Theoretical and experimental Study of water transfer in cylindrical wood samples

Kawtar EL MOKHTARI*a, Abdelhak EL BROUZIb, M’hammed EL KOUALI a, Mohammed TALBI a

*a Research Laboratory of Analytical Chemistry and Physical Chemistry of Materials, University Hassan II of Casablanca, Faculty of Sciences Ben M’sik, Casablanca, B.P 7955, Morocco
b Research Laboratory of Physical Chemistry and Bioorganic, University Hassan II of Casablanca, Faculty of Sciences and Technology Mohammedia, BP 146, Morocco

ARTICLE INFO
Received July 20th, 2019
Received in revised form September 5th, 2019
Accepted 5th, September 2019

Keywords:
Diffusion,
Wood,
Water,
Mathematical model,
Transfer,
numerical method.

ABSTRACT

The diffusion phenomenon is one of the properties bound to the transfers of material within the wood. This phenomenon describes the capacity of this material to let migrate the water or water vapor. This paper aims to study kinetics absorption of water by wooden cylinders. This process is studied by immersing the sample in water.

In this regard, a mathematical model based on numerical method with finite differences was elaborated. This method has been successfully used to describe the absorption step for a period of time after which, absorption equilibrium is achieved.

The mathematical model takes into account the transport of water by transient diffusion with constant diffusivity; not only this model gave us the kinetics of absorption of water in good agreement with experiments carried out with wood sample immersed in water, but it is also able of calculating the profiles of concentration of water developed within the sample, and so affording a further insight on the process of water absorption.

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1. Introduction:

Wood is a hygroscopic material, which means it naturally takes on and gives off water to balance out with its surrounding environment. [1-3] Similarly, the wood absorbs water when it is immersed in liquid water; however, scientists consider that the transfer of water in this material is controlled by the diffusion phenomenon. Three principal axes of diffusion are observed along the main axes of the wood: longitudinal, tangential and radial.

In fact, the hygroscopicity of the wood, which brings it to balance with its environment, is an additional reason justifying the study of diffusion phenomenon, always present in the life of this material. In general, water can move through wood in three forms; as water vapor through the cell cavities, as bound water in the wood substance and as free liquid water. [4] Beyond the fiber saturation point (FSP), additional water is in the free liquid form. However, the presence of free water in wood drastically increases its decay susceptibility as well as the possibility of fungal attacks. [5]

Free-water in wood is in liquid form in the voids of the wood. The amount of free water which can be held is limited by the porosity of the wood.

Studies aimed to describe the circulation of water below the fiber saturation point have been made, but the problem of water transfer above the fiber saturation point is also of interest, and especially in the following two cases: [6]

1) When the wood is in contact with a surrounding atmosphere around 100% and the liquid water condenses on the wood surface. This condensation occurs very often during the night at lower temperature in the form of dew.
2) When the wood is immersed into liquid water.

The first purpose in this work was to study the absorption process in transient conditions, when the wood sample is immersed in liquid water, for a specified time, in order to contribute in the understanding of absorption process. For this, the radial direction in the wood was selected.

The second aim of this study was to build up and test a numerical model able to describe the process of absorption when the water content is above the fiber saturation point during the whole process.

2. Theoretical part:

2.1. Assumptions:

In order to clarify the problem, the following assumptions are made:
- The water is transferred through the radial direction of the wood.
- The water transport is expressed by transient diffusion with the constant diffusivity obtained from experiments during absorption of water.

2.2. Mathematical treatment:

The concentration of diffusing substance is a function of radius \( r \) and time \( t \), following Fick's second law.

\[
\frac{\partial C}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left( r D \frac{\partial C}{\partial r} \right) \tag{1}
\]

With constant diffusivity and radial diffusion, Fick's equation can be written:

\[
\frac{\partial C}{\partial t} = D \left[ \frac{\partial^2 C}{\partial r^2} + \frac{1}{r} \frac{\partial C}{\partial r} \right] \tag{2}
\]

By using the method of separation of the variables, with:

\[
C_{r,t} = U_r T_t \tag{3}
\]

It is found for the function \( T \):

\[
T = \exp(-\alpha^2 D t) \tag{4}
\]

Where \( \alpha \) is a constant, and \( U \), which is a function of \( r \) only, is a solution of:

\[
\frac{d^2 U}{d r^2} + \frac{1}{r} \frac{d U}{d r} + \alpha^2 U = 0 \tag{5}
\]

The preceding equation is Bessel's equation of order zero. The general solution is:

\[
C_{r,t} = \sum_{n=1}^{\infty} A_n J_0(\alpha_n r) \exp(-\alpha_n^2 D t) \tag{6}
\]

Where \( J_0 \) is the Bessel function of the first kind of order zero. Roots are given in tables of Bessel functions.

The initial and boundary conditions are:

Initial \( t = 0 \) \( 0 < r < R \) \( C = C_i \) \( \tag{7} \)

Boundary \( t > 0 \) \( r = R \) \( C = 0 \) \( \tag{8} \)

The boundary condition is satisfied by equation (6), provided that:

\[
J_0(\alpha_n R) = 0 \tag{9}
\]

And the \( \alpha_n \)s are roots of equation (9)

The initial condition is given by equation (10):

\[
C_i = \sum_{n=1}^{\infty} A_n J_0(\alpha_n r) \tag{10}
\]

And \( A_n \) is determined by multiplying both sides of equation (10) by \( rJ_0(\alpha n r) \) and integrating from 0 to \( R \). By considering that:

\[
\int_{0}^{R} r J_0(\alpha r).d r = \frac{R}{\alpha} J_1(\alpha R) \tag{11}
\]

And

\[
\int_{0}^{R} r J_0^2(\alpha r).d r = \frac{R^2}{2} J_1^2(\alpha R) \tag{12}
\]

Where \( J_1(\alpha R) \) is the Bessel function of the first order, the concentration \( C_{r,t} \) is given by:
\[
\frac{C_{r,t} - C_i}{C_{ext} - C_i} = 1 \frac{2}{R} \sum_{n=1}^{\infty} J_0(\alpha_n r) \frac{\exp(-\alpha_n^2 Dt)}{\alpha_n J_1(\alpha_n R)}
\]

(13)

And the amount of diffusing substance \( M \), which has entered (or left) the cylinder in time \( t \), is expressed in terms of the corresponding quantity after infinite time, \( M_\infty \), by:

\[
\frac{M_\infty - M_t}{M_\infty} = \sum_{n=1}^{\infty} \frac{4}{R^2 \alpha_n^2 \exp(-\alpha_n^2Dt)}
\]

(14)

Another expression is also given for the concentration (Where \( r/R \) is not small) \( C_{r,t} \):

\[
\frac{C_{r,t} - C_i}{C_{ext} - C_i} = \left( \frac{r}{R} \right)^{0.5} \text{erfc} \left( \frac{R-r}{2(Dt)^{0.5}} \right) + \frac{(R-r)(R\alpha)^{0.5}}{4 R^{1.5}} \text{erf} \left( \frac{R-r}{2(Dt)^{0.5}} \right) \text{erf} \left( \frac{R-r}{2(Dt)^{0.5}} \right) + \frac{(9 R^2 - 7 r^2 - 2 R r) t}{32 R^{1.5} r^{2.5}} t^2 \text{erfc} \left( \frac{R-r}{2(Dt)^{0.5}} \right) + ...
\]

(15)

And for the amount of diffusing substance which has entered (or left) the cylinder in time, \( M_t \):

\[
\frac{M_t}{M_\infty} = \frac{4}{\pi^{0.5} R^{2.5}} \frac{D t}{R^2} - \frac{1}{3 \pi^{0.5}} \left( \frac{D t}{R^2} \right)^{1.5}
\]

(16)

**Remark 1:** Case of Long Time: In the case of long time, equation (14) can be reduced to the first term of the series: Equation (17) is of interest for determination of diffusivity, from the slope of the line obtained by plotting \( \log (M_\infty - M_t/M_\infty) \) as a function of time:

\[
\frac{M_\infty - M_t}{M_\infty} = \frac{4}{R^2 \alpha_1^2} \exp(-\alpha_1^2Dt)
\]

(17)

**Remark 2:** Case of Short Time: In the case of short time, equation (16) can be reduced to:

\[
\frac{M_t}{M_\infty} = \frac{4}{R} \left( \frac{Dt}{\pi} \right)^{0.5}
\]

(18)

Expressing a relationship between the amount of matter transported \( M_t \) and the square root of time.

### 2.3. Numerical analysis:

The wood cylinder sample is divided into \( N \) slices of constant thickness \( \Delta r \). The concentration is determined on position during the increment of time \( \Delta t \).

The center is defined by the two integers \( i, j \) defined as follows:

Center of cylinder \( i=1 \):

\[
C(1,j) = C(1,j-1) + \frac{4}{M} \left( C(2,j-1) - C(1,j-1) \right)
\]

(19)

Within the cylinder: \( 1 < i < N Tr \) (\( N Tr \): number of slices):

\[
C(i,j) = \frac{(C(i-1,j-1) + (M-2)C(i,j-1) + C(i+1,j-1))}{M} + \frac{(C(i+1,j-1) - C(i-1,j-1))}{M \Delta i}
\]

(20)

Surface of cylinder \( i=N Tr \):

\[
C(i,j) = C_{eq}
\]

With:

\[
M = \frac{\Delta r^2}{D \Delta t}
\]

### 3. Material and methods:

#### 3.1. Materials:

A sample of cylindrical picea wood was used in the study, with dimensions of 9 cm in length and 0.25 cm in radius. Diffusion through the radial direction of this sample has been studied by immersing the sample in liquid water. During the contact, the sample has been weighed with a balance (sensibility \( 10^{-3} g \)) at intervals in order to obtain the kinetics of water absorption.

#### 3.2. Methods:

The weight of the sample previously dried to constant weight at 103°C has been taken as the reference weight for all experiments. After equilibrating the sample at 103°C, the sample was immersed in water. The kinetics of absorption was followed by weighing the sample at intervals. At the end of the first 4 hours of absorption an equilibrium seemed to be reached, and the sample was removed from the water.

### 4. Results and discussion:

As some data are needed in studying the absorption process with the help of the mathematical model, the following three parts of interest have been successfully studied:
- The kinetics of water absorption and the determination of data, as diffusivity of water in the radial direction of the wood and water content in the sample were carried out.
- The validity of the model was tested by comparing the kinetics obtained by experiments and by calculation made with the numerical model.
- Prediction of the profiles of concentration of water developed throughout the sample during the process of absorption.

### 4.1. Determination of data:

#### 4.1.1. Water content:

The amount of water absorbed at equilibrium is obtained at the end of the absorption stage after an immersion time of the cylinder wood in liquid water of 4 hours. As shown in Fig.1, an equilibrium is attained after this absorption time. From the weight of the sample measured at intervals, the water content has been determined as a function of time. The following equation is used to calculate the percentage of mass variation of a wood sample after a contact with liquid water according to the contact time. [15-17]

\[
\Delta m = \frac{m_t - m_0}{m_0} \times 100
\]

(21)

Where:
- \(m_0\): initial mass of the sample
- \(m_t\): the mass of the sample after a contact time \(t\).

As a result from these measurements, when they are obtained, the kinetics of water absorption has been drawn in Fig.1.

#### 4.1.2. Diffusivity:

Diffusivity is determined by using the "short test" method (Vergnaud 1984) run under transient conditions. [18-23]

The diffusivity was calculated from the slope of the straight line obtained by plotting the amount of water absorbed as a function of the square root of time, its value is easily obtained by using Equation (18). (Crank 1975). [23-27]

\[
\frac{M_t}{M_\infty} = \frac{4}{R} \left( \frac{D t}{\pi} \right)^{0.5}
\]

(18)

Where \(M_t\) and \(M_\infty\) are the amounts of water absorbed at time \(t\) and at equilibrium, respectively; and \(R\) is the radius of the sample.

By making these experiments, the diffusivity has been obtained for radial direction.

Radial: \(D_r = 2.3311 \times 10^{-6}\) cm²/s

#### 4.1.3. Absorption kinetics:

On the Fig.1, The curve is obtained by plotting the amount of water transported as a function of the square root of time. The results shown in Fig.1 indicate that the wood sample presented the characteristic water absorption behavior.

![Figure 1: Experimental kinetics of water absorption into the wood.](image)
In this figure it is clearly shown that the amount of water in the wood increases according to the time of contact until achieving equilibrium. However, the Water transfer process in the wood is governed by the diffusion phenomenon.

4.2. Validity of the model:
The validity of the model, based on the assumption of the transient diffusion, by considering a constant diffusivity has been tested by comparing the experimental and calculated kinetics when the equilibrium is attained in the case of absorption. The data in Fig. 2 shows that the kinetics calculated by using the numerical model is in good agreement with the experimental results which proves the validity of the model.
It is difficult to determine the water content in the sample experimentally. The model is able to give information on the water content in the sample at any place and any time.

The following conclusions can be drawn:
1. The numerical model describes the process of absorption during the whole process, when the water content is above the fiber saturation point.
2. An equilibrium seems to be reached after 4 hours of absorption of water.
3. The assumption that the diffusivity is constant and is the same at the stage of absorption is confirmed by the fact that the calculated kinetics are in good agreement with experiments.

4.3. Profiles of concentration of water developed in the sample:
It is of interest to draw the profiles of water content developed through the material at various times as they are obtained by calculation. It is difficult to measure the water content inside the sample.
It has been easy to draw the profiles of concentration of water developed through the thickness of wood cylinder when the water was transferred along radial direction.
Figures 3 and 4 show the profiles of the water content developed through the sample radius (r = 0.25 cm) during the step of absorption.
These profiles can be calculated by using the numerical model, whatever are the initial conditions, in the case of the absorption. These profiles are carried out for a time of absorption of 4 hours.

- The Models simulate transfers allowing to obtain the concentration profile at a point in the cylinder which is difficult to determine experimentally. We can think that the profiles obtained by the calculation are close to real profiles.
- These Concentration profiles tell us about the concentration inside the wood, so we can determine for each point of our sample the concentration easily.

5. Conclusion:
The main objective of this study is to develop a methodology based on a coupling of experimental results and mathematical treatments (numerical model).

According to the results of this study, the method coupling experiments and modeling of the process was efficient for studying the transport of water throughout the wood for the stage of absorption. Water absorption was studied above the fiber saturation point (FSP), by immersion the wood sample in water for 4 hours until achieving of equilibrium. The kinetics of absorption was followed by weighing the sample at intervals.

The results obtained in this study show a good agreement between experiments and calculated values in case of absorption of water in wood.
In this regard, the numerical model is able not only to describe the kinetics of absorption of water but also to obtain the profiles of concentration of water developed throughout the sample. Moreover, the model is able to attain the profiles of concentration of water developed throughout the sample. This information enables one to gain a fuller insight on the process. For this, the numerical model is capable of providing more information than do the experiments.

References