A Genetic Algorithm Approach to Shariah-Compliant Portfolio Optimization in a Fuzzy Environment with Transaction Costs Considerations

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Abstract

In this article, a novel framework is put forth for the optimization of portfolios that adhere to Shariah principles and involve fuzzy variables. Various approaches have been examined to construct financial portfolios that are compliant with Shariah principles. The semi-spread of the fuzzy portfolio return is regarded as a risk measure in Islamic finance, encompassing two parameters that represent the investor's level of risk aversion. By utilizing fuzzy variables, the optimization model becomes more realistic and enables the consideration of uncertainty. Different types of transaction costs have been modeled and taken into account in the optimization model. We employed a genetic algorithm (GA) to solve the portfolio optimization problem. The proposed model simplifies the computational complexity and facilitates the future development of research on the optimization model for Shariah-compliant fuzzy pricing.

1. Introduction

Islamic finance has established a strong presence in the global financial landscape and is expected to continue growing in the coming years [1]. However, in order for Islamic finance to maintain its long-term growth, it is crucial to provide a unique value proposition that meets the needs of customers seeking Shariah-compliant products. This presents an opportunity to attract socially responsible investors who are interested in expanding their ethical investments due to shared universal values. Another area of research in Islamic finance that is experiencing rapid growth is the examination of Shariah-compliant financial assets, particularly Shariah-compliant stocks. This research focuses on two main aspects: the performance of Shariah-compliant stocks compared to conventional stocks, and the dynamic interactions and contagion between them [2].

Portfolio selection involves choosing a combination of securities from a large pool of options to achieve a desired level of investment return [3]. Markowitz's groundbreaking work [4,5] introduced the principle that investors should strike a balance between maximizing return and minimizing risk when making investment decisions. This can be achieved by maximizing return for a given level of risk or minimizing risk for a predetermined level of return. Various researchers, such as Sharpe [6,7], Stone [8], Sengupta [9], Best and

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Grauer [10], have contributed to this field by using different approximation schemes [11]. Fuzzy approaches are generally more suitable than probabilistic approaches when incorporating subjective opinions into the decision-making process, considering the numerous non-probabilistic factors that impact financial markets and introduce fuzzy uncertainty in the returns of risky assets. Some researchers have explored portfolio selection problems in a fuzzy environment using fuzzy set theory [12,13].

While many portfolio selection models have been proposed within the framework of fuzzy set theory, most of them only consider market risk, specifically the risk associated with portfolio returns. In this paper, we will focus on Shariah-compliant investment strategies and introduce Shariah compliance into a portfolio optimization model based on the semi-spread of portfolio returns, which is treated as a fuzzy variable.

The structure of this paper is as follows: In Section 2, we provide a review of Islamic Shariah-compliant principles and the basics of portfolio optimization to enhance understanding. Section 3 highlights how Shariah-compliance can be incorporated into the portfolio optimization framework. Next, in Section 4, we propose a Fuzzy expectation-semi-spread model for portfolio optimization that incorporates Shariah compliance. Finally, we conclude the paper in Section 5.

2. Core concepts

2.1. Shariah compliant principles

Shariah governs all aspects of Islamic matters including faith, worship, economic, social, political and cultural aspects of Islamic societies. The rules and laws are derived from three important sources, namely the Holy Quran, sunna (the practice and tradition of the Prophet Muhammad s.a.w.) and ijtihad (the reasoning of qualified scholars) [14]. To apply these Shariah compliance functions, it is necessary to further elaborate on the concept of public interest (maslahah al-ammah). Public interest constitutes protection of everything that is useful for whole or most of the community, and doesn’t concern only individuals but considers those individuals as member of the whole. One example is the safeguarding of everything that possesses economic value from natural or human-induced destruction. It is clear that all Shariah-compliant investments need to be in line with Shariah objectives and public interest. Therefore, all contemporary guidelines for Shariah investment must be developed bearing in mind the underlying Shariah concepts as just mentioned. Consequently, the five basic driving principles of Islamic activities that influence Islamic investment are [1]:

- Prohibition of payment of interest (riba)
- Prohibition of uncertainty (gharar) in contracts and prohibition of gambling (qimar or maysir)
- Prohibition of dealing with non-permissible activities
- Application of trading and commerce (al-bay) and obligation of risk sharing
- The obligation to back any financial transaction with assets

2.2. Principles of Optimal Portfolio Selection
Portfolio theory focuses on the identification of portfolios that maximize expected returns while maintaining acceptable levels of risk at the individual level. By employing quantitative models and historical data, portfolio theory establishes the concepts of "expected portfolio returns" and "acceptable levels of portfolio risk" and provides guidance on the construction of an optimal portfolio. The primary objective of portfolio selection is to create portfolios that optimize expected returns while adhering to individually acceptable levels of risk [15].

The actual return on a portfolio of assets during a specific time period can be determined by calculating the weighted average of the returns on the individual assets within the portfolio. This calculation is relatively straightforward and can be accomplished using the following formula.

\[ R_p = x_1R_1 + x_2R_2 + \ldots + x_nR_n \]

where:
- \( R_p \) is the rate of return on the portfolio over the period,
- \( R_n \) is the rate of return on asset g over the period,
- \( x_n \) is the weight of asset i in the portfolio (i.e., market value of asset i is a proportion of the market value of the total portfolio) at the beginning of the period, and n is the number of assets in the portfolio.

3. Principles for Constructing Shariah-Compliant Portfolios

The Quran and the hadith provide the basis for the common rules concerning permissible financial activities in Islam (Khan, 2010) [16]. A key characteristic of Islamic Banking is its emphasis on risk sharing among the investor, financial agent, and fund user [17].

Due to the Shariah's prohibition of interest-based assets (Mosler and Scarsini, 1991) [18], it is necessary to establish specific guidelines that introduce additional constraints into models for constructing Shariah-compliant portfolios (Whitmore and Findlay, 1978) [19]. These guidelines are derived from the Quran, Hadith, and Ijtihad, which are the main sources of Shariah. In the following section, we will outline how Shariah compliance can be incorporated into the framework of portfolio optimization.

Suppose that the investment universe \( \Omega \) consists of \( n \) financial assets \( \Omega = \{1, \ldots, n\} \) from which a portfolio can be constructed.

The result of an investment decision is a portfolio, the composition of which is denoted by \( x_i = x_1, x_2, \ldots, x_n \), where \( x_i \) is the portfolio weight corresponding to the i-th instrument, and we assume that the total capital has to be integrally invested and short-sellings are not allowed, which is realistic since short-sellings are generally not allowed under Shariah.

we also assume the objective function \( H(x) \) by which the performance of a portfolio is measured and a set of constraints stemming from investment guidelines other than Shariah guidelines, legal guidelines for instance.
The objective function is reflecting the investment strategy as well as the return/risk tradeoff and could stem for instance from the mean-variance model (Markowitz, 1952; Dentcheva and Ruszczynski, 2006) [4,20].

Before, discussing the Shariah-compliance let consider the following conventional portfolio optimization model:

\[
\begin{align*}
\text{Min} & \quad H(x) \\
\text{subject to} & \quad \sum_{i=1}^{n} x_i = 1 \\
& \quad x_i \geq 0 \quad \forall i \in \Omega
\end{align*}
\]

As a sector standards have to be fulfilled on a single assets or a whole portfolio, the compliance is considered as one of them.

Assumed an economic guideline \( w \in W \) we have to determine a financial ratio \( r_i(w) \) for each asset \( i \in \Omega \), which measures the level of participation in a non-compliant financial activity, and to evaluate the value with a maximum permissible value \( P(w) \), the so called threshold value. Thus, we have to take into consideration set of constraints of the following type:

\[
r_i(w) \leq P(w)
\]

Definitely, the said guidelines occur in an extra decrease of the asset universe and thus their completion can be safe in a pre-processing phase similarly to the sector standards. Therefore, Shariah-fulfilment of a portfolio can be operationally generated by a pre-processing stage in which the asset universe for a conventional portfolio optimization model is specified.

However, the ineffectual way to model compliance regarding a guideline is to formalize conditions (2) as a set of mathematical inequalities, which are introduced as constraints into the portfolio model.

Conceptually, a financial rule can be modeled by a set of logical constraints of the following type:

\[
x_i = 0 \quad \text{if} \quad r_i(w) > P(w) \quad \forall i \in \Omega
\]

The set of inequalities (3) can be replaced with a set of mathematical inequalities as follows:

Let \( z_i \) be a binary variable defined for each \( i \in \Omega \)

\[
\begin{align*}
\text{with} & \quad z_i = \begin{cases} 
1 & \text{if } i \text{ is compliant} \\
0 & \text{otherwise}
\end{cases} \\
\text{and we set the constraints} & \quad x_i \leq z_i \quad \forall i \in \Omega \\
& \quad r_i(w) \times z_i \leq P(w) \quad \forall i \in \Omega
\end{align*}
\]
Constraints (5) ensure that for every asset $i \in \Omega$ $x_i > 0$ if $z_i = 1$. The constraints (6) ensure that $z_i$ is 0 if the guideline (4) is not satisfied (Derigs and Marzban, 2009) [21].

3.1. Modelling Shariah compliance strategies

Let $S$ be a set of Shariah strategies and $W$ a set of financial Shariah guidelines, then a given Shariah compliance strategy is defined by a specific subset $W_s$. A portfolio $X$ is compliant with a strategy $s$ if $X$ satisfies all the guidelines $W_s$, i.e., the sum of the ratio values over all assets $i \in \Omega$ weighted by their share values $x_i$ is less than the threshold (7).

$$\sum_{i \in \Omega} r_i(w)x_i \leq P(w) \quad \forall w \in W_s$$

We formulate then the following optimization model:

$$\text{Min} \quad H(x)$$

subject to

$$\sum_{i \in \Omega} r_i(w)x_i \leq P(w) \quad \forall w \in W_s$$

$$\sum_{i = 1}^{n} x_i = 1$$

$$x_i \geq 0 \quad \forall i \in \Omega$$

Compliance strategies can now be modeled as follows:

- **Best of Shariah strategy: Top Shariah Strategy**

  In this strategy, the optimization model (8) has to be solved for each strategy $s \in S$. Furthermore, the optimal portfolio related to this strategy is the one issued by the model and then providing the best portfolio achievement.

- **Consensus / Ijmaa Shariah strategy: Common Shariah Strategy**

  In this strategy, Shariah compliance is defined regarding all its strategies $s \in S$. Under the Ijmaa strategy, we have to replace (7) by:

  $$\sum_{i \in \Omega} r_i(w)x_i \leq P(w) \quad \forall s \in S, \forall w \in W_s$$

- **Liberal strategy: Free Shariah Strategy**

  In this strategy we set the binary variables for model as the following:

  $$z\theta(s) = \begin{cases} 
  1 & \text{if the portfolio } X \text{ is complaint for } s \in S \\
  0 & \text{otherwise}
  \end{cases}$$

  then (7) have to be substituted with

  $$z\theta(s)(\sum_{i \in \Omega} r_i(w)x_i) \leq P(w) \quad \forall s \in S, \forall w \in W_s$$
In order to get a solvable optimization model, the set of nonlinear constraints (11) should be linearized as follows:

\[ \sum_{s \in S} r_i(w)x_i \leq (1 - z\theta(s)).M + P(w) \quad \forall s \in S, \forall w \in W_s \]  

(13)

Since \( M \) is a large number, and according to (12) there is at least for one strategy \( s \in S, s^0 \) we obtain \( z\theta(s^0) = 1 \)

- **Majority strategy**: Most of Shariah Strategy

We substitute in the model for the liberal strategy for the majority strategy by

\[ \sum_{s \in S} z\theta_s \geq \left[ \frac{|S|}{2} \right] \]  

(14)

Which confirm that at least the majority of main principal strategies \( s \in S \) states as the portfolio as being compliant.

4. Shariah-compliant portfolio with fuzzy variables

4.1. Fuzzy Portfolio Optimization

Let \( \xi \) be a fuzzy variable with a possibility distribution function \( \mu \). Using L-S integral the spread of the fuzzy variable \( \xi \), which was first defined as \( Sp[\xi] \), i.e.,

\[ Sp[\xi] = \int_{(-\infty, +\infty)} (r - E[\xi])^2 d\Phi(r) \]

(15)

where \( \Phi(r) \) is the credibility distribution of the fuzzy variable \( \xi \).

Portfolio is to deal with the problem of how to allocate wealth among several assets. The portfolio optimization problems have been one of the important research fields in modern risk management. Consider we have \( n \) risky assets. \( \xi_i \) denotes the fuzzy return in the next time period on investment \( i, i = 1, \ldots, n \). The random return associated with the portfolio \( X = x_1, x_2, \ldots, x_n \) is thus \( R_p = \sum_{i=1}^{n} x_i \xi_i \) which is also a fuzzy variable.

The expected return associated to the portfolio is \( E[R_p] = E[\sum_{i=1}^{n} x_i \xi_i] \)

In generally, an investor always prefers to have the return on their portfolio as large as possible. At the same time, he also wants to make the risk as small as possible. However, a high return always accompanied with a higher risk.

Next, we will define the risk associated with an investment to be the semi-spreads \( Ssp_{\alpha, \beta}[R_p] \) of the fuzzy return \( R_p \), which is a convex combination of the two spreads (below and above the expected value).

\[ Ssp_{(\alpha, \beta)}[\xi] = \alpha \int_{(-\infty, +\infty)} \min\{0, r_i - E[\xi]\}^2 d\Phi(r) + \beta \int_{(-\infty, +\infty)} \max\{0, r_i - E[\xi]\}^2 d\Phi(r) \]

(16)
Where \(\alpha\) and \(\beta\) are two positive parameters representing the degree of risk aversion of the investor.

with \((\alpha + \beta = 1)[22]\).

The spread (15) of the fuzzy variable \(\xi\) is equivalent to (17) [23]:

\[
Sp[\xi] = \sum_{i=1}^{n}(r_i - E[\xi])^2 (\Phi(r_i) - \Phi(r_i^-))
\]  

The semi-spread of the fuzzy variable \(\xi\) is therefore

\[
Sp_{(\alpha,\beta)}[\xi] = \alpha \sum_{i=1}^{n} \min\{0, r_i - E[\xi]\}^2 (\Phi(r_i) - \Phi(r_i^-)) + \beta \sum_{i=1}^{n} \max\{0, r_i - E[\xi]\}^2 (\Phi(r_i) - \Phi(r_i^-))
\]  

In case of \(\alpha = \beta\), the equation (18) is equivalent to (17)

**Proof**

Let \(\lambda(\xi) = r_i - E[\xi]\)

\[
|\lambda_+| = \max(0, \lambda) = \begin{cases} 
\lambda & \text{if } \lambda \geq 0 \\
0 & \text{otherwise}
\end{cases}
\]

with

\[
|\lambda_-| = \min(0, \lambda) = \begin{cases} 
0 & \text{if } \lambda \geq 0 \\
-\lambda & \text{otherwise}
\end{cases}
\]

then

\[
Sp_{(\alpha,\beta)}[\xi] = \alpha \sum_{i=1}^{n} |\lambda_-| (\Phi(r_i) - \Phi(r_i^-)) + \beta \sum_{i=1}^{n} |\lambda_+|^2 (\Phi(r_i) - \Phi(r_i^-))
\]

\[
|\lambda_-| = \frac{|\lambda| - \lambda}{2} \quad \text{et} \quad |\lambda_+| = \frac{|\lambda| + \lambda}{2}
\]

\[
Sp_{(\alpha,\beta)}[\xi] = \alpha \sum_{i=1}^{n} \left(\frac{|\lambda| - \lambda}{2}\right) (\Phi(r_i) - \Phi(r_i^-)) + \beta \sum_{i=1}^{n} \left(\frac{|\lambda| + \lambda}{2}\right) (\Phi(r_i) - \Phi(r_i^-))
\]

\[
= \alpha \sum_{i=1}^{n} \left\{2\lambda^2 - 2\lambda|\lambda|\right\} (\Phi(r_i) - \Phi(r_i^-)) + \beta \sum_{i=1}^{n} \left\{2\lambda^2 + 2\lambda|\lambda|\right\} (\Phi(r_i) - \Phi(r_i^-))
\]

\[
= \alpha \sum_{i=1}^{n} \left\{2\lambda^2 - 2\lambda|\lambda|\right\} (\Phi(r_i) - \Phi(r_i^-)) + \beta \sum_{i=1}^{n} \left\{2\lambda^2 + 2\lambda|\lambda|\right\} (\Phi(r_i) - \Phi(r_i^-))
\]

In case of \(\alpha = \beta\), we have

\[
Sp_{(\alpha,\beta)}[\xi] = \frac{\alpha + \beta}{2} Sp[\xi]
\]

In other words, minimizing the risk of a portfolio using a convex combination of the two spreads is equivalent to that done using its spread.
Here we formulate the whole optimization model with budgeting constraint and restriction over the short sales

\[
\min Ssp(\alpha, \beta) \left[ \sum_{i=1}^{n} x_i \xi_i \right]
\]

subject to

\[
E \left[ \sum_{i=1}^{n} x_i \xi_i \right] \geq \rho
\]

\[
\sum_{i=1}^{n} x_i = 1
\]

\[
x_i \geq 0 \quad i = 1, \ldots, n
\]

### 4.2. Modelling Transaction Costs

In practice, each security must be acquired in multiples of a minimum transaction unit. Let \((x_1, \ldots, x_n)\) be the investor portfolio. \(x_i\) is defined as the proportion of the capital \(C\) invested in security \(i\) (\(x_i\) is a multiple of the minimum transaction unit of security \(i\)). Let \(c_j\) be the cost of one transaction unit, the number of securities \(j\) acquired is

\[
n_i = \frac{c x_i}{c_i} \quad \text{for} \quad j = 1, \ldots, n
\]

Let \(p_j\) and \(q_j\) be the purchase (respectively, the sale) cost of one transaction unit of security \(i\) \((i = 1, \ldots, n)\) and \(x^0(x_1^0, \ldots, x_n^0)\) be the initial holding portfolio. In the case of \(x^0 = 0\), the investment in security \(i\) is obtained by multiplying the variable \(x_i\) and the cost of one transaction unit. Formally we have

\[
p_i x_i \quad \text{if purchase}
\]

\[
q_i x_i \quad \text{if sale}
\]

In case of \(x^0 \neq 0\) the investment in security \(j\) will be

\[
p_i \max(0, x_i - x_i^0) = p_i |x_i - x_i^0|_+ \quad \text{if purchase}
\]

\[
-q_i \min(0, x_i - x_i^0) = q_i |x_i - x_i^0|_- \quad \text{if sale}
\]

or more generally

\[
a_i = p_i |x_i - x_i^0|_+ - q_i |x_i - x_i^0|_- \quad (20)
\]

The suggested optimization program will be subject to proportional transaction purchase or sale costs \(d_i\) or \(d_i'\) respectively. Here we suppose the existence of a set \(I_k\) \((k \in K)\) of securities characterized by a fixed transaction cost \(d_k''\).

Let \(\alpha_i\) a variable which is defined as follow

\[
\alpha_i = \max(0, x_i - x_i^0) - \min(0, x_i - x_i^0)
\]

\[
= |x_i - x_i^0|_+ + |x_i - x_i^0|_- \quad (21)
\]

we introduce a binary variable \(\theta_k\) taking the value 1 when at least one security with fixed transaction cost is acquired and the value 0 otherwise

\[
\theta_k = \begin{cases} 1 & \text{if } \sum_{i \in I_k} \alpha_i > 0 \\ 0 & \text{otherwise} \end{cases} \quad (22)
\]
formally (22) can be rewritten as a set of constraints (23)

\[ \sum_{i \in F_K} a_i \leq M \theta_k \]

\[ \theta_k \leq M \sum_{i \in F_K} a_i \]

\[ \theta_k \in \{0, 1\} \]

where \( M \) is a positive number taking an arbitrary large value

The fixed transaction costs can be expressed as fellow

\[ \sum_{k \in K} d^p_k \theta_k \]

moreover, the constraint on the capital to be invested can be written as fellow

\[ \sum_{i=1}^{n} \left[ (1 + d_i) p_i |x_i - x_i^0|_+ + (d_i' - 1) q_i |x_i - x_i^0|_- \right] + \sum_{k \in K} d^p_k \theta_k \leq C \]

(24)

in practice, the right term of the constraint (24) is the real invested capital, denoted \( \tilde{C} \) which is less or equal to the holding capital \( C \) \( (\tilde{C} \leq C) \)

we have

\[ \tilde{C} = \sum_{i=1}^{n} \left[ (1 + d_i) p_i |x_i - x_i^0|_+ + (d_i' - 1) q_i |x_i - x_i^0|_- \right] + \sum_{k \in K} d^p_k \theta_k \]

(25)

formally the constraint on the minimum rate of return will be given by

\[ E[R(x)] \geq \rho \tilde{C} \]

(26)

where the expected rate of return of the portfolio \( E[R(x)] \) can be estimated using \( a_i \) (20) by

\[ \sum_{i=1}^{n} \xi_i a_i \]

(27)

the constraint on the minimum rate of return becomes

\[ \sum_{i=1}^{n} \left[ (\xi_i - \rho - \rho d_i) p_i |x_i - x_i^0|_+ + (\rho - \xi_i - \rho d_i') q_i |x_i - x_i^0|_- \right] - \sum_{k \in K} \rho d^p_k \theta_k \geq 0 \]

(28)

4.3. Shariah-Compliant Portfolio Optimization Model with Fuzzy variables

Ulrich, D., Marzban, S. (2011) [24] used the binary variable \( z_i \) to express the criterion of Shariah-compliance in portfolio optimization.

\[ z_i = \begin{cases} 1, & \text{if } i \text{ is compliant} \\ 0, & \text{otherwise} \end{cases} \]

We can model Shariah-compliance then in our optimization model (19) using the semi-spread of the fuzzy portfolio return.

\[ Min \ Ssp(\alpha, \beta) \left[ \sum_{i=1}^{n} z_i x_i \xi_i \right] \]

(29)
subject to \[
\sum_{i=1}^{n} \left[ (\xi_i - \rho d_i) p_i |x_i - x_i^0|_+ + (\rho - \xi_i - \rho d_i) q_i |x_i - x_i^0|_+ \right] - \sum_{k \in K} \rho d_k^* \theta_k \geq 0 \\
\sum_{i=1}^{n} x_i = 1 \\
x_i \geq 0 \quad i = 1,...,n \\
\theta_k \in \{0,1\} \\
z_i \in \{0,1\}
\]

where \( \rho \) is a parameter prescribing the minimum expected return level that the investors can accept.

5. The proposed Genetic Algorithm (GA) for portfolio optimization

In this section, we employ a genetic algorithm (GA) to solve the portfolio optimization problem, aiming to minimize risk while considering different constraints, including Shariah compliance. Genetic algorithms are a subset of evolutionary algorithms used for search and optimization problems and are particularly suited for complex problems where traditional optimization techniques may falter.

- **Input Configuration and Parameters:**
  - **Population Size:** We set the population size to 50, which refers to the number of individual solutions (portfolios) considered in each generation. This size was chosen to balance computational efficiency with the ability to explore a diverse set of solutions.
  - **Number of Generations:** The GA iterated through 40 generations. This number allows sufficient evolution of the population towards optimal solutions while maintaining a manageable computational load.
  - **Crossover Probability (cxpb):** Set at 0.7, this parameter determines how frequently two individuals (portfolios) will be selected for crossover, combining their traits (asset weights) to produce offspring for the next generation.
  - **Mutation Probability (mutpb):** Set at 0.2, this parameter influences how often random changes (mutations) will be applied to individual portfolios, introducing variation and helping the algorithm to explore new areas of the solution space.
  - **Selection Method:** We employed tournament selection with a tournament size of 3. This method involves randomly selecting three portfolios and choosing the best out of these to become a parent for the next generation, ensuring that stronger solutions have a higher chance of propagation.

**Constraints:**

- **Return Constraint:** Each portfolio must achieve a minimum expected return, varied linearly from 5% to 15% across different runs to generate the efficient frontier.
- **Investment Constraint:** Total asset weights in each portfolio must sum to 1, ensuring full capital allocation.
• **Shariah Compliance**: For the Shariah-compliant model, portfolios must only include assets marked as compliant; non-compliant assets are excluded by setting their weights to zero.

• **Short Selling**: Prohibited by ensuring all asset weights are non-negative.

**Fixed Transaction Costs**: These are costs incurred regardless of the transaction size. They could represent brokerage fees, ticket charges, or other flat fees associated with executing trades. In our simulations, fixed transaction costs were assumed for simplicity to fall within a uniform range for all assets. Specifically, we generated fixed transaction costs uniformly at random for each asset, ranging from 0.005 to 0.01. This range was chosen to reflect a scenario where each transaction incurs a small but significant cost, mimicking real-world conditions where investors face minimal charges for each trade executed, regardless of the volume or value of the transaction.

**Variable Transaction Costs**: Unlike fixed costs, variable transaction costs are proportional to the size of the trade. These costs often represent a percentage of the total transaction value and can include brokerage commissions, bid-ask spreads, and impact costs due to market liquidity. In our model, variable transaction costs were also generated randomly for each asset, ranging from 1% to 5% of the transaction value. This range was selected to illustrate the impact of costs that scale with investment size, which can become substantial for larger trades or in less liquid markets.

**Genetic Algorithm Workflow**:

The GA begins with a randomly generated initial population of portfolios. Each portfolio’s fitness is evaluated based on its risk (objective to minimize) and whether it satisfies the constraints. Through the processes of selection, crossover, and mutation, new generations of portfolios are created and evaluated. The algorithm iterates through generations, each time improving the population’s overall fitness.

We applied this GA separately under two scenarios: one adhering to Shariah compliance and the other without such constraints. This approach allowed us to compare the impact of Shariah compliance on the efficient frontier, visualized by plotting the risk-return profiles of the optimized portfolios from each scenario.

6. **Numerical Study**

In this numerical study, we utilized real financial data sourced from the Morocco Stock Exchange to model the market conditions, asset returns, and Shariah compliance status. The data reflects the unique characteristics and dynamics of the Moroccan financial market. The study analysed weekly returns from 20 selected assets listed on the Morocco Stock Exchange over a period of approximately two years (2021-2022). This timeframe allowed us to capture a variety of market conditions, including fluctuations and trends typical of the Moroccan market. Weekly returns were chosen as the unit period to align with the typical investment review cycle in regional trading practices and to smooth out the high-frequency volatility seen in daily returns.

6.1. **Results and Discussion**:
Figure 1 represents the efficient frontiers generated under both scenarios with and without Shariah compliance constraints. The genetic algorithm's flexibility and adaptability make it an effective tool for exploring the complex solution space of portfolio optimization, particularly under non-linear constraints and uncertainties inherent in financial markets.

![Comparison of Efficient Frontiers (GA)](image.png)

**Figure 1: Efficient frontiers for both Shariah-compliant and Non-Shariah compliant portfolios**

As we can mention in figure 1, the Shariah-compliant frontier lies below the non-Shariah-compliant one, this indicates that, for a given level of risk, Shariah-compliant portfolios typically offer a lower expected return due to the restrictions placed on permissible investments. Furthermore, by examining the two frontiers diverge, we can assess the impact of Shariah constraints on portfolio performance. Significant divergence indicates that Shariah principles restrict access to high-return, high-risk investments, or conversely, protect against overly risky assets.

The analysis offers practical insights for Shariah-compliant investors, indicating the trade-offs involved and helping in the selection of portfolios that align with both their financial goals and religious principles.

7. Conclusion

In this study, we have explored the integration of Shariah-compliant principles within the framework of optimal portfolio selection, challenging the traditional reliance on historical data to predict future stock market conditions. Given the inherent volatility and unpredictability of financial markets, this research has adopted fuzzy set theory to enhance the modeling of portfolio selection, treating the portfolio return as a fuzzy variable to better manage uncertainty [25]. Notably, the research has been extended to include various types of
transaction costs within the optimization model, enhancing its practical applicability and realism. This incorporation is particularly significant as it directly affects investment strategies and outcomes within the context of Islamic finance. The development of a semi-spread portfolio selection model that adheres to Shariah-compliant investment principles, combined with the implementation of fuzzy programming, represents a significant advancement. The model's unique risk measure, influenced by two parameters that depict the investor's risk aversion, allows for tailored investment control. Moreover, by integrating compliance within the optimization process itself, the model ensures Shariah adherence throughout the investment decision-making process, contrasting with traditional methods where compliance is often addressed separately. This approach not only aligns with Islamic financial derivatives but also sets a foundation for future developments in Shariah-compliant fuzzy optimization in finance.

By employing genetic algorithms and comparing efficient frontiers for portfolios with and without Shariah compliance constraints, the study provides insights into the trade-offs between ethical considerations and financial performance in an emerging market setting.

This approach allows investors to understand the implications of Shariah-compliant investing in Morocco and aids in the development of strategies that align with both financial goals and ethical standards. The real data from the Morocco Stock Exchange ensures that the findings and conclusions are directly applicable and relevant to investors interested in this market.

References