:

$$\frac{<S^z>}{S} = 1 - \frac{1}{S}\frac{1}{N}\frac{v}{(2\pi)^2}\sum_{l=1}^{N}\int_{BZ} <a^+_{k,l}, a_{k,l}> dk_x dk_z \quad (15)$$

where $v$ is the unit cell surface and $<a^+_{k,l}, a_{k,l}>$ is the average spin wave occupation number:

$$<a^+_{k,l}, a_{k,l}> = \left[\exp\left(\frac{E_l(k)}{kT}\right) - 1\right]^{-1} \quad (16)$$

leading to the magnetization expression :

$$\frac{<S^z>}{S} = 1 - \frac{1}{S}\frac{1}{N}\sum_{l=1}^{N}\frac{v}{(2\pi)^2}\int \frac{1}{\exp\left(\frac{E_l(k)}{kT}\right) - 1} dk_x dk_z \quad (17)$$

Noting that only the matrix A diagonal terms, given by (5-a) and intervening in the $E_l(k)$ expressions, depend on the wave vector k=($k_x$,$k_z$) and only the spin waves corresponding to the low frequencies are excited at the low temperatures, we developed A(k) in the vicinity of k=(0,0).

Let us put : $\eta = k_x a$ and $\zeta = k_z a$ with □□□1□□□1 such as □□□□□□, then:

$$1 - \cos\frac{k_x a}{2} \approx \frac{1}{2}(\eta/2)^2 \; ; \quad 1 - \cos\frac{k_z a}{2} \approx \frac{1}{2}(\zeta/2)^2 \text{ and :}$$

$$A(k) = 8J_1 S(\delta - 1) + J_1 S\eta^2 + 2J_2 S\zeta^2 + 2J_\perp S + h \quad (18)$$

Then, the dispersion relation by plane $l$ is:

$$E_l(k) = 8J_1 S(\delta - 1) + (J_1 S + 2J_2 S)\rho^2 + 2J_\perp S(1 + \varepsilon_l) + h \quad (19)$$

In the polar coordinates, and after integration of (17), we have :

$$\frac{<S^z>}{S} = 1 + \frac{1}{4\pi}\frac{1}{N}\frac{kT}{2(J_1 S + 2J_2 S)}\sum_{i=1}^{N} Ln\left\{1 - \exp\left[-(8J_1 S(\delta - 1) + h + 2J_\perp S(1 + \varepsilon_i))/kT\right]\right\} \quad (20)$$

For a semi infinite super-lattice ($N \to \infty$), the N planes system becomes continuous, and then:

$2J_\perp S(1 + \varepsilon_i) \to 4SJ_\perp(1 - \cos k_y a)$ and $\frac{1}{N}\sum_{i=1}^{N} \to \frac{a}{2\pi}\int_{-\frac{\pi}{a}}^{\frac{\pi}{a}} dk_y$. For the excited frequencies (□□$k_y$a<<1), 1-cos$k_y$a≅(□)$^2$/2 and the magnetization is :

$$\frac{<S^z>}{S} = 1 + \frac{1}{4\pi N}\frac{kT}{2S(J_1 + 2J_2)}\int_0^{\pi} \ln\left[1 - \exp\left(-\frac{(\Delta + h + 2J_\perp S\xi^2)}{kT}\right)\right] d\xi$$

where $\Delta$=8S$J_1$($\delta$-1) (21)

from which we deduced the two limit cases:

a- For kT<< $\Delta$' where $\Delta$'=$\Delta$+h, we have $x = \exp\left\{-\frac{\Delta' + 2SJ_\perp\xi^2}{kT}\right\}$ <<1. Then, using the $Ln(1+x)$ development, and tending the upper limit to ∞, the integral calculation gives :

$$\int_0^\infty \ln\left[1-\exp\left(-\frac{(\Delta'+2J_\perp S\xi^2)}{kT}\right)\right] = \frac{-1}{2\pi}\sum_{n=1}^{\infty}\frac{e^{-n\Delta'/kT}}{n}\int_0^\infty d\xi \exp\left(\frac{-2nSJ_\perp\xi^2}{kT}\right)$$
$$= \frac{-1}{4\pi}g_{3/2}\left(e^{\frac{\Delta'}{kT}}\right)\left[\frac{\pi kT}{2nSJ_\perp}\right] \qquad (22)$$

where $g_{3/2}\left(e^{\frac{\Delta'}{kT}}\right)$ is Bose's function, that $g_{3/2}(\Delta'=0)=\zeta(3/2)=2.612$. Finally:

$$\frac{<S^Z>}{S} = 1-g_{3/2}\left(e^{\frac{\Delta'}{kT}}\right)\frac{1}{8\pi^{3/2}}\frac{1}{(2SJ_\perp)^{1/2}}\frac{1}{S(J_1+2J_2)}(kT)^{3/2} \qquad (23)$$

b- For kT>>□'; the integration of (21) gives :

$$\frac{<S^z>}{S} = 1-\frac{1}{4\pi}\frac{1}{N}\frac{kT}{2(J_1S+2J_2S)}Ln\left\{\frac{kT}{\Delta'+4SJ_\perp}\frac{2}{1+\left[1-\left(1-4SJ_\perp/(\Delta'+4SJ_\perp)\right)^2\right]^{1/2}}\right\} \qquad (24)$$

**3.3- Discussion :**

*3.3.1- Excitation spectra :*

On the figures1, we represented the excitation spectrum of the various studied systems corresponding to N=3,5 and 10 atomic planes.

We note thus that for the 3 treated cases, the excitation modes number is equal to the number of planes engaged in the super-lattice for any value of the wave vector $k(k_x, k_z)$ as obtained in previous works[12 ], reflecting that these N planar modes are coupled. We also note that the excitation spectrum having lowest energy corresponding to the surface modes (l=1 and N) is the same one for the three cases; i.e. : E(k)=A(k)-W. Consequently, the energy gap □' which is given compared to the lower energy mode, is also the same; i.e. .:□'=$8J_1S$(□-1)+h.

In the absence of the field (h=0), this gap depending only on the in-plane anisotropy □ and the exchange $J_1$ would be independent on the super-lattice thickness. For an isotropic system (□=1), this value of the gap is null reflecting thus the important role played by the anisotropy in the magnetic long range order stability for quasi-two-dimensional. This significant result agree with former works[3,13]. In particular the Mermin-Wagner theorem[13] provides that the ferromagnetic order of a two-dimensional system cannot exist in absence of the magnetic field only in the presence of the anisotropy. The experimental results obtained by BLS-technique on two-dimensional cobalt showed also that the existence of a ferromagnetic phase is primarily due to the presence of a strong uniaxial anisotropy[3].

In the presence of the external magnetic field (h≠0), the role of this anisotropy is reinforced and the magnetic order stability of the super-lattice is even accentuated.

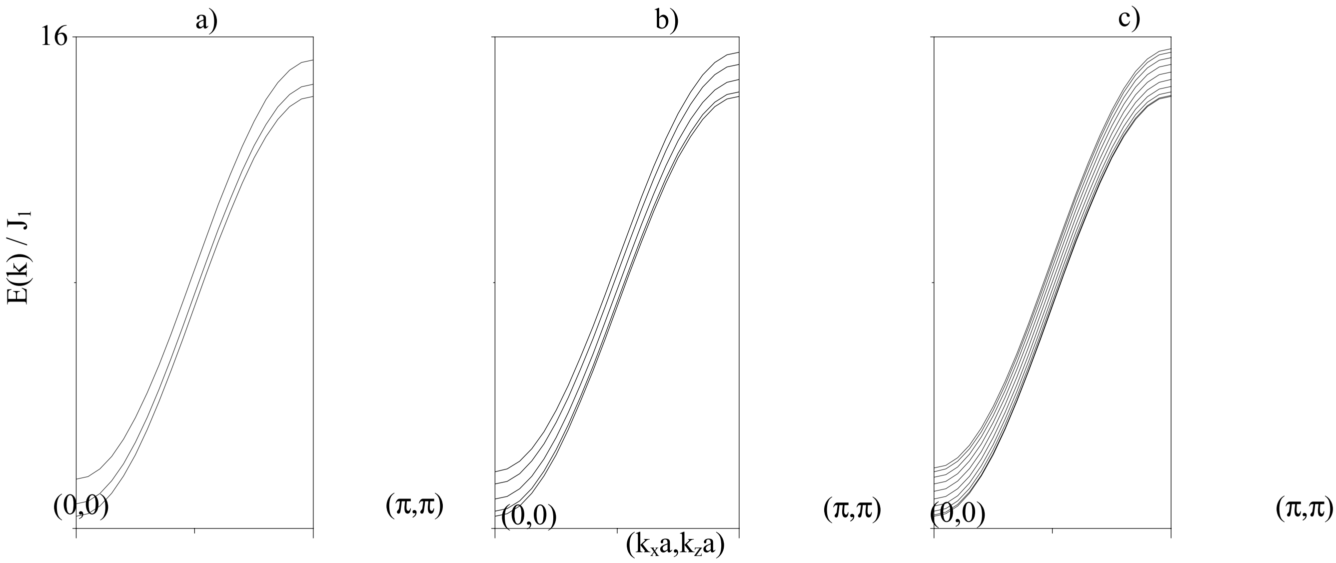


*Fig. 1*

Excitation spectrum of a super-lattice made up of: a) 3 atomic planes, b) 5 planes and c) 10 planes. The parameters are $J_2$=1/2.83 $J_1$, $J_\perp$=0.2$J_1$ and δ=0.05$J_1$[11].

*3.3.2- Magnetization :*

From the magnetization expressions (23) and (24), we note that :

a- For low temperatures compared to the gap, the thermal evolution of the magnetization deviation finds the $T^{3/2}$ behavior usually observed for bulk samples.

b- When the thermal effect is more significant than the magnetic order stability gap, the magnetization expression depends on the temperature in TlogT, which corresponds to a behavior characteristic of the quasi-two-dimensional systems. The logarithmic factor is a consequence of the coupling between the atomic planes. Indeed; in absence of the anisotropy, this factor depends only on the exchange integral between the ferromagnetic planes J□. Consequently, one can suggest that it would correspond to a magnetization contribution due to the coupling between these planes. In addition, in absence both of this coupling and the anisotropy (□=1), the logarithmic factor above diverges for any temperature ($T\neq0$) indicating that this type of system can have a ferromagnetic order only in the presence of the anisotropy confirming the result deduced above from the gap analysis.

Thus, the figure (2-a) represents the magnetization by spin thermal variation. We note that the three curves are very slightly different. Indeed, on average, each super-lattice site can be considered, in first approximation, as having the same average contribution to the total magnetization for any finite number of planes engaged in the super-lattice. On the other hand the total magnetization can only increased with the increasing number of planes as showen on the figure 2-b representing the magnetization per number of sites by plane and where the difference between the three systems is quite clear.

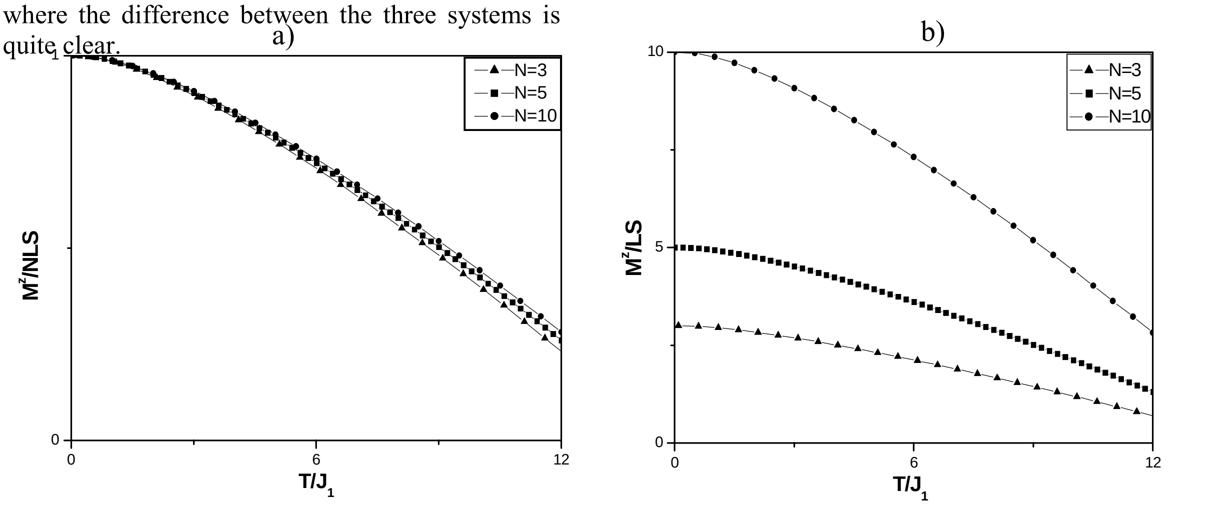


*Fig. 2*

Thermal variation of the magnetization for the 3 systems :
a) magnetization per spin. b) global magnetization.

Otherwise, to investigate the high frequencies spin waves excitations effect on the magnetization, we represented the latter in both cases, the first(fig. 3-a), we contented with a low frequencies development (eq.20), and the second (fig.3-b) correspond to the magnetization where all excitations in the BZ are considered (eq.17). We note that more the temperature increases more the high frequencies spin waves effect becomes more significant, it cannot be neglected. A result which is awaited since this kind of excitation tends to throw the system into disorder and consequently to decrease the transition temperature.

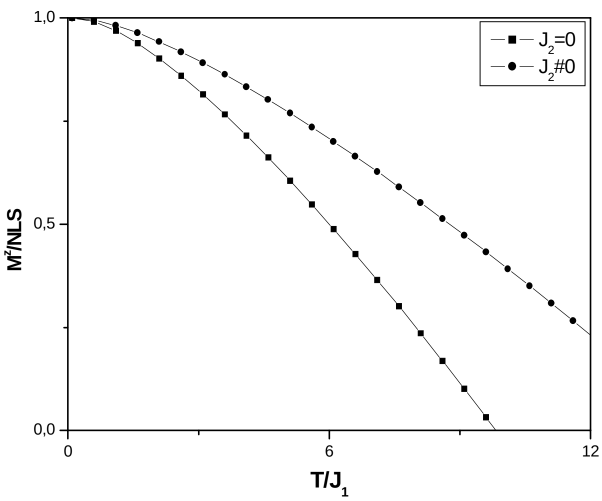


*Fig. 4*

Thermal variation of the magnetization.
The parameters are the same as fig. 1.

*3.3.3-Next nearest exchange interaction effect :*

To show the next nearest exchange interaction effect we represented the magnetization expressed by (21), on figure 4 in presence (fig.4-a) and absence (fig.4-b) of this interaction. Comparing the two curves, we deduce that the NN exchange interactions reinforce the ferromagnetic order of the super-lattice by causing a displacement of the transition temperature towards higher temperatures.

*3.3.4- External magnetic field effect :*

The figure 5 represents the variation of the reduced magnetization $M^z$/(NLS) according to the external magnetic field h. Null at high temperatures and for the weak fields, $M^z$(h)/(NLS) increases as and when h increases.
As this could be expected, the effect of the field h increasing is to align moments more and more and thus to reach the saturation magnetization value for sufficient h values

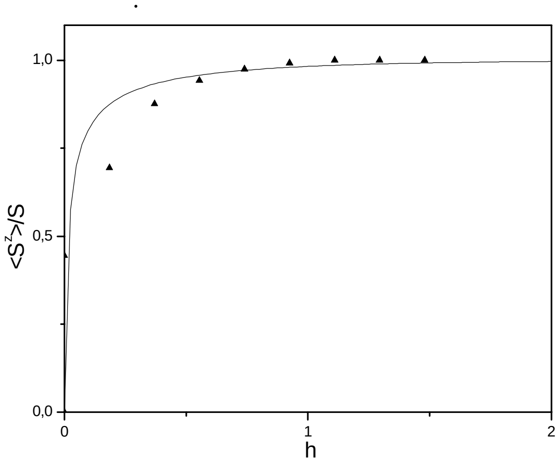


*Fig. 5*
Magnetization variation as a function of the applied field solid line analytical calculation, triangles: experimental data[4].

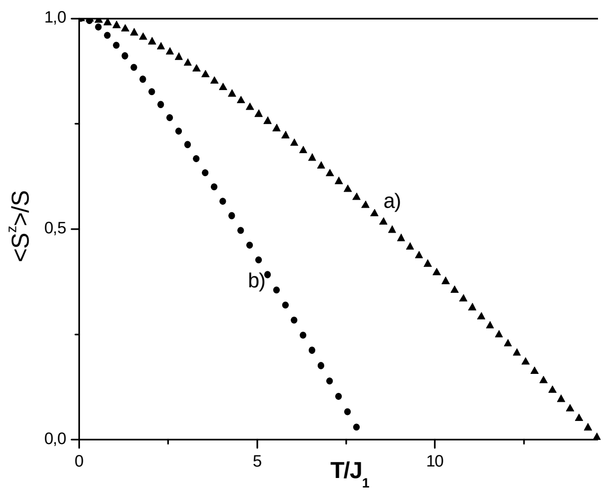


*Fig.3*
Magnetization thermal variation : a) from expression (20), b) from expression (17).

The adjustment of the magnetization variation as a function of the applied field calculated from (17) (fig.5, solid line) with that measured at T=300K (fig. 5, triangles), allowed us to determine an approximate value of the exchange integrals $J_1$ and $J_□$as $J_1$≈43,81 erg/cm$^2$ $J_□$≈0.22erg/cm$^2$, values which belong to the range of the exchange interaction given in experiment work [14]. A transition temperature $T_c$=1185K is also obtained which is inferior, as can be expected, to that corresponding to the three-dimensional cobalt, $T_c$=1388K[15]. The agreement between these calculated and experimental exchange forces $J_i$ values prove that the used super-lattice modelling describes enough the magnetic elementary excitation evolution.

## 4- DIPOLAR INTERACTION AND SURFACE ANISOTROPY EFFECTS:

To analyze the dipolar interactions and the surface anisotropy effects on the system magnetic state in the previous paragraph, it is necessary to start again from the system (7) resolution with □≠0 and D≠0. The spin waves average number intervening in the magnetization expression is calculated using spectral theorem[16]:

$$< a^{+}_{k_{//},l}, a_{k_{//},l} >= -2\int \frac{\mathrm{Im} << a^{+}_{k_{//},l}, a_{k_{//},l} >>}{e^{\beta E_l(k)} - 1} \tag{25}$$

The Green's function $G_{ll}$=<<$a^{+}_{k,l}$,$a_{k,l}$>> are expressed here according to the elements of the passage matrix and its reverse (P and P$^{-1}$ resp.) which diagonalize the Hamiltonian and (25) becomes :

$$< a^{+}_{k,l}, a_{k,l} >= \sum_{n=1}^{N} \left[ \frac{P_{\ln} P^{-1}_{nl} - P_{\ln'} P^{-1}_{n'l}}{e^{\beta E_l(k)} - 1} - P_{\ln'} P^{-1}_{n'l} \right]$$

avec $n' = N+n$
(26)

Consequently, the super-lattice reduced magnetization is:

$$\frac{M^z}{NLS} = 1 - \frac{1}{S}\frac{1}{N}\frac{v}{(2\pi)^2}\sum_{l=1}^{N}\sum_{n=1}^{N}\int_{BZ}\left[\frac{P_{\ln} P^{-1}_{nl} - P_{\ln'} P^{-1}_{n'l}}{e^{\beta E_l(k)} - 1} - P_{\ln'} P^{-1}_{n'l}\right] dk_x dk_z$$

(29)

The exact analytical resolution of (7) and consequently the calculation of the integral above are more complex than in case □=D=0, we thus carried out a numerical resolution allowing to obtain the excitation spectra E(k) and magnetization by spin $M^z$/NLS. The effect of □ and D on the mode frequencies doesn't produce great modifications compared to the case (□=D=0), we omitted to present these spectrum, while this effect

can be more important on the magnetization. Thus, on figure 6, we represented the magnetization thermal variation compared to the obtained one when the dipolar interaction and the surface anisotropy intervene with the same weight ($\alpha$=D) in the magnetic order stability. It comes out that for very low temperatures lower than a “threshold temperature” $T_s$(N), these two parameters don’t have an effect (see fig. 6). We will qualify this situation as “standard” state where these two effects contribute with the same weight ($\alpha$=D) to the super-lattice magnetic properties. However the decreasing with N shown by the “threshold temperature” $T_s$(N) suggests the presence of these effects increasingly significant that the super-lattice becomes wide. An asymmetry of their influence is also recorded. The anisotropy would be heavier in the evolution of the magnetic state of the super-lattice. Its impact is indeed perceived more rapidly than the dipolar interaction, when T increases. Furthermore, the competitive character between these two effects is clearly put in obviousness. When the anisotropy overcomes ($\alpha$/D>>1), the magnetization decreases, while if the dipolar interaction is dominant ($\alpha$/D<<1), the magnetization increases compared with the “standard” case ($\alpha$=D). Indeed, dipolar interaction tends to align magnetic moments in the film plane and consequently to reinforce the magnetization in the plane, while the surface anisotropy tends to align moments perpendicularly to the plane, and as a result to decrease the magnetization.

## 5- CONCLUSION :

In this work, we have studied an elementary magnetic excitation contribution to a super-lattice properties having bcc structure by using the linear spin wave theory. The excitation spectrum calculated show the existence of N planar modes coupled for all wave vector k value in the film plane of the super-lattice to N planes. The obvious dependence of a gap on the surface anisotropy shows the role played by this anisotropy in the existence of a magnetic order. A competition between surface anisotropy and dipolar interaction effects on the stability of this long-range magnetic order is also shown.

*References*

[1] A. Ercole, W. S. Lew, G. Lauhoff,E. T. M. Kernohan, J. Lee and J. A. C. Bland, Phys. Rev. **62**, 6429(2000)

[2] A. Yoshishire, Y. Suzuki and T. Katayama, JPSJ. **65**, 1469(1996)

[3] D. Kerkmann, J.A. Wolf, D. Pescia, D. Woike and P. Grünberg, Solid State Commun. **72**, 963(1989).

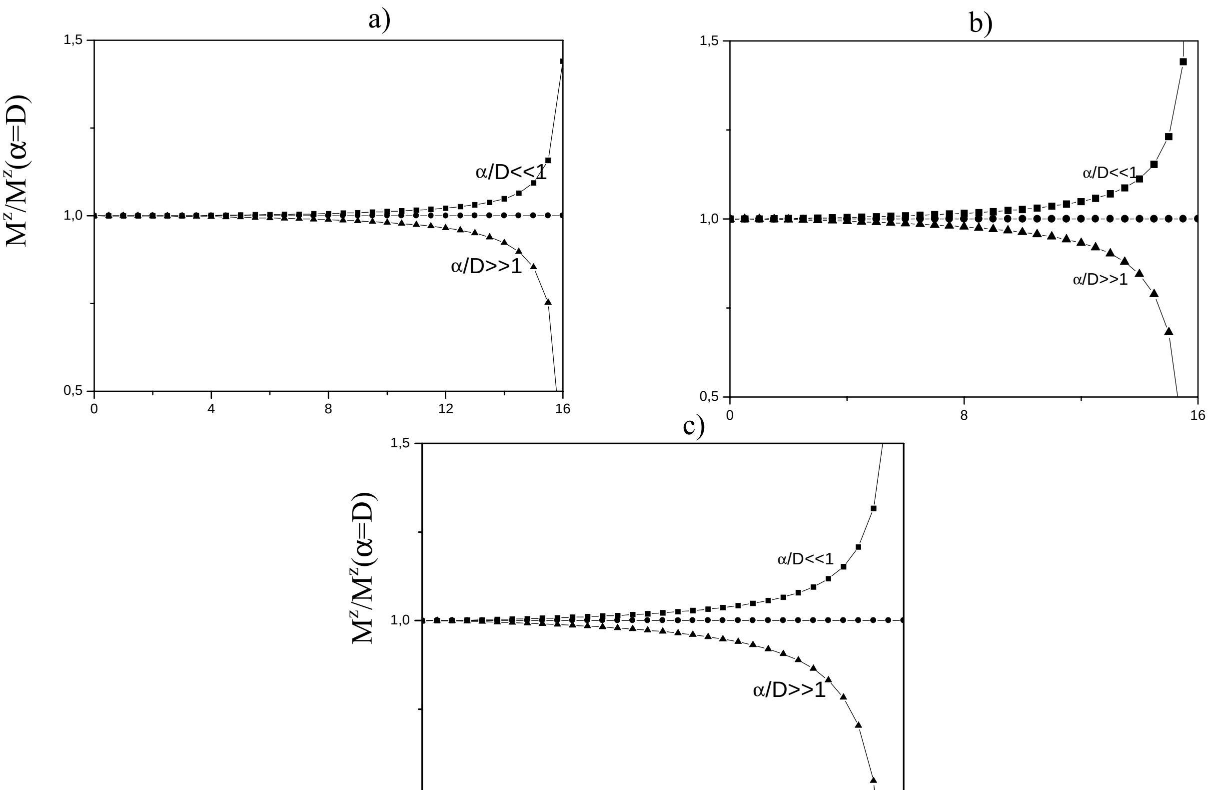


*Fig. 6*

Thermal variation of the standard magnetization. a) N=3, b) N=5 et c)N=10, the other parameters are the same as fig.1

[4] D. Meziane Mtalsi, M. Elharfaoui, A. Qachaou, M. Faris, J. Benyoussef, H. Le Gall and M. Boutarfass, accepted in Annals de chimie, Sciences des Matériaux (2003)
D. Meziane Mtalsi, PhD thesis (2002).
[5] V. Strom,B. J. Johnson, K. V. Rao, D. Dahlberg. J; Appl. Phys.**81**, 8 (1997)
[6] D. S. Xue, M. Z. Gao, J. B. Yang, Y. Kong and F. S. Li, Phys. Stat. Sol. B **227**, 563(2001)
[7]J. L. Dunn and C. A. Bates, J. Pyhs. Cond. Matter. **1**, 375 (1989)
[8]T. Holstein and H. Primakoff, Phys. Rev. **58**, 1908 (1940)
[9]J. Barnas and R. Swirkowics, Phys. Stat. Sol. B **206**,787(1998)
[10] RNDr. S. Krupicka, CSc. And RNDr. J. Sternberg, CSc "Elements of Theoretical Magnetism", edition: Iliffe Books Ltd.
[11] P. Bruno, Phys. Rev. B**43**, 6015 (1991)
[12] J. Barnas and R. Swirkowicz, Phys. Stat. Sol. (b) 206, 787 (1998)
[13] P. Politi, M. G. Pini and A. Rettori, Phys. Rev. B **46**, 8312(1992)
[14] G. Rupp, E. Schester, J. Magn. Magn. Matter 121 (1993)416
[15] Ch. Kittel, Physique de l'état solide, édition : Dunod université (1983).
[16] M. Maccio, M. G. Pini, P. Politi, and A. Rettori, Phys. Rev. B **46**, 8276(1992)