

Passivity Based Control of Dc-Dc Converters Operating In Discontinuous Conduction Mode

Mohamed Bendaoud^{a*}, Saad Ladide^b and Hicham Hihi^c

^aLaSTI Laboratory, National School of Applied Sciences of Khouribga, Sultan Moulay Slimane University-Morocco

^bLGECOS Laboratory, National School of Applied Sciences Marrakech, Cadi Ayyad University, Morocco

^cNational School of Applied Sciences, Fez, Sidi Mohamed Ben Abdellah University, Morocco

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Corresponding author.

Name

E-mail address: m.bendaoud@usms.ma

ABSTRACT

This paper discusses the extension of the passivity based control approach to DC-DC converters operating in discontinuous conduction mode (DCM) such as buck, boost and buck-boost. To do this, a valid average model for these power converters is presented. The proposed regulator is based on a “damping injection” scheme, achievable through nonlinear dynamical feedback. The idea of this strategy is to synthesize the control law in order to ensure that the closed loop system is passive and therefore asymptotically stable. The performances of the proposed passivity-based control are evaluated for the buck converter. The regulating feedback law, derived on the basis of a buck model composed of ideal switches and ideal circuit components is assessed, via computer simulations. The paper, unlike other studies in the literature, shows that the proposed control provides excellent dynamic performances for the DC-DC converters operating in DCM. It is capable to provide a good regulation of the output voltage in a wide range of the load resistance.

KEYWORDS: Passivity Based Control; DC-DC Converters; Discontinuous conduction mode (DCM)

1. Introduction

Recently, power converters DC-DC have attracted more attention for applications such as power supplies for mobile phone (Hossain, 2018), renewable energy (Changizian, 2017; Jyotheeswara Reddy, 2018; Bendaoud, 2019), electric vehicles (Jyotheeswara Reddy, 2018), industrial systems, and many more due to their advantages in weight, size and efficiency. These converters can operate in either conduction continuous mode (CCM) or discontinuous conduction mode (DCM), depending on the choice of the switching frequency, the inductance and the resistance of the load. In practice, the converters operating in DCM mode are designed for low power applications in order to avoid the reverse recovery problem of the diode. DCM operation has also been considered as a possible solution to minimize the total harmonic distortion (THD) if a technique of power factor correction (PFC) is applied for the converters composed of an uncontrolled rectifier in series with a boost converter operating in DCM (Bendaoud, 2019).

Over the last three decades, much progress has been made in the development of control approaches for DC-DC converters in order to obtain an effective regulation. Among these methods, the sliding mode control (Venkataramanan, 1985; Tan, 2007), Adaptive Tracking Control (Nam, 2018), Fuzzy Logic Controllers (Perry, 2007) etc.

The passivity based control (PBC), originally developed by R. Ortega (Ortega Romeo, 1998), is another approach widely cited and used in the literature. This method of control has also been successfully applied on various DC-DC converters such as buck, boost and buck-boost (Sira-Ramirez, 1997; Ortega Romeo, 1998; Ortega, 2002; Cormerais, 2008). In this approach, the average model concept should be used and a pulse width modulation is implemented

to generate the Boolean variables that will be sent to the switches of the converter. However, this method of control has not yet been extended to converters operating in DCM, which motivates our present research. It should be noted that the average model for the converters operating in CCM proves to be unsuitable for the converters operating in DCM. Therefore, several studies have been carried out in order to develop the average models for converters operating in DCM (Cuk, 1977; Maksimovic, 1989, 1991; Sun, 1998; Jian, 2001). These are classified in two categories, and the generated equations are often identical or equivalent: reduced-order models (Cuk, 1977; Jian, 2001) and full-order models (Maksimovic, 1989, 1991; Sun, 1998).

The principle of the reduced order models is to consider the state variable which becomes null during a time interval as a dependent variable, therefore it does not appear as a state variable. Hence the reduction of system order leads to a lack of precision.

With regards to full-order models, they maintain all state variables of the converter, including the discontinuous inductor current. Thus, they show an improved precision compared to reduced order models. This constitutes the critical reason why the full-order models are used in this paper, and more precisely the model proposed by (Maksimovic, 1991).

This paper discusses the extension of the passivity based control approach to DC-DC converters operating in DCM. To do this, the modeling of the converters operating in DCM is presented in the second section. The conception method of the passivation control is described in the third section. The fourth section gives an example of the buck Converter that illustrates the proposed method. The fifth section shows the simulation results using Matlab/Simulink. The sixth and last section of this paper gives a brief summary of the findings and highlights directions for future research.

Nomenclature

T_s	Switching period
i_{pk}	Peak value of the inductor current
μ_c	Duty ratio
ρ_c	Equivalent duty cycle
\bar{i}_L	Average inductor current
X	State vector of the average model
J	Skew-symmetric interconnection matrix
R_a	Symmetric dissipation matrix
H_a	Energy stored in the system
G	Power input matrix
E	Power input
L	Buck inductance
C	Capacitance
R	Resistance of the load
R_1	Damping parameter
v_{od}	Desired output voltage

2. Modeling of converters operating in DCM

The DCM operation of a buck, boost and buck-boost converters is characterized by an additional third interval in each switching cycle which neither the switch nor the diode conduct current. The converter is described by the following state equations:

$$\begin{cases} \dot{x}(t) = A_1 x(t) + B_1 E \text{ for } t \in [0, d_1 T_s] \\ \dot{x}(t) = A_2 x(t) + B_2 E \text{ for } t \in [d_1 T_s, (d_1 + d_2) T_s] \\ \dot{x}(t) = A_3 x(t) + B_3 E \text{ for } t \in [(d_1 + d_2) T_s, T_s] \end{cases} \quad (1)$$

With T_s is the switching period, E is the input voltage of the converter.

Thereafter, d_1 and d_2 will be used to denote the duty ratio of the first and the second interval, respectively, as shown in Fig.1.

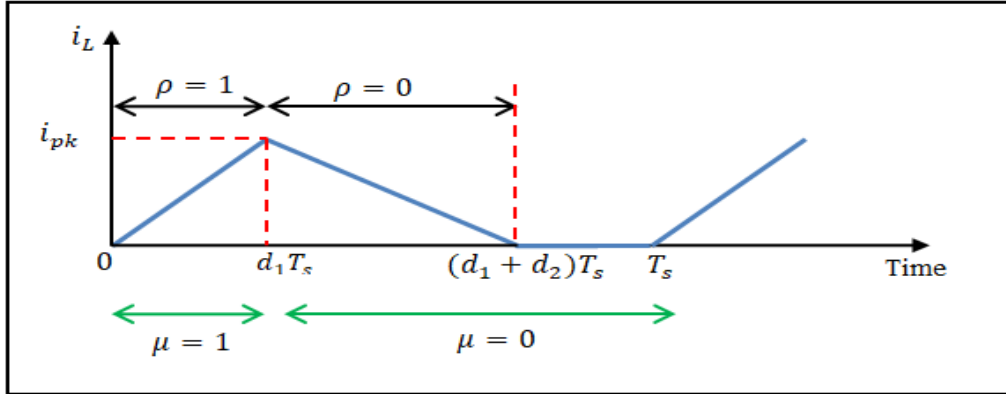


Fig. 1 Inductor current waveform of the converter in DCM

With i_{pk} is the peak value of the inductor current.

From Fig.1, the average value of the control signal is given by:

$$\mu_c = \frac{d_1 T_s}{T_s} = d_1 \quad (2)$$

The method presented in (Maksimovic, 1991) consists in establishing the average model of the converter on the interval $[0, (d_1 + d_2)T_s]$:

$$\dot{X} = [\rho_c A_1 + (1 - \rho_c) A_2] X + [\rho_c B_1 + (1 - \rho_c) B_2] E \quad (3)$$

The model obtained is equivalent to the average model of the converter operating in continuous conduction as a function of the equivalent duty cycle ρ_c . The latter is defined as the ratio of the average diode voltage v_D , over the average voltage v_{12} (Maksimovic, 1991).

$$\rho_c = \frac{\overline{v_D}}{v_{12}} \quad (4)$$

Applying Kirchoff's voltage law to circuit presented in Fig.2, yields $v_L = v_D + v_{23}$.

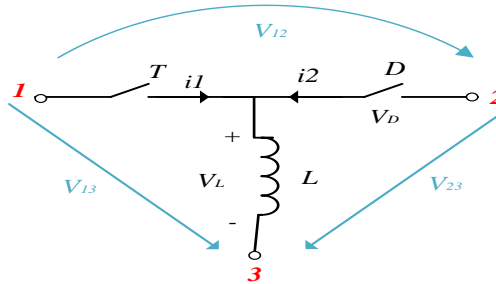


Fig.2 PWM switch cell for DCM (Tymerski, 1986)

Considering that the average voltage of v_L is zero, so the last equation becomes:

$$\overline{v_{23}} = -\overline{v_D} \quad (5)$$

From Fig.2, the following differential equations can be written for the instantaneous inductor current by using (5):

$$\begin{cases} L \frac{di_L}{dt} = \overline{v_{12}} + \overline{v_{23}} = \overline{v_{12}} - \overline{v_D} & t \in [0, d_1 T_s] \\ L \frac{di_L}{dt} = \overline{v_{23}} = -\overline{v_D} & t \in [d_1 T_s, (d_1 + d_2) T_s] \\ i_L = 0 & t \in [(d_1 + d_2) T_s, T_s] \end{cases} \quad (6)$$

The average inductor current is given by:

$$\overline{i_L} = \frac{d_1 + d_2}{2} \cdot i_{pk} = \frac{d_1 + d_2}{2} \cdot \frac{\overline{v_{12}} - \overline{v_D}}{L} d_1 T_s \quad (7)$$

On the other hand, the average voltage across the inductor L is zero implies:

$$d_1 (\overline{v_{12}} - \overline{v_D}) = d_2 \overline{v_D} \text{ so } d_2 = d_1 \cdot \frac{\overline{v_{12}} - \overline{v_D}}{\overline{v_D}} \quad (8)$$

Substituting the expression of d_2 into (7) yields:

$$\overline{i_L} = \frac{d_1^2 T_s}{2L} \cdot \overline{v_{12}} \cdot \left(\frac{\overline{v_{12}}}{\overline{v_D}} - 1 \right) = \frac{d_1^2}{2L f_s} \cdot \overline{v_{12}} \cdot \left(\frac{1}{\rho_c} - 1 \right) \quad (9)$$

From equations (9) and (2) the equivalent duty cycle can be defined as:

$$\rho_c = \frac{\mu_c^2}{\mu_c^2 + 2LF_s(L/\overline{v_{12}})} \quad (10)$$

So, the equivalent duty ratio becomes dependent on the external variables. Thereafter, the converter is treated as if it works in the continuous conduction mode with the duty ratio of the switch ρ_c defined by (10).

3. Passivity Based Control (PBC)

The control by passivation often named PBC «Passivity Based Control» uses the notion of system passivity and energy dissipation. Qualitatively, a system is dissipative (or passive) if the power stored in this system remains always inferior to the supplied power. The idea is to take into account these principles energies to synthesize the control law. The objective is to ensure passivity of the closed-loop system by changing the energy function and adding damping so as to achieve asymptotic stability of the system.

Damping parameters are used to set the controller in order to adjust its performances (Sira-Ramirez, 1997; Ortega Romeo, 1998; Ortega, 2002). The controller is presented in the form of a dynamic feedback, providing control laws in the continuum $[0, 1]$. It is therefore appropriate to deduce the equivalent Boolean control via an additional Pulse Width Modulation (PWM) (see Fig.3).

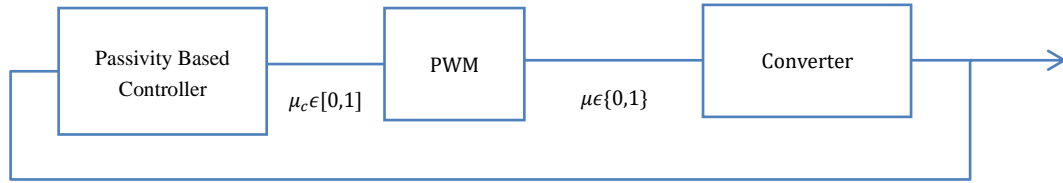


Fig.3 Structure of the passivation control

PBC is a continuous control method that requires knowledge of the average model. Now, the model of this type of system can be represented either by using Euler Lagrange's formalism (EL) or Hamiltonian formalism (PCH) (Ortega, 2002). This latter will be used in this work.

The state representation of the average model of a DC-DC converter in standard PCH formulation has the following form:

$$\dot{X} = [J(\mu_c) - R_a(\mu_c)] \frac{\delta H_a(X)}{\delta X} + G(\mu_c)E \quad (11)$$

Where X is the state vector of the average model, μ_c is the duty ratio, J is the skew-symmetric interconnection matrix, R_a is the symmetric dissipation matrix, H_a is the energy stored in the system, also called Hamiltonian of the system, G is the power input matrix and E is the power input.

If the constitutive relations of the storage elements are linear, the Hamiltonian of the system is:

$$\frac{\delta H_a(X)}{\delta X} = FX \quad (12)$$

Where $F = F^T > 0$ is a diagonal matrix. So the Hamiltonian of the system is given by:

$$H_a = \frac{1}{2} X^T F X \quad (13)$$

In order to develop the control law, an error vector denoted by \tilde{X} is introduced (Sira-Ramirez, 1997):

$$\tilde{X} = X - X_d \quad (14)$$

With X_d is the desired value of X to be defined.

Using this error vector, the average model can be rewritten as:

$$\dot{\tilde{X}} - [J(\mu_c) - R_a(\mu_c)]F\tilde{X} = G(\mu_c)E - [\dot{X}_d - (J(\mu_c) - R_a(\mu_c))FX_d] \quad (15)$$

The principle of the PBC method is to inject the damping under the form of a matrix R_{D1} ($R_{D1} > 0$) in equation (15). Therefore, the following equation can be deduced:

$$\dot{\tilde{X}} - [J(\mu_c) - (R_a(\mu_c) + R_{D1})]F\tilde{X} = \underbrace{G(\mu_c)E - [\dot{X}_d - (J(\mu_c) - R_a(\mu_c))FX_d - R_{D1}F\tilde{X}]}_{\psi} \quad (16)$$

In order to ensure an asymptotic stability of the system, the right side of the equation (16) has to be null ($\psi = 0$), thus the following condition is deduced:

$$\dot{X}_d - (J(\mu_c) - R_a(\mu_c))FX_d - R_{D1}F\tilde{X} = G(\mu_c)E \quad (17)$$

Equation (Eq.17) gives an implicit definition of the control law. As μ_c is included in the matrices J , R_a and G , the resulting system of n equations with $n+1$ unknowns must be solved, to obtain an explicit definition of the control law. This procedure will be illustrated later in the case of an application for the buck converter.

4. Example: buck converter

To illustrate the results of the previous section, we consider the following example of a buck converter (Fig.4).

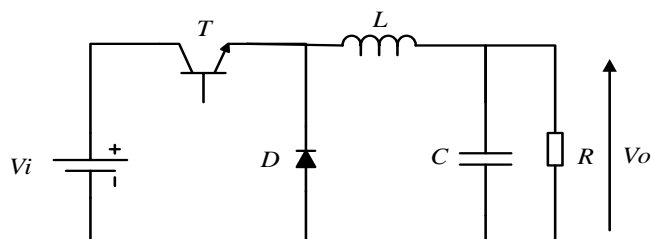


Fig.4 Buck converter

The average model under the Hamiltonian form for the buck converter operating in DCM is given by:

$$\dot{X} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ 0 & \frac{1}{R} \end{bmatrix} \begin{bmatrix} \frac{1}{L} & 0 \\ 0 & \frac{1}{C} \end{bmatrix} X + \begin{bmatrix} \rho_c \\ 0 \end{bmatrix} V_i \quad (18)$$

With ρ_c is the equivalent duty ratio which is rewritten by using $\bar{v}_{12} = V_e$ for buck topology:

$$\rho_c = \frac{\mu_c^2}{\mu_c^2 + 2Lf_s(L/V_i)} \quad (19)$$

According to (17), the following system of equations is obtained:

$$\begin{cases} L\dot{x}_{1d} + x_{2d} - R_1(x_1 - x_{1d}) = \rho_c V_i \\ C\dot{x}_{2d} - x_{1d} + \frac{1}{R}x_{2d} = 0 \end{cases} \quad (20)$$

By substituting the expression of the equivalent duty ratio ρ_c in (20), we obtain:

$$\begin{cases} L\dot{x}_{1d} + x_{2d} - R_1(x_1 - x_{1d}) = \frac{\mu_c^2}{\mu_c^2 + 2Lf_s(L/V_i)} V_i \\ C\dot{x}_{2d} - x_{1d} + \frac{1}{R}x_{2d} = 0 \end{cases} \quad (21)$$

Setting $x_{2d} = v_{od}$ and $x_1 = i_L$, the control law is deduced:

$$\mu_c = \sqrt{\frac{v_{od} - R_1(i_L - v_{od}/R)}{V_i - (v_{od} - R_1(i_L - v_{od}/R))}} \cdot 2Lf_s(L/V_i) \quad (22)$$

With v_{od} is the desired output voltage.

4. Simulation results and discussions

In order to verify the performance of the proposed method, a simulation was carried out by using MATLAB/Simulink. The parameters of the buck converter and its corrector are given in Table 1.

The structure of the buck converter and its PBC corrector shown in Figure 5 has three main blocks:

- The PBC corrector modeled by equation (Eq.22).
- The PWM which performs the transition between the continuous control variable resulting from the corrector PBC and the Boolean command applicable to the converter.
- The buck converter.

Table 1 Parameters of the buck converter

Variable	Parameter	Value
L	Inductance	22 μ H
C	Capacitance	150 μ F
R	Load resistance	30 Ω
V_e	Input voltage	24V
f_s	Switching frequency	200KHz
R₁	Damping parameter	0.4

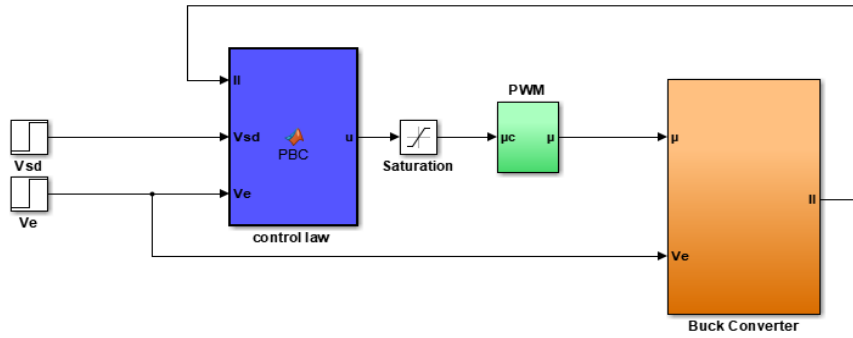


Fig.5 Simulink block of the closed loop control of buck converter

Fig.6 shows the steady-state behaviour of the control signal μ_c with the input ramp, the generated gate pulse μ , the inductor current i_L and the output voltage V_o for the buck converter operating in DCM. For these tests, the reference of the desired output voltage is 12V.

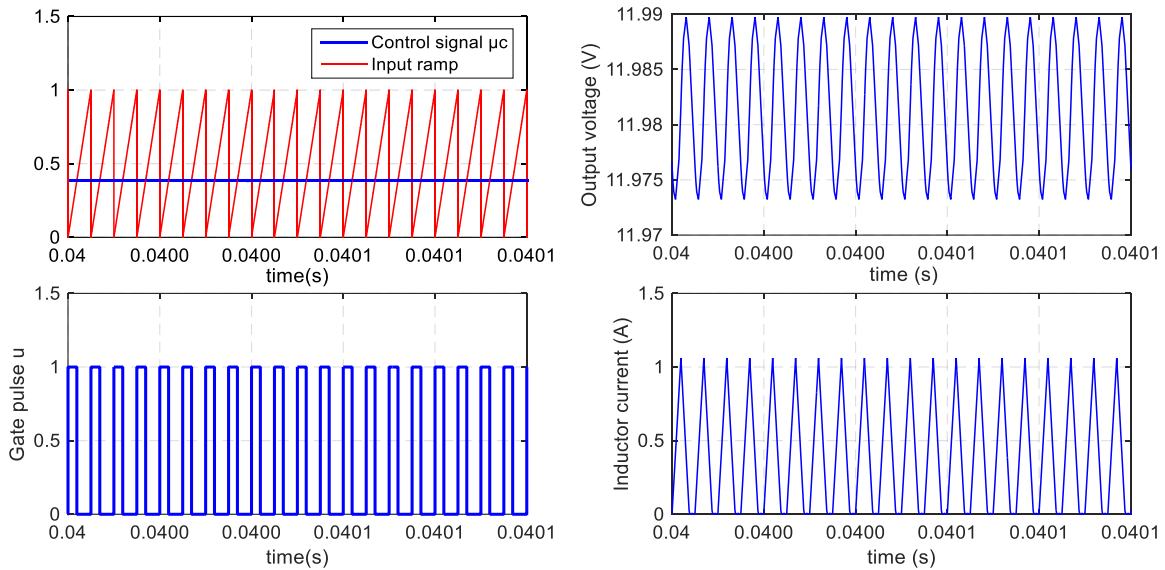


Fig.6 Waveforms of control signal μ_c with input ramp, generated gate pulse μ (left), and inductor current i_L and output voltage V_o (right) for the buck converter operating in DCM

Fig. 7 shows the output voltage waveform in response to a variation of the desired voltage. In terms of performance, the analysis of this figure shows that the output voltage follows the desired voltage quickly without overshoot, with a response time less than 3 ms. The system response has an error less than 0.02V, if compared with the sliding mode approach in (Tan, 2007), where the error is equal to 0.08V, the difference can be seen.

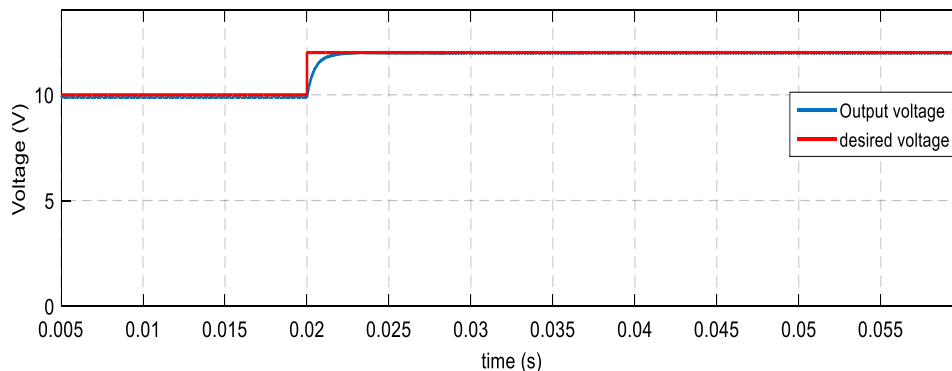


Fig.7 Output voltage waveform for buck converter

Fig.8 shows the performances of the buck converter operating in DCM at step load changes alternating between $30\ \Omega$ and $50\ \Omega$. The step change responses are critically damped. The settling times of the system are 5 ms (load resistance changes from $30\ \Omega$ to $50\ \Omega$) and 13 ms (load resistance changes from $50\ \Omega$ to $30\ \Omega$).

Fig.9 shows a plot of the DC output voltage against the different operating load resistances. The results show a good load regulation property for the load range $30\ \Omega \leq R \leq 100\ \Omega$, with only a 0.12 V deviation.

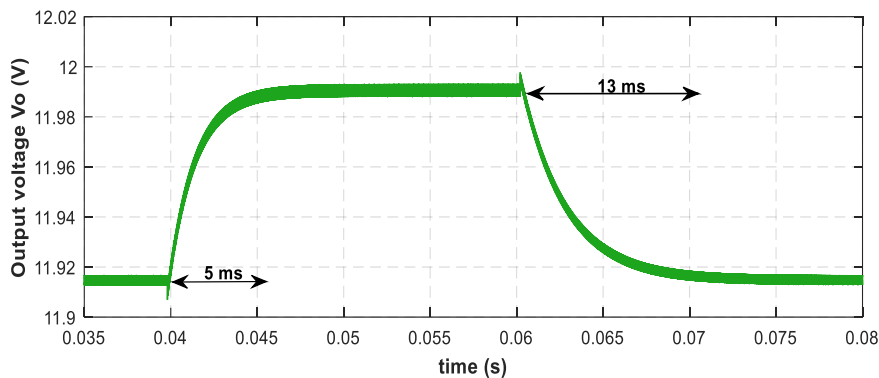


Fig.8 Output voltage waveforms in response to load variations

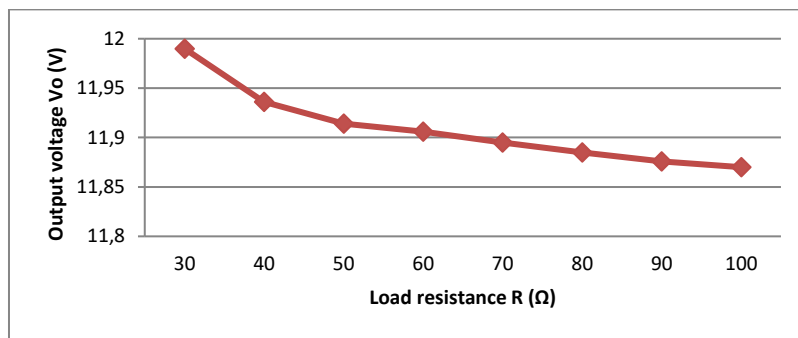


Fig.9 Output voltage Vo against load resistance R

4. Conclusion

In this paper, an extension of the passivity based control approach to DC-DC converters operating in discontinuous conduction mode (DCM) such as buck, boost and buck-boost is presented. A method of developing the average model of these converters in DCM operation required for the controller design is described. Control law obtained by applying the proposed approach is derived and verified through computer simulation for the buck converter. These findings, compared to other results in the literature, show that the proposed controller gives satisfactory results in terms of stability, precision and rapidity. It is capable to provide a good regulation of the output voltage in a wide range of the load resistance.

Future studies will focus principally on extending these results to other types of converters such boost and buck-boost.

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